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Operator Schur Algorithm and Associated Functions

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1. The aim of this paper is to point out an operatorial version of a classical algorithm due to I. Schur in [17]. This algorithm is the precursor of the layer-peeling procedure that is applied in a broad class of problems: in filter synthesis, to inverse scattering, to the partial realization problem... (see for instance a special issue of "Integral Eq. and Operator Theory" on this theme). But, the motivation of Schur's paper came from classical extrapolation theory (Caratheodory — Fejer and trigonometric moment problem), providing the structure of an analytic bounded function in the unit disc.

We note now some of the important moments in the evolution of this last point of view, of course, in connection with our interest: an operatorial approach of the Carathéodory — Fejér and Nevanlinna — Pick problems is done in [16]; the Sz.-Nagy — Foias theorem of lifting of commutants (see [18]) can be viewed as a generalized abstract formulation of these problems in [1]; the Nehari problem is treated [3], ends a series of papers containing generalizations of Schur's paper [17] for the set of the liftings of a commutant. The main algorithm in [3] connects the observable sequence with its choice sequence (in the scalar case of the Carathéodory — Fejér problem this is exactly the connection between the Taylor coefficients of a given bounded analytic function in the unit disc and its Schur coefficients), but it is not obtained by a direct generalization of the Schur algorithm.

Returning to the present note, we develop an operatorial version of the Schur algorithm by adapting the initial argument in [17]. But our principal interest is to introduce some associated functions and to point their role in replacing the Szegö polynomials.

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2. In this section we obtain an operatorial version of the Schur algorithm. The scalar case was done in [17] in the following way: let f be an analytic function in the unit disc $\mathrm{D}=\{|z|<1\}$ and:

$$\begin{cases} f_1=f, \quad g_n=f_n(0), & n\geqq 1 \\ f_{n+1}(z)=\dfrac{f_n(z)-g_n}{z(1-\bar{g}_n f_n(z))}, & |z|<1 \end{cases} \tag{2.1}$$

then $|f(z)|\leqq 1$ in D if and only if $|g_n|\leqq 1$ for all $n\geqq 1$ and if $|g_{n_0}|=1$ for some $n_0\in\mathrm{N}$, then $g_n=0$, $n>n_0$ and f is a rational function with the polles on $|z|=1$.

The point here is that the one-to-one analytic mappings of the unit disc onto itself are given by $c\frac{z-a}{1-\bar{a}z}$, $|c|=1$, $|a|<1$. Now, we develop an operatorial variant of this fact. For two separable Hilbert spaces $\mathscr{H}$ and $\mathscr{K}$ define

$$\mathscr{B}(\mathscr{H},\ \mathscr{K})=\{f:\mathrm{D}\to\mathscr{L}(\mathscr{H},\ \mathscr{K})/f \text{ analytic in D and } \|f(z)\|\leqq 1 \text{ for } z\in\mathrm{D}$$

(here $\mathscr{L}(\mathscr{H},\mathscr{K})$ denotes the set of the linear bounded operators from $\mathscr{H}$ into $\mathscr{K}$ and $\|.\|$ is the operatorial norm in $\mathscr{L}(\mathscr{H},\ \mathscr{K})$).

For a contraction $T\in\mathscr{L}(\mathscr{H},\mathscr{K})$ (i.e. $\|T\|\leqq 1$), we consider $D_T=(I-T^*T)^{1/2}$ and $\mathscr{D}_T=\overline{D_T\mathscr{H}}$, and we define the operator function

$$(2.2)\qquad \begin{cases} U(z,T):\mathscr{H}\oplus\mathscr{D}_{T*}\to\mathscr{K}\oplus\mathscr{D}_{T}, & |z|<1.\\ U(z,T)=\begin{bmatrix} T, & zD_{T^*}\\ D_T, & -zT^*\end{bmatrix} \end{cases}$$

and for $f\in\mathscr{B}(\mathscr{H},\ \mathscr{K})$ we consider the linear fractional map

$$(2.3)\qquad C_U(f)(z)=T+zD_{T^*}f(z)(I+zT^*f(z))^{-1}D_T:\mathscr{H}\to\mathscr{K}.$$

One obtains the formula:

$$(2.4)\qquad I-C_U(f)(z)^*C_U(f)(\zeta)$$

$$=D_T(I+\bar{z}f^*(z)T)^{-1}(1-\bar{z}\zeta f^*(z)f(\zeta))(I+\zeta T^*f(\zeta))^{-1}D_T$$

which proves in particular that $C_U(f)\in\mathscr{B}(\mathscr{H},\ \mathscr{K})$ (in the matricial case, such kind of linear fractional maps are intensively studied, for instance in [15, 11, 14,...].

We need now to recall some remarks regarding the canonical decomposition of the functions in $\mathscr{B}(\mathscr{H},\ \mathscr{K})$ (see [18]). Thus, for a contraction $T\in\mathscr{L}(\mathscr{H},\ \mathscr{K})$ we have

$$(2.5)\qquad T=T_U\oplus T_p:\ker\ D_T\oplus\mathscr{D}_T\to\ker\ D_{T^*}\oplus\mathscr{D}_{T^*},$$

where T_U is a unitary operator and T_p is a pure contraction in the sense that $\|T_ph\|<\|h\|$, $h\neq 0$. For $f\in\mathscr{B}(\mathscr{H},\ \mathscr{K})$, the maximum moduls principle leads to the structure

$$(2.6)\qquad f(z)=T_U\oplus f_p(z):\ker D_T\oplus\mathscr{D}_T\to\ker D_{T^*}\oplus\mathscr{D}_{T^*},$$

where $T=f(0)$, T_U is the unitary part of T and $f_p(z)$ are pure contractions.

2.1. Theorem. *For $f\in\mathcal{B}(\mathcal{D}_T, \mathcal{D}_{T^*})$, where $T=f(0)$, the equation*

$$f=C_{U(\cdot, T)}(F)$$

has a unique solution $F\in\mathcal{B}(\mathcal{D}_T, \mathcal{D}_{T^*})$.

Proof. We devide the proof into steps.

Step 1. $I-f^*(0)f(z)$ is one-to-one for all $z\in\mathbf{D}$.

The statement is clear for $z=0$, so for $z\neq 0$, let $h\in\mathcal{D}_T$ with

$$(I-f^*(0)f(z))h=0,$$

consequently,

$$\|h\|^2=(h, f^*(0)f(z)h)=(f(0)h, f(z)h).$$

By the maximum modulus principle applied to the function $f_h(z)=f(z)h$ it results

$$\|h\|^2\leqq\|f(0)h\|\,\|f(z)h\|\leqq\sup_{|\zeta|\leqq|z|+\varepsilon}\|f_h(\zeta)\|\,\|f(z)h\|\leqq\|h\|^2.$$

Thus, either $f(z)h=0$, so $h=0$ or $\|f(0)h\|=\sup_{|\zeta|\leqq|z|+\varepsilon}\|f(\zeta)h\|$ and invoking again the maximum moduls principle, we conclude that f_h is a constant function, consequently $h=0$.

In a similar way we can deduce that $I-f^*(z)f(0)$ is one-to-one for all $z\in\mathbf{D}$, consequently, $(I-f^*(0)f(z))\mathcal{D}_T$ is dense in $\mathcal{D}_T$.

Step 2. Let $f(z)=T+T_1z+T_2(z^2)+\dots$ be the Taylor expansion of f.

It easily results that $T_n=D_{T^*}T_n'D_T$, $n\geqq 1$, so

$$I-T^*f(z)=D_T(I-zT^*T_1'-z^2T^*T'_2-\dots)\,D_T$$

and as D_T is one-to-one on $\mathcal{D}_T$, if we consider

$$H_1(z)=I-zT^*T_1'-z^2T^*T_2'-\dots:\mathcal{D}_T\to\mathcal{D}_T,$$

then H_1 is analytic on $\mathbf{D}$. We also define the analytic function on $\mathbf{D}$, $H(z)=H_1(z)D_T:\mathcal{D}_T\to\mathcal{D}_T$ then $H_1(z)\mathcal{D}_T$ and $H(z)\mathcal{D}_T$ are dense in $\mathcal{D}_T$ for all $z\in\mathbf{D}$.

To prove this, let $h_0\in D_T\mathcal{H}$, $h_0 DH(z)\mathcal{D}_T$ so, there exists $h_1\in\mathcal{D}_T$ such that $(D_Th_1, H(z)h)=0$, for all $h\in\mathcal{D}_T$; hence, $((I-f^*(z)f(0))h_1, h)=0$, $h\in\mathcal{D}_T$ and by the first step, $h_1=0$.

Step 3. Fix $z\in\mathbf{D}$. Defining the linear transformation by

$$\text{“}-T+D_{T^*}f(z)(I-T^*f(z))^{-1}D_T\text{”}: H(z)\mathscr{D}_T\to\mathscr{D}_{T^*}$$

$$(\text{“}-T+D_{T^*}f(z)(I-T^*f(z))^{-1}D_T\text{”})H(z)h=-TH(z)h+D_{T^*}f(z)h;\ h\in\mathscr{D}_T,$$

we readily check that it is contractive, hence we can uniquely extend it to a contraction from $\overline{H(z)\mathscr{D}_T}\doteq\mathscr{D}_T$ into $\mathscr{D}_{T^*}$ and we denote this extension by $F_1(z)$.

Step 4. By the definition,

$$(I+T^*F_1(z))H(z)=D_T$$

consequently $(I+T^*F_1(z))H_1(z)=I$. Analogously, $H_1(z)(I+T^*F_1(z))=I$, and since H_1 is analytic in D, $I+T^*F_1$ is also analytic in D

Step 5. We verify by a direct computation the formula

$$T+D_{T^*}F_1(z)(I+T^*F_1(z))^{-1}D_T=f(z).$$

Step 6. According to the Step 5, $D_{T^*}F_1(z)(I+T^*F_1(z))^{-1}D_T$ is anlytic in D, $F_1(z)(I+T^*F_1(z))^{-1}$ is analytic in D. As $F_1(z)=F_1(z)(I+T^*F_1(z))^{-1}H_1(z)$ we obtain that F_1 is analytic in D, $\|F_1(z)\|\leqq 1$ for $z\in$ D and $F_1(0)=0$.

Using Schwartz Lemma, it follows that there exists a function $F\in\mathscr{B}(\mathscr{D}_T,\mathscr{D}_{T^*})$ such that $F_1(z)=zF(z)$ for $z\in$ D, and $f=C_U(F)$. □

Now, we can describe the Schur algorithm for $\mathscr{B}(\mathscr{H},\mathscr{K})$: Let $f\in\mathscr{B}(\mathscr{H},\mathscr{K})$, $T=f(0)$ and $f=T_U\oplus f_p$ is the canonical decomposition of f. Define $f_1=f_p\in\mathscr{B}(\mathscr{D}_T,\mathscr{D}_{T^*})$ and by Theorem 2.1, there exists a unique $f_2\in\mathscr{B}(\mathscr{D}_T,\mathscr{D}_{T^*})$ with $f_1=C_{U(\cdot,T)}(f_2)$.

Define $G_1=T$. Repeating this procedure, we have

$$\text{(2.7)}\qquad\begin{cases} f_1=f_p\in\mathscr{B}(\mathscr{D}_T,\mathscr{D}_{T^*}), & G_n=f_n(0),\quad n\geqq 1,\\ f_{n,p}=C_{U(\cdot,G_n)}(f_{n+1}), & f_{n+1}\in\mathscr{B}(\mathscr{D}_{G_n},\mathscr{D}_{G_n^*}).\end{cases}$$

2.2. Theorem. *There exists a one-to-one correspondence between the set $\mathscr{B}(\mathscr{H},\mathscr{K})$ and the set of sequences of contractions*

$$\{G_n\}_{n=1}^{\infty},\ G_1\in\mathscr{L}(\mathscr{H},\mathscr{K}),\ G_n\in\mathscr{L}(\mathscr{D}_{G_{n-1}},\mathscr{D}_{G^*_{n-1}}),\ n>1.$$

Proof. By taking into account the Taylor series, we readily check that the Taylor coefficients of f are given by

$$\text{(2.8)}\quad T_n=T_n(G_1,\dots,G_n)=\text{form}\,(G_1,\dots,G_{n-1})+D_{G_1^*}\dots D_{G^*_{n-1}}G_nD_{G_{n-1}}\dots D_{G_1},$$

where *form* $(G_1,\dots,G_n)$ denotes a formula depending only on $G_1,\dots,G_n$. □

2.3. Theorem. *The sequence $\{G_n\}_{n=1}^{\infty}$ given by the algorithm* (2.7) *coincides with the choice sequence associated to a corresponding lifting of the commutant T of the contractions $O_{\mathscr{H}}$ and O_K by the algorithm in* [3].

Proof. The algorithm (2.7) describes the set $\mathscr{B}(\mathscr{H}, \mathscr{K})$ in such a way that the Taylor coefficients belong to a certain "operatorial disc", exactly as in the algorithm in [3]. Consequently, the centers and left and right radii coincide. □

3. From (2.1) we obtain by induction

$$f(z)=\frac{z\mathscr{A}_n^*(z)f_{n+1}(z)+\mathscr{B}_n^*(z)}{z\mathscr{B}_n^*(z)f_{n+1}(z)+\mathscr{A}_n^*(z)},\tag{3.1}$$

where $\mathscr{A}_n$ and $\mathscr{B}_n$ are polynomials of degree $n-1$,

$$\mathscr{A}_1(z)=1,\quad \mathscr{B}_1(z)=g_1,$$

$$\mathscr{A}_{n+1}(z)=\mathscr{A}_n(z)+zg_{n+1}\mathscr{B}_n(z)\tag{3.2}$$

$$\mathscr{B}_{n+1}(z)=z\mathscr{B}_n(z)+\bar{g}_{n+1}\mathscr{A}_n(z);\tag{3.3}$$

for a polynomial p of degree n, we use the notation $p^*(z)=z^n\bar{p}(1/z)$, $\bar{p}(z)=\overline{p(\bar{z})}$.

There is a close connection between the polynomials $\mathscr{A}_n$ and $\mathscr{B}_n$ and orthogonal polynomials on the unit circle.

Thus, let μ be a positive measure on the unit circle $\mathsf{T}=\{|z|=1\}$ with $\mu(1)=1$ and

$$\mathscr{F}(z)=\frac{1}{2\pi}\int_0^{2\pi}\frac{e^{it}+z}{e^{it}-z}d\mu(t)$$

is an anlytic function in D with $Re\mathscr{F}(z)\geqq 0$. If we define $f(z)=(\mathscr{F}(z)-1)/(z(\mathscr{F}(z)+1))$, then f is a contractive analytic function in D. Applying the Schur algorithm (2.1) to f, one can state the following classical results (see [12])

$$\mathscr{F}(z)=\frac{z\psi_n(z)f_{n+1}(z)+\psi_n^*(z)}{-z\varphi_n(z)f_{n+1}(z)+\varphi_n^*(z)},\tag{3.4}$$

where $\{\varphi_n\}_{n=0}^{\infty}$ are the orthogonal polynomials associated to the measure μ and $\{\psi_n\}_{n=0}^{\infty}$ are the so-called polynomials of the second kind ([9, 12]). Moreover, for $n\geqq 1$

$$\Phi_n(z)=z\mathscr{A}_n^*(z)-\mathscr{B}_n(z)\tag{3.5}$$

$$\Psi_n(z)=z\mathscr{A}_n^*(z)+\mathscr{B}_n(z),\tag{3.6}$$

where $\Phi_n(\Psi_n)$ is obtained by dividing $\varphi_n(\psi_n)$ by its highest coefficient.

The aim of this section is to find the correspondents of these considerations for the situation analysed in Section 2. First, we recall the so-called Redheffer product ([15]). For

$$L=\begin{bmatrix} A, & B \\ C, & D \end{bmatrix} \quad \text{and} \quad L_1=\begin{bmatrix} A_1, & B_1 \\ C_1, & D_1 \end{bmatrix}$$

the Redheffer product is

$$L_1 * L=\begin{bmatrix} X, & Y \\ Z, & W \end{bmatrix}$$

where

$$X=A_1+B_1A(I-D_1A)^{-1}C_1$$

$$Y=B_1A(I-D_1A)^{-1}D_1B+B_1B$$

$$Z=C(I-D_1A)^{-1}C_1$$

$$W=C(I-D_1A)^{-1}D_1B+D$$

whenever the relevant inverse exists. We have $C_{L_1} \cdot C_L = C_{L_1 * L}$.

Consequently, from the algorithm (2.7) we deduce that

$$f=C_{U(\cdot,G_1)}C_{U(\cdot,G_2)}\cdots C_{U(\cdot,G_{n-1})}(f_n)$$

and if we define $\tilde{U}_n=U(\cdot, G_1)*U(\cdot, G_2)*\ldots*U(\cdot, G_n)$ then $\tilde{U}_n$ is an inner function in D (because $U(\cdot, G_n)$ are inner functions) and

$$f=C_{\tilde{U}_{n-1}}(f_n). \tag{3.7}$$

Moreover, if

$$\tilde{U}_n=\begin{bmatrix} A_n, & B_n \\ C_n, & D_n \end{bmatrix} : \mathcal{H} \oplus \mathcal{D}_{G_n^*} \to \mathcal{K} \oplus \mathcal{D}_{G_n}$$

then

$$A_1(z)=G_1, \; B_1(z)=zD_{G_1^*}, \; C_1(z)=D_{G_1}, \; D_1(z)=-zG_1^*$$

and, for $n \geqq 1$,

$$A_{n+1}(z)=A_n(z)+B_n(z)G_{n+1}(I-D_n(z)G_{n+1})^{-1}C_n(z) \tag{3.8}$$

$$B_{n+1}(z)=zB_n(z)G_{n+1}(I-D_n(z)G_{n+1})^{-1}D_n(z)D_{G_{n+1}^*}+zB_n(z)D_{G_{n+1}^*} \tag{3.9}$$

$$C_{n+1}(z)=D_{G_{n+1}}(I-D_n(z)G_{n+1})^{-1}C_n(z) \tag{3.10}$$

$$(3.11)\qquad D_{n+1}(z)=z(-G^*_{n+1}+D_{G_{n+1}}(I-D_n(z)G_{n+1})^{-1}D_n(z)D_{G^*_{n+1}})$$

(it is easy to see that $I-D_n(z)G_{n+1}$ is bounded invertible in D).

Let us state an important property of the functions $\{A_n\}_{n=1}^{\infty}$.

3.1. Theorem. *$A_n \to f$ as $n\to\infty$, and the convergence is uniform on the compact subset of the unit disc.*

Proof. As A_n is analytic and contractive in D let

$$A_n(z)=\sum_{k=1}^{\infty} A_k^{(n)}z^{k-1}$$

be its Taylor expansion. We will prove (and this will suffice) that

$$(3.12)\qquad A_k^{(n)}=T_k,\quad 1\leqq k\leqq n,\quad n\geqq 1.$$

We have

$$f(z)-A_n(z)=C_{U_{n-1}}(f_n)(z)-C_{U_{n-1}}(G_n)(z)$$

$$=B_{n-1}(z)(f_n(z)(I-D_{n-1}(z)f_n(z))^{-1}-G_n(I-D_{n-1}(z)G_n)^{-1})C_{n-1}(z).$$

But it is easy to see that $B_n(z)=z^n\tilde{B}_n(z)$, where $\tilde{B}_n$ is an analytic function in D and $\tilde{B}_n(0)\neq 0$ and, as $f_n(0)=G_n$, we get one more z in $f_n(z)(I-D_{n-1}(z)f_n(z))^{-1}-G_n(I-D_{n-1}(z)G_n)^{-1}$ in order to obtain (3.12). □

3.2. Remark. Theorem 3.1 can be viewed as an approximative (and also partial) Darlington synthesis for functions in $\mathcal{B}(\mathcal{H},\mathcal{K})$ (see, on this subject [6, 4],..., for exact Darlington synthesis see [2, 8]). □

3.3. Remark. We have to explain now (3.7) reduces to (3.1). That is when D_{G_n} are invertible operators for all $n\geqq 1$, left operatorial orthogonal polynomials are associated with the semispectral measure F on T determined by $\{G_n\}_{n=1}^{\infty}$ (see [5], also for additional references on matricial orthogonal polynomials).

Defining operatorial polynomials $\mathcal{A}_n$ and $\mathcal{B}_n$ by the formulas (3.5) and (3.6), we easily check that $\mathcal{A}_n$ are invertible operators for $z\in$ D, $A_n(z)=\mathcal{B}^*_n(z)\mathcal{A}_n^{-1}(z)$ and so on... □.

4. In this section we briefly describe an extension of the context when a Schur algorithm works. We mention some other extension in [7, 13].

Thus, we first note that to give an element $f\in\mathcal{B}(\mathcal{H},\mathcal{K})$ is equivalent to give a contractive analytic Toeplitz operator T_f of symbol f. S_+ is the right unilateral shift on $l^2(\mathrm{N},\mathcal{D}_{T^*})(T=f(0))$.

Then, the first step in the Schur algorithm (2.7) can be rewritten in the form:

$$(4.1)\qquad T_{f_1}=C_{R_D(G_1)}(T_{f_2})$$

where

$$(4.2) \qquad R_D(\hat{G}_1)=\begin{bmatrix} \hat{G}_1, & S_+\hat{D}_{G_1^*} \\ \hat{D}_{G_1}, & -S_+\hat{G}_1^* \end{bmatrix}$$

and $\hat{G}_1$ means the diagonal operator on $l^2(\mathsf{N}, \mathscr{H})$ with values into $l^2(\mathsf{N}, \mathscr{K})$, the diagonal element being G_1.

Now, let us consider the contraction

$$(4.3)\, C=\begin{Bmatrix} T_{00}, & 0, & \dots & \\ T_{01}, & T_{11}, & 0, & \dots \\ T_{02}, & T_{12}, & T_{22}, & \dots \end{Bmatrix} : l^2(\mathsf{N}, \bigoplus_{i=0}^{\infty} \mathscr{H}_i)\to l^2(\mathsf{N}, \bigoplus_{i=0}^{\infty} \mathscr{K}_i)$$

A Schur algorithm for this operator is the following:

$$(4.4) \qquad \begin{cases} C_1=C, \ \hat{G}_n = \text{the 0-principal diagonal of } C_n, \ n\geqq 1 \\ C_n=C_{R_D(\hat{G}_n)}(C_{n+1}) \,. \end{cases}$$

We obtain an analogue of Theorem 2.2.

4.1. Theorem. *There exists a one-to-one correspondence between the set of the contractions* C *given by (4.3) and the set of the sequences of contractions* $\{G_{ij}/_{i=1,\infty}, \alpha\geqq i\}$ $G_{ii}\in \mathscr{L}(\mathscr{H}_i, \mathscr{K}_i)$, $G_{ij}\in \mathscr{L}(\mathscr{D}_{G_{i+1,j}}, \mathscr{D}_{G^*_{i,j-1}})$, for $j>i$. □

4.2. Remark. We can obtain an explicit connection between T_{ij} and G_{ij}, also in the form that T_{ij} belong to certain operatorial balls.

We do not give here these formulas because they readily follows from the analysis in [5]. □

4.3. Remark. The operators of the form (4.3) appear as transfer operators for time-variant linear systems (with the time given by N) (see, for instance [10]).

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