BENEFITS OF USING VIRTUAL MANIPULATIVES IN MATH LESSONS IN PRIMARY SCHOOL[*]

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The mission of the school of the 21st century is to achieve a 90–95% success in the state curriculum. In addition, schools must incorporate ICT (information-communication technology) into the curriculum. Therefore, the emphasis in this paper is placed on the importance of incorporating virtual manipulatives (interactive applets) in math lessons in primary school to increase the percentage of success. Our research was conducted in two classes of students of the 7th grade in primary school and in an optional online class of ICT college students

The topic was the Pythagorean Theorem. In both 7th grade classes, instruction was enhanced using manipulatives. In one class, we used actual manipulatives (paper and scissor constructions) and in the other class, we used a virtual manipulative created with the free interactive dynamic mathematics software GeoGebra. Then, we compared the understanding acquired in both of the classes. In the online class, we gave the virtual manipulative to these (good and highly motivated) students and challenged them to create their own constructions using GeoGebra.

In 7th grade, we found a distinct measurable improvement in the understanding of this topic by the students from the class using ICT and virtual manipulative over the class using classical methods of teaching and in the online class, we saw both how little understanding was retained about this important theorem and the resulting ah-ha factor of doing and understanding this construction.

1. Introduction. Traditionally, technology has **not** played an **important** role in the teaching and learning of mathematics. Free dynamic software for mathematics allows both teachers and students to actively explore mathematics – not just by seeing, but by doing.

2. Methodology in the 7th grade classroom. This research was made among two different classes of students in the 7th grade in a primary school. Class 1 and Class 2 were divided into smaller groups numbering 4–5 students. In both classes, the teacher guides the students to understand the Pythagorean Theorem and bring the conclusions by themselves using an activity called Pythagorean Puzzle.

2.1. Interactive activity for learning the Pythagorean Theorem. Students from the groups of Class 1 were working with different puzzles printed on paper. They cut out the small and medium square (see Figure 1) along the numbered parts and then arranged the pieces they cut to fit into the larger square (see Figure 2).

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[*]**Key words:** ICT, GeoGebra, primary, manipulative, Pythagoras.
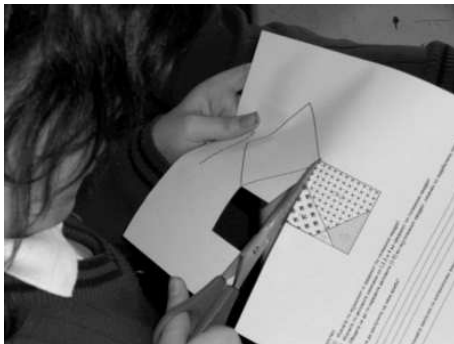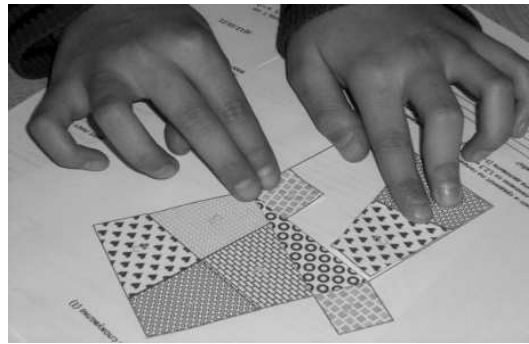
Fig. 1. Cutting the squares

Fig. 2. Fitting the pieces

The most important problem that appeared in Class 1 was the lack of enough time to fulfill the activities in the time allotted. Because of this, the students became confused, uncoordinated and noisy.

Student groups from Class 2 did this activity using GeoGebra. That is, students from the groups of Class 2 were working with a virtual manipulative – in this case an interactive computer applet created with free dynamic software GeoGebra. As you see below, there are several benefits of the proposed methodology. Firstly, the teacher must know how to construct the applet, perhaps increasing his/her understanding. Secondly, the student understands that there is nothing magic in creating this construction (unlike the most cut-out activities where the lines must be carefully drawn ahead of time so that the puzzle fits together properly). Thirdly, the student can create the construction himself at home or in school.

In our case here, this activity was done within the same time framework as in Class 1 – both because of the time constraints of the state curriculum and in order to compare the results from both classes.

Firstly, the teacher demonstrates to the students the already created interactivity on the video projector (see Fig. 3). She shows them how clicking and dragging the blue point is completely free and determines the length of the green square (and hypotenuse). She showed them that they can move the red point, but the triangle is always a right triangle. She asked them to think about how the red point moves so that the triangle is
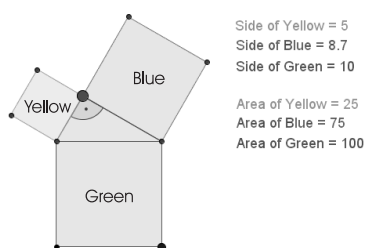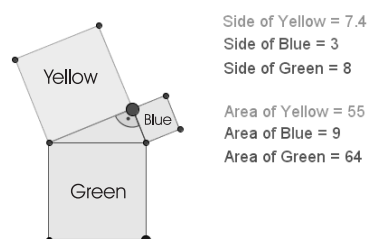

Fig. 3. Initial screen of applet

Fig. 4. Another screen

always a right triangle. She drags the blue point to some easy length and point. She asks them to think about what lengths they can now choose for the yellow and blue sides.

(We point out that it is by intention that approximate values are used for the sides. Most mathematics teachers (and hence ready-to-use) interactive applets use "exact" square root notation and GeoGebra can certainly write this notation. However, (a) our extensive experience in teaching college students has shown that exact symbolic values (e.g. $\sqrt{55}$) have little meaning to the majority of students (and to people in general), whereas an approximate number value (e.g. 7,4) does, and (b) using approximate values allows a dynamic construction that is continuous. Thus by using approximate values, the lengths and areas have meaning and the continuity of the construction is visible.)

She shows them that the information at the right is dynamic, i.e. it changes as the graphic changes. The teacher now asks them whether they see any relationships between the values on the screen? She says "We want to relate this to Pythagoras' theorem." and asks them to think about the values for the areas of the squares to see if they see a relationship. (They should see that the sum of the 1st and 2nd areas is the 3rd area.) She can prompt "How big is the Area of the Yellow Square?". "How is it calculated?" ...(see Fig. 4)

Secondly, the teacher actually creates the construction in front of the class. She started by drawing a big "dynamic square" that will be the "hypotenuse square". Dynamic meaning that the size of the square can be changed (by dragging the blue point), but the figure is always a square. Then, she talks to them about how to create the red point which is the third vertex of the triangle. She shows them how it moves and asks them to think about what points always works. If there is time, she lets the student move to themselves the red point to see how it is restricted. The objective here is to get the students to remember Thales' theorem on their own – thus reinforcing and connecting previous learning. The teacher constructs the semicircle and takes the red point on it and, then, hides the semicircle (and the midpoint used to construct it). In this way the students see that there were "hidden objects". They are not magic, just hidden. Then, she constructs the other squares and colors them. She adds the dynamic texts and had the students check everything. Again we emphasize that with this, the class is *not* just "show and tell" as with a ready-to-use demonstration applet. Not only does the teacher completely understand the dynamics of this construction, but the students see how the construction works and can do it themselves. Of course, with more time for this activity, much more explorations are possible including getting the students to create the applet themselves.

In the online class, the ICT college students were challenged to do just this. These ICT college students consider themselves to be "good" at mathematics, they had used GeoGebra in several previous exercises and they are good students. Exactly the parts that the 7th grade teacher emphasized in the above, were the points where these students had difficulty, indicating non-mastery of both Pythagoras' and Thales' theorems. Just creating the dynamic texts, i.e. understanding how the picture matched the maths, was challenging for them. However, their satisfaction in creating and understanding the worksheet and how it works was evident at the end of the class.

3. Results. In the 7th grade classes, an appropriate check list was used to assess the activities of the student groups in each of the classes. The check lists were indicators of the understanding of the students. The diagrams below show that the group with the

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highest result from Class 1 (see Figure 5) had the same points as the group with the lowest result in Class 2 (see Figure 6). The success of Class 2 is obvious.
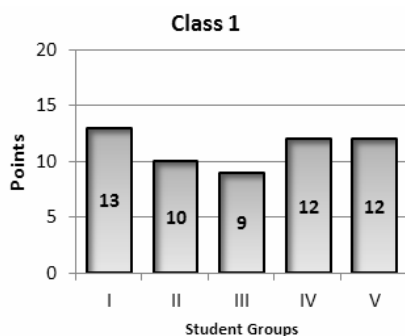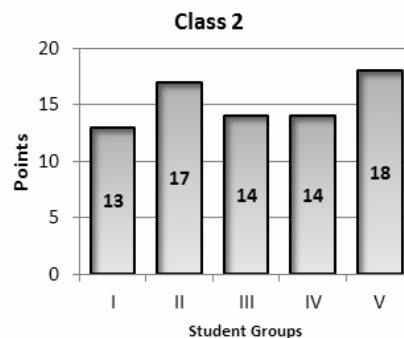

Fig. 5. Results from Class 1

Fig. 6. Results from Class 2

Also, our impression was that the students from Class 2 were more interested, organized, motivated and more stimulated in doing their activities.

Are the results objective? We think they are. The groups in the 7th grade classes were formed as heterogeneous with students of varying scholastic quality. Grading was done at the group level, which also contributes to the objectivity.

4. Conclusion. In this paper, we presented an example and opportunity for actively using GeoGebra in primary schools to explore some basic concepts in geometry. By using GeoGebra, teachers can escape the appearance of abstraction and theory without practice and yet not be simply "show and tell" drama, but actually help students to visualize different math processes and explore connections between math symbols and visual representations themselves.

The statistical analysis suggests that the use of these manipulatives has a positive effect on the increased knowledge of the students. They seem more able to explore mathematical relations that were difficult to explore before and visualize relationships that were just formulas. It places emphasis and understanding by doing both on the part of the teacher and on the part of the student. Therefore, the main emphasis in this paper is placed on the importance of using a free interactive mathematics software such as GeoGebra as a tool for creating virtual manipulatives in the math lessons in primary school.

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ПРЕДИМСТВА ОТ ИЗПОЛЗВАНЕТО НА ВИРТУАЛНИ МАНИПУЛАТИВИ В УРОЦИТЕ ПО МАТЕМАТИКА В ОСНОВНОТО УЧИЛИЩЕ

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Мисията на училището в 21-и век е да се постигне 90–95% успеваемост по държавните учебни програми. Освен това училищата трябва да прилагат ИКТ (информационни и комуникационни технологии) в учебната програма. Поради това ударението в тази статия е поставено върху значението на използването на виртуални манипулативи (интерактивни аплети) в уроците по математика в основното училище за повишаване процента на успеваемост. Изследването е проведено с два класа ученици от 7 клас в основно училище и с един допълнителен онлайн клас от студенти по ИКТ в колеж.

Темата беше Питагоровата теорема. И в двата 7-и класа обучението беше проведено с използване на манипулативи. В единия клас бяха използвани манипулативи (построения с хартия и ножица), а в другия клас бяха използвани виртуални манипулативи, създадени с безплатната интерактивна софтуерна програма за динамична математика GeoGebra. След това сравнихме постигнатото разбиране у учениците от двата класа. В онлайн класа дадохме виртуални манипулативи на (онези добри и силно мотивирани) студенти и ги предизвикахме да създадат свои собствени конструкции с GeoGebra.

В 7-и клас констатирахме ясно измеримо подобрение в разбирането на темата от учениците от класа, използващ ИКТ и виртуални манипулативи, спрямо класа, използващ класическите методи на преподаване, а в онлайн класа видяхме както то, че задържаното разбиране по тази толкова важна теорема е твърде слабо, така и достигане до фактора „аха-а-а-а" при създаване и разбиране на чертежа.

Ключови думи: ИКТ, GeoGebra, основно училище, манипулативи, Питагор.