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**MONITORING IN EDUCATIONAL PROCESS VIA
STUDENTS GRADE POINTS***

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This paper considers stability of an educational process via student assessment progress in studied disciplines. The most popular tools on quality control of manufacturing production and services are applied to the average grade points of a group of twenty students in seven subjects in one academic year to investigate the process normality. The observed results are analyzed and statistical inference is interpreted in the context of the educational process.

1. Introduction. The goal of education in any discipline is to teach students in subject matter, so that they can acquire, transform and apply achieved knowledge and skills in other fields of studies and practice. An educational process (EP) consists of providing information, training, clarification and testing [3]. In all training – instructor-lead, Web or computer based, the teacher, lecturer or other can provide the information to learners through various educational media, like course exercises, homeworks and others. Discussions and answers to the exercises are then given with explanations to clarify the tasks and their solutions.

The last step in the EP is a verification of learning and understanding via assessment of students in various forms, such as presentations, tests, exams and others [7]. A good assessment measures learning outcomes and is easily administered, scored and interpreted. The grade points inform teachers about student course progress and experiences, provide a feedback to the learner and represent a tool for evaluation of a learning process.

Thus the EP, measured by student point grades, can be considered as a special technological process, and for its monitoring we can apply most popular statistical techniques for quality control (QC) of manufacturing production [10] to review student progress, teaching methods and other sub-processes and elements. Statistical tools, such as check sheets, control cards, histograms, cause-and-effect diagrams, Pareto charts and scatter plots, are popular for testing quality (Q) of any process, because they are graphical tools, applicable for vast Q related processes, suitable for users with basic prerequisites in statistics and available to implement in administrative control.

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2. Statistical processing with assessment data. Monitoring a program, course or other activity is a systematic process of examining and testing students' achievements and teaching strategies. The main goals of this process are: collecting information, analyzing and evaluating obtained data, and taking action to improve student performance, see [2].

2.1. Primary data processing. During a quarter, semester or others academic management's receives by instructors, teachers etc, information about students achievements. Data can be represented per students and subjects as a matrix $X_{n \times k} = \{x_{ij}\}$, where x_{ij} is a final mark (grade points) of student i on subject D_j ; $i = 1, 2, \dots, n$; $j = 1, 2, \dots, k$; or a table, see Table 1. For subject D_j the average (arithmetic mean) grade X_j , variance S_j^2 of students' grade and coefficient of skewness SK_j , are introduced as follows:

$$(1) \quad X_j = \sum_{i=1}^n x_{ij}/n, \quad S_j = \sqrt{\sum_{i=1}^n (x_{ij} - X_j)^2 / (n-1)}, \quad SK_j = \sum_{i=1}^n (x_{ij} - X_j)^3 / (nS_j^3).$$

The next statistical procedures are most popular as a graphical tools and easy to apply.

2.2. Check sheets and histograms. A check sheet for student grade points is represented as a table and is used to construct the corresponding frequency histogram.

Check sheets are documents (blank forms) that are used for collecting data, the time and location, where the data is gathered. A shape of the document is a table with frequencies of marks of students' assessment by subjects D_j . Such tables are called frequency tables, if marks are in ascending order per magnitude, and desired information is marked on by "tally – |", when the event occurred.

Histograms are graphical representations of a distribution of collected data and their frequency densities. A histogram provides information about: density's modes, the symmetry of the distribution, and for unimodal distribution what is its skewness (asymmetry) and kurtosis ("peakedness"). For symmetric unimodal densities we have equal mode, mean and median, as it fulfilled for normal distribution. If the histogram is closed to normal distribution, i.e. $SK_j \approx 0$, see Figure 2, then the EP is stable and the assessment is normal, [4]. If the mode is shifted to the left of median, i.e. the mass of the histogram is concentrated on the left of the figure, then he have right-skewed, $SK_j > 0$, and there are downgrading, thus the EP is instable, otherwise – we have

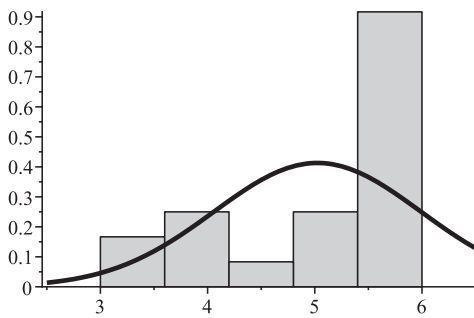


Fig. 1. Histograms of marks per subject D_1

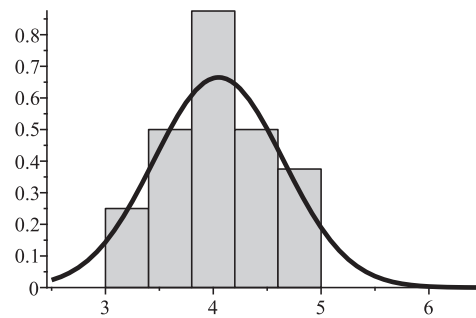


Fig. 2. Histograms of marks per subject D_2

left asymmetry, $SK_j < 0$, and hence there is a raising of marks, see Figure 1. In such cases the curriculum, teaching process and assessment on D_1 should be administratively controlled.

2.3. Control Charts. Control charts are statistical tools for monitoring the Q of productions and services, and for detecting the variation of process Q by tracking samples Q for non-standard behavior, [8], [9], [10]. If the process variation is due only to random events the process is said to be stable or in control, otherwise if it includes both random and special influence of variation, then the process is said to be instable or out-of-control. To monitor the EP via students grade some Shewhart charts can be used: \bar{X} -bar chart (X chart), see Figure 3, which controls the mean value of a process; S chart (Figure 4) manages the variability of a process; np chart and C chart govern numbers of defectives (non-successive marks). All control charts are established under the assumptions that all measurement data are normally distributed and independent of each other, i.e. all x_{ij} are independent observation and $x_{1j}, x_{2j}, \dots, x_{nj}$ are normally distributed with parameters: mean μ_j and variance σ_j^2 , which is denoted as $x_{ij} \sim N(\mu_j, \sigma_j^2)$, $i = 1, 2, \dots, n$.

Constructing a 3-sigma control chart. The lower and upper control limits (LCL, UCL) and the center line (CL) for monitoring the process mean and variability of data are possible to have been established by a-priori investigations. If such data are not available, by using sample data the CL, then LCL and UCL are calculated. For the X and S chart the CL of an EP for given period is set as an average \bar{X} of subject means X_j , and the average standard deviation \bar{S} is defined as arithmetic mean of standard deviations per subjects, i.e.

$$(2) \quad \bar{X} = \sum_{j=1}^k X_j/k, \quad \bar{S} = \sum_{j=1}^k S_j/k.$$

It is known, e.g. [5], if random variables are independently identically distributed via normal law, $x_{ij} \sim N(\mu_j, \sigma_j^2)$, $i = 1, 2, \dots, n$, then $X_j \sim N(\mu_j, \sigma_j^2/n)$ and standard deviation is scaled chi-distributed, i.e. $(n-1)S_j^2/\sigma_j^2$ is chi-squared distributed with $n-1$ degree of freedom. Therefore, $\bar{X} \sim N(\bar{\mu}, k\bar{\sigma}^2/n)$, where $\bar{\mu} = \sum_{j=1}^k X_j/k$, $\bar{\sigma}^2 = \sum_{j=1}^k S_j^2/k$. So, it is possible to find any confidence intervals for \bar{X} by using quantiles of $N(\bar{\mu}, k\bar{\sigma}^2/n)$. A distribution of \bar{S}^2 , under some additional assumptions, can be approximated by gamma distribution and then derive a distribution of \bar{S} . In practice, 3-sigma rules proceeding are used. By Chebishev inequality

$$(3) \quad P\{|X - EX| \leq l DX\} \geq 1 - 1/l^2,$$

where: the random variable X has mean EX , variance DX , and l is a positive integer, we can derive control intervals for the process mean. The inequality (3) yields, that at least $(1 - 1/l^2) \%$ of means are in $[\bar{X} - lS, \bar{X} + lS]$, and this interval is called l -sigma interval. For $l = 3$ the LCL and UCL for X -chart are defined by (4), respectively

$$(4) \quad L_{X,l} = \bar{X} - lS_{\bar{X}}, \quad U_{X,l} = \bar{X} + lS_{\bar{X}}, \quad S_{\bar{X}} = \sqrt{\sum_{i=1}^k (X_i - \bar{X})^2 / (k-1)}.$$

The warning (W) LCL and UCL lines are obtained by (4), respectively, for $l=2$. For the S , the CL for monitoring the variability of the assessment process is defined by (2). Hence, LCL, UCL, WLCL and WUCL for average of k standard deviations are set to

be, see [6],

$$(5) \quad L_{S,l} = \bar{S} - l \bar{S} \sqrt{1 - c^2}/c, \quad U_{S,l} = \bar{S} + l \bar{S} \sqrt{1 - c^2}/c,$$

where:

$$l = 3, 2, \quad c = \frac{\sqrt{2/(k-1)}(k/2-1)!}{((k-1)/2-1)!}, \quad (k/2)! = (k/2)(k/2-1) \dots (1/2)\sqrt{\pi}.$$

The most popular stopping rules in a controlled process were suggested by the “Western Electric Company” (WECO), and mean for the EP the followings.

Rules for control cards. If one point (D_j, X_j) exceeds the control limits, then the process has to be stopped, i.e. administrative actions are needed to remove causes putting the process out-of-control.

If all points are near to one of the 3 sigma control line, then the EP should be regulated by administrative enrollment controls.

If most points are disposed in upper or lower 2 sigma and 3 sigma band with significant dispersion, then the process is instable and the administration has to eliminate the reason of instability.

If all points are in the admissible 2 sigma zone but above or below the CL, then the process is in control but slightly displaced, no actions needed only warning.

If all points are in the 2-sigma zone and are symmetrically disposed about CL, then the process is stable.

3. Numerical example. A group of $n = 20$ students for one academic year passed $k = 7$ exams on disciplines: D_1, D_2, \dots, D_7 , with credits d_j . Average scoring of i -th student on subject j is x_{ij} , via 6 scale grade (excellent = 6, failed = 2) is calculated as average grade of: course projects, tests, exams and etc. Input data and results of statistical processing are given in Table 1, the frequency tables are given in Table 2, and they are used for histograms’ drawing.

To design the control X chart by (2) and (4), we derived respectively:

$$\bar{X} = 4.861, \quad S_{\bar{X}} = \sqrt{\sum_{j=1}^7 (X_j - \bar{X})^2 / 6} = 0.494.$$

Control and warning limits are:

$$L_{X,3} = 3.38, \quad U_{X,3} = 6.34, \quad L_{X,2} = 3.87, \quad U_{X,2} = 5.85.$$

Obviously, the UCL being grater than maximal mark via 6-th grading system points out that there are some problems in the EP. The X control card is given on Figure 3.

To construct S -control chart, applying (2) and (5) we calculate:

$$\bar{S} = 0.818, \quad c = 0.959$$

and

$$L_{S,3} = -0.07, \quad U_{S,3} = 1.70, \quad L_{S,2} = 0.23, \quad U_{S,2} = 1.41.$$

The students’ average grade is between WLCL and WUCL, the same is valid for the standard deviation, see Fig. 4. The EP is considered to be a stable one.

In Figures 1 and 2 are shown frequency histograms of grades and curves of probability density functions of corresponding normal distribution $N(X_j, S_j^2)$ for marks on D_1 and D_2 , respectively. On x -axes are presented subjects with step one. Figure 1 illustrates a fact that in the first subject students’ marks have been over-graded from the average. The

Table 1. Students' marks per subjects

No of student	Grade marks per subjects						
	D_1	D_2	D_3	D_4	D_5	D_6	D_7
1	6	3	6	5	6	5.5	4
2	5	4	5	4.5	6	6	5
3	5.5	4.5	5.5	5	5.5	6	6
4	3	5	5	4	5	4.5	3
5	6	4.5	6	4	6	5.5	6
6	5	4	5	4	6	6	5.5
7	4.5	3	5.5	4.5	5.5	5.5	4
8	3	4	4.5	3.5	5	5	3.5
9	6	4	5	5	6	5.5	6
10	5	5	5.5	4.5	6	6	5
11	4	3.5	5	5.5	4.5	4.5	4.5
12	4	4.5	4.5	3	5	4.5	3
13	5.5	5	6	6	4	6	4.5
14	6	4	6	4.5	6	6	4.5
15	5.5	3.5	5.5	6	5.5	5.5	5
16	6	4	3.5	3.5	3.5	3.5	3
17	5.5	3.5	6	5	6	6	5.5
18	5.5	4.5	5.5	5	6	5.5	5
19	5.5	3.5	5	5	5.5	5.5	5
20	4	4	4	3	3.5	4.5	3
Average grade X_j	5.025	4.050	5.200	4.525	5.325	5.350	4.550
Standard deviation S_j	0.966	0.605	0.698	0.866	0.847	0.709	1.037
Coefficient of skewness SK_j	-0.858	-0.011	-0.731	-0.137	-1.070	-1.017	-0.239
Credits d_j	3	10	4	8	5	5	6

normal distribution fitted sufficiently well the relative frequencies of grades in subject D_2 , therefore the teaching process and assessment are normal. Table 1 shows that in D_1 we have negative skewness ($SK_{1j} < 0$), thus there is a tendency for raising marks. The same is observed in subjects D_3 , D_5 and D_6 , which should be a warning for administration.

Table 2. Frequency tables per subjects

Grade	Frequencies of marks per subjects						
	D_1	D_2	D_3	D_4	D_5	D_6	D_7
3	2	2	—	2	—	—	4
3.50	—	4	1	2	2	1	1
4	3	7	1	3	1	—	2
4.50	1	4	2	4	1	4	3
5	3	3	6	6	3	1	5
5.50	6	—	5	1	4	7	2
6	5	—	5	2	9	7	3

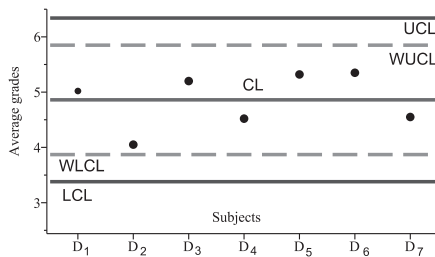


Fig. 3. X-chart of marks per subject D_2

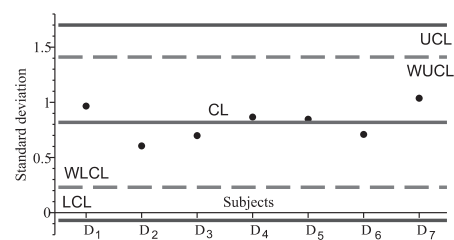


Fig. 4. S-chart of marks per subject D_2

4. Conclusion. Students' assessments per subjects can be analyzed by checking sheets and frequency histograms of received marks. The EP monitoring can be based on X , R , S , C or np control cards. Possible factors for receiving unsatisfactory grades in some discipline may be detected by student feedback and Pareto charts. The relationship between cause effecting instability of the EP can be established via scatter graph. If there is a-priori information about behavior of the EP, then the measure limits from previous periods can be used as control limits or use some standard measure limits can be used.

Most of the monitoring EP charts can be prepared for small groups of students by hand, otherwise they can be generated by simple spreadsheet programs, such as OpenOffice.orgCalc, MS Excel, any statistical software package or online quality charts generators.

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КОНТРОЛИРАНЕ НА УЧЕБНИЯ ПРОЦЕС ПОСРЕДСТВОМ УСПЕВАЕМОСТТА НА СТУДЕНТИТЕ

Йорданка А. Ангелова, Петър М. Халачев

Настоящият доклад разглежда устойчивостта на учебният процес посредством успеваемостта на студентите по изучаваните дисциплини. За изследване нормалното протичане на процесът на обучение, в група от двадесет студента по седем предмета за една академична година, към средната успеваемост са приложени най-популярните процедури от качественият контрол на промишлена продукция и услуги. Наблюдаваните резултати са анализирани и статистическите изводи са интерпретирани спрямо контекстът на образователния процес.