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Zeros of Quasi-Polynomials and Linear Time-Optimal Control Problem

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An algorithm is presented for finding all the sign changes of the real-valued quasi-polynomial $f(t)=\sum_{k=1}^{n} c_k \exp(\lambda_k t)$ in a given interval, where c_k and λ_k are complex numbers. Such a question arises in numerical solving a linear time-optimal control problem when determining switches of controls. Particular attention is paid to the localization of the desired points. The idea of the algorithm is based on the fact that by means of factorization applied to the appropriate differential operator the problem can be reduced to a problem of finding the zeros of quasi-polynomials with smaller number of summands.

1. We consider a linear time-optimal control problem in the following form: to synthesize a control $u=u(t)\in U$, which steers in a minimal time t_1 the solution of the equation

$$\dot{x}(t)=Ax(t)+Bu(t), \quad x(0)=x_0(\neq 0)$$

to the origin: $x(t_1)=0$. Here U is a parallelepiped in R^r, defined by the inequalities

$$(0>)\alpha_i \leqq U^i \leqq \beta_i \ (>0), \ i=1, 2, \ldots, r;$$

A and B are constant $n\times n$ and $n\times r$ matrices, respectively; $x(t)$ is an n-dimensional vector-valued function. It is assumed that the normality condition holds [1] and x_0 belongs to the controllable set (which is automatically fulfilled in case of a stable A). It is well-known that under these conditions the desired optimal control exists, that it is unique (to within a set of measure zero) and is represented by piecewise constant function, whose values coincide with the vertices of U. The points t in which $u(t)$ has jumps are called switches. According to the Pontryagin Maximum Principle $u(t)$ is uniquely determined by the condition

$$\psi(t)Bu(t)=\max_{u\in U}\psi(t)Bu,$$

where $\psi(t)$ is the solution of the adjoint system $\dot{\psi}=-A^*\psi$ with some initial condition $\psi(0)=\psi_0$. The value of ψ_0 is a priori unknown and this is the main

Thus principal directions and principal curvatures in p and q are related by

(2) $$0=\langle v_p, v_q\rangle(k_q-k_p),$$

implying that principal directions of different principal curvatures in p and q are perpendicular to each other.
Restricting our considerations to a small neighbourhood of a deliberately chosen point in M, we can assume that there are no orthogonal pairs of tangent planes in this neighbourhood. Now take an orthonormal system $e_1,\ldots, e_{n-1}$ of principal directions of M in p and add $e_n:=N_p$. Then (2) implies

(3) $$v_q=\sum_{i\in I} c_ie_i+c_ne_n,$$

where the sum is taken over all i, such that e_i has k_q as eigenvalue. Since by the assumptions on the tangent planes made above v_q cannot be proportional to e_n, we see that the multiplicities of k_q as principal curvature of M in p and q are equal. Furthermore taking a second principal curvature $\tilde{k}_q$ of M in q with principal direction $\tilde{v}_q$ we get from $k_q\neq\tilde{k}_q$ and (3)

$$0=\langle v_q, \tilde{v}_q\rangle=c_n\tilde{c}_n.$$

If $N_p\neq N_q$, this implies that the orthogonal projection of N_q-N_p onto the tangent plane of M in p is proportional to a principal direction of M at p, i.e., the orthogonal projection on this hyperplane of the local image of the Gauss map around p spans a starlike subset of the set of eigenvectors of A_p. Then the projected Gauss image must contain a submanifold of dimension μ and 0 is principal curvature of M in p of multiplicity $n-\mu-1$. Therefore the starlike set defined above must be the eigenspace of the uniquely defined second principal curvature of M in p of multiplicity μ in the case $\mu\neq 0$. Considering the other properties of the principal curvatures and directions given above we have that M is umbilical or has exactly two constant principal curvatures of constant multiplicity, one of them being 0. Standard arguments ([3], [4]) lead to the product decomposition proposed in the theorem. The other direction of the proof is obvious.

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