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OPTIMAL CHOICE OF A FIRM IN CONDUCTING AUCTION*

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An important aspect of activity in the Ministry of Regional Development and Public Works is working on the Operational Programme "Regional Development" with a priority axis "Sustainable and Integrated Urban Development" on an operation "Improvement of Physical Environment and Risk Prevention". This operation involves 86 municipalities. The financial resource for this operation is 238 589 939 euro, of which 202 801 448 euro are European financing [1]. Each of this 86 municipalities has to solve a problem to assign a public order to a fixed firm. In fact, this problem is a problem for holding public auction for a choice of executive firm. The optimal choice of executive firm is very important. We formulate this problem as a problem of the multi-criteria decisions making and using appropriate methods and criteria we transforme it to an one-criterion optimization problem.

1. Introduction. Similar problems in the area of sport and human resources are investigated in [3], [4] and [5].

Here we describe a generalized mathematical model for auction. Therefore, we need a few experts in the proper aspects to set the parameters, which will be taken into account. These parameters are appropriate and suitable for each auction. The model consists of two main stages – data processing; creating criteria and their unification, which helps to convert the multi-criteria decision problem into a problem of one criteria linear programming optimization.

2. Optimal choice of a firm at an auction. The aim of the problem is to make maximum efficiency by using minimal money resources.

The organizer of the auction has certain requirements to the firms who are going to participate. Each requirement has its own significance and is estimated by the proper area experts. This process is described by the table below:

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Table 1

AUCTION				
	Requirement 1	***Requirement 2***	**. . .**	***Requirement n***
Significance	ω_1	ω_2	$\dots$	ω_n
Estimation	p_1	p_2	$\dots$	p_n

Let us have in the auction: k firms, $\Phi_1, \Phi_2, \dots, \Phi_k$, $k \in N$. Requirements are n, $n \in N$. Experts determine significance of each one, by weight vector: $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$, $0 < \omega_i \le 1$, $\sum\limits_{i=1}^{n} \omega_i = 1$.

The determination of the weight vector is very important. The greatest is the weight coefficient of the most important of the proper requirement.

Each firm describes how will cover each of the necessary requirements and the experts estimate them.

Let the experts have set the weight coefficients, characterizing significance of the proper requirements and have estimated each firm:

Table 2

Firm		***req. 1***	***req. 2***	**. . .**	***req. n***	***Requirement***
		ω_1	ω_2	$\dots$	ω_n	***Significance***
Φ_1	x_1	p_{11}	p_{21}	$\dots$	p_{n1}	***Estimation***
Φ_2	x_2	p_{12}	p_{22}	$\cdots$	p_{n2}	
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	
Φ_n	x_n	p_{1n}	p_{2n}	$\cdots$	p_{nk}	

So we have the matrix P:

$$(1) \qquad P = \begin{pmatrix} p_{11} & p_{21} & \cdots & p_{n1} \\ p_{12} & p_{22} & \cdots & p_{n2} \\ \cdots & \cdots & \cdots & \cdots \\ p_{1k} & p_{2k} & \cdots & p_{nk} \end{pmatrix}, \quad 0 < p_{ij} \le b, \quad b = \text{const}.$$

The vector Q=$P.\omega$ is created by the efficiencies of each firm participating in the auction:

$$(2) \qquad Q = \begin{pmatrix} p_{11} & p_{21} & \cdots & p_{n1} \\ p_{12} & p_{22} & \cdots & p_{n2} \\ \cdots & \cdots & \cdots & \cdots \\ p_{1k} & p_{2k} & \cdots & p_{nk} \end{pmatrix} . \begin{pmatrix} \omega_1 \\ \omega_2 \\ \cdots \\ \omega_n \end{pmatrix} = \begin{pmatrix} \sum\limits_{i=1}^{n} p_{i1}.\omega_i \\ \sum\limits_{i=1}^{n} p_{i2}.\omega_i \\ \cdots \\ \sum\limits_{i=1}^{n} p_{ik}.\omega_i \end{pmatrix} = \begin{pmatrix} Q_1 \\ Q_2 \\ \cdots \\ Q_k \end{pmatrix}.$$

The problem for winning the auction may be formulated as a knapsack problem with maximum common efficiency with limited resources – money resources. Let these limited money resources be C, $C =$const. Also the firms $\Phi_1, \Phi_2, \dots, \Phi_k$ give proper prices for the offered material resources or services: $c_1, c_2, \dots, c_k$, $k \in N$, $c_j > 0, j = 1, \dots, k$.

Mathematical model – Knapsack problem [2]:

$$\begin{array}{ll} & \max_X \left\{ Z(X) = \sum_{j=1}^{k} Q_j x_j = \sum_{i=1}^{n} \sum_{j=1}^{k} \omega_i p_{ij} x_j \right\} \\ (3) & subject\_to: \\ & \sum_{j=1}^{k} c_j x_j \le C, \quad x_j \in \{0,1\}, \quad j = 1, \ldots, k, \end{array}$$

where:

$$0 < \omega_i < 1, \quad i = 1, \ldots, n, \quad \sum_{i=1}^{n} \omega_i = 1,$$
$$p_{ij} \ge 0, \quad i = 1, \ldots, n, \quad j = 1, \ldots, k,$$
$$0 < c_j, \quad j = 1, \ldots, k.$$

The function Z presents the common efficiency in the auction, and the variable x_j, $j = 1, 2, \ldots, k$, is 1, if j^{th} firm wins the auction and 0 (zero) – if it does not win the auction.

3. Solution. The solution of the problem includes the following stages: calculation of data of the estimations; description of general criterion for global efficiency of the firms; solutions of the optimization problems. These problems are solved by *MAPLE* – for multiplication of the vectors and the matrixes and *LINDO* – for solution of the linear problem – Knapsack problem, respectively.

4. Example: Optimal choice of a firm in conducting auction using exact data. In small municipality on the Operational Programme 'Regional Development' with a priority axis 'Sustainable and Integrated Urban Development' on an operation 'Improvement of Physical Environment and Risk Prevention', 12 firms applied for participation in auction for choice to construction works.

The municipality has certain requirements for firms who are going to participate [1]. The experts determine significance by weight vector:

$\omega = (0.02, 0.10, 0.08, 0.06, 0.05, 0.05, 0.04, 0.03, 0.03, 0.03, 0.02, 0.09, 0.08, 0.07, 0.06, 0.05, 0.05)^T$ [1].

The experts have set the weight coefficients, characterizing significance of the proper requirements and have been estimated – matrix P, each firm by requirements [1] (Table 3).

Then the experts provided 2 matrixes (P and ω) and $Q = P.\omega$:

Every coordinate of the vector Q is corresponding to medium estimation for every firm.

The numerical calculations were obtained by *MAPLE*.

The problem is to obtain maximum efficiency by using limited money resources – 76 773.76 euro (Table 5).

We obtain a result by solving the knapsack problem: Firm Φ_6 wins in the auction with efficiency – medium estimation 3.60 and money resource 69 753.12 euro.

The numerical solutions were obtained by *LINDO*.

Table 3

Requirement		*req.1*	*req.2*	*req.3*	*req.4*	*req.5*	*req.6*	*req.7*	*req.8*	*req.9*
Significance		0.12	0.10	0.08	0.06	0.05	0.05	0.04	0.03	0.03
Firm		*Estimation*								
Φ_1	x_1	2	5	3	2	4	2	3	4	4
Φ_2	x_2	2	3	4	4	2	2	4	3	3
Φ_3	x_3	3	4	2	2	2	3	3	3	4
Φ_4	x_4	3	3	2	4	2	4	4	3	3
Φ_5	x_5	2	2	2	3	3	4	3	4	2
Φ_6	x_6	3	3	4	4	5	4	5	5	3
Φ_7	x_7	3	2	2	3	4	4	5	5	4
Φ_8	x_8	3	3	3	4	5	5	4	4	4
Φ_9	x_9	2	3	2	2	3	3	3	3	4
Φ_{10}	x_{10}	3	3	3	3	2	4	4	5	4
Φ_{11}	x_{11}	2	3	3	3	3	3	4	2	4
Φ_{12}	x_{12}	3	3	2	3	4	4	4	4	3

Requirement		*req.10*	*req.11*	*req.12*	*req.13*	*req.14*	*req.15*	*req.16*	*req.17*
Significance		0.02	0.02	0.09	0.08	0.07	0.06	0.05	0.05
Firm		*Estimation*							
Φ_1	x_1	5	3	2	2	3	3	4	4
Φ_2	x_2	3	4	2	3	3	4	5	5
Φ_3	x_3	5	5	4	4	4	2	3	3
Φ_4	x_4	3	3	4	4	5	2	4	4
Φ_5	x_5	3	3	4	4	4	3	3	4
Φ_6	x_6	3	4	4	4	2	3	3	4
Φ_7	x_7	2	4	3	3	3	3	4	4
Φ_8	x_8	4	3	2	2	3	3	4	2
Φ_9	x_9	5	4	4	4	3	4	3	4
Φ_{10}	x_{10}	4	3	3	3	4	2	3	4
Φ_{11}	x_{11}	3	4	4	4	3	3	3	4
Φ_{12}	x_{12}	3	4	5	2	4	4	2	3

Table 4

Firm		*Medium estimation*
Φ_1	x_1	3.05
Φ_2	x_2	3.15
Φ_3	x_3	3.20
Φ_4	x_4	3.37
Φ_5	x_5	3.04
Φ_6	x_6	3.60
Φ_7	x_7	3.19
Φ_8	x_8	3.21
Φ_9	x_9	3.11
Φ_{10}	x_{10}	3.21
Φ_{11}	x_{11}	3.16
Φ_{12}	x_{12}	3.29

Table 5

Firm		*Money resources*
Φ_1	x_1	82 564.24 euro
Φ_2	x_2	78 356.76 euro
Φ_3	x_3	81 164.78 euro
Φ_4	x_4	73 542.97 euro
Φ_5	x_5	78 157.78 euro
Φ_6	x_6	69 753.12 euro
Φ_7	x_7	76 156.42 euro
Φ_8	x_8	68 452.76 euro
Φ_9	x_9	67 153.45 euro
Φ_{10}	x_{10}	70 439.12 euro
Φ_{11}	x_{11}	76 682.44 euro
Φ_{12}	x_{12}	71 855.22 euro

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ОПТИМАЛЕН ИЗБОР НА ФИРМА ПРИ ПРОВЕЖДАНЕ НА ТЪРГ

Силвия К. Баева, Цветана Хр. Недева

Важен аспект в системата на Министерството на регионалното развитие и благоустройство е работата по Оперативна програма "Регионално развитие" с приоритетна ос "Устойчиво и интегрирано градско развитие" по операция "Подобряване на физическата среда и превенция на риска". По тази програма са включени 86 общини. Финансовият ресурс на тази операция е на стойност 238 589 939 евро, от които 202 801 448 евро са европейско финансиране [1]. Всяка от тези 86 общини трябва да реши задачата за възлагане на обществена поръчка на определена фирма по тази операция. Всъщност, тази задача е задача за провеждане на общински търг за избор на фирма-изпълнител. Оптималният избор на фирма-изпълнител е много важен. Задачата за провеждане на търг ще формулираме като задача на многокритериалното вземане на решения, като чрез подходящо изграждане на критерии и методи може да се трансформира до задача на еднокритериалната оптимизация.