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The Fréchet Space Structure of Global Sections of Certain Coherent Analytic Sheaves

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In this work we obtain some results on the Fréchet space structure of sections of coherent analytic sheaves on Stein spaces. In particular we are interested in sequence space representations of global sections of certain coherent analytic sheaves.

The solution of the basic problem for nuclear Fréchet spaces in the negative by B. S. Mitiagin and N. M. Zobin [18] (c.f. also [7]) and the recent progress made in the structure theory of nuclear Köthe spaces ([31]) initiated the study of sequence space representations of "concrete" spaces ([27], [28], [30], [2], [4], [16]). This problem has two parts. The first part deals with the question of existence of bases and, if this is answered in the positive, the second part deals with the determination of the matrix of the associated Köthe sequence space. In this work we obtain sequence space representations of global sections of certain coherent analytic sheaves.

After establishing terminology and notation, in the first section we obtain some results on the Fréchet space structure of sections of coherent analytic sheaves on Stein spaces. We introduce the concept of a sheaf of type (DN) (I. Definition) and give a characterization of Stein spaces for which the structure sheaf is of type (DN). We also give some examples of coherent analytic sheaves of type (DN) and determine the associated exponent sequences (see section 0) for this class.

In the second section we deal with sheaves on Stein spaces X for which $\mathcal{O}(X)$ is isomorphic to a finite type power series space. Stein manifolds which admit a proper negative plurisubharmonic function are in this class. We obtain a sequence space representation for global sections of sheaves of type (DN) over such spaces.

In the third section we consider locally free sheaves and subsheaves of the structure sheaf on Stein spaces X for which $\mathcal{O}(X)$ is isomorphic to an infinite type power series spaces. The main tool used in this section is the generalized decomposition method given in [4]. We give a sequence space representation of global sections of such sheaves and consider some examples. This research is partially supported by the Turkish Scientific and Technical Research Council.

§0. We shall use the standard terminology and notation of the theory of locally convex spaces as in ([15]), and ([23]). For an infinite matrix $A=(a_{j,k})_{j,k\in\mathbb{N}}$,

$0 \leqq a_{j,k} \leqq a_{j,k+1}$, $\sup_k a_{j,k} > 0$ for all j, k, the Köthe sequence space $\lambda(A)$ is defined by

$$\lambda(A) = \{x = (x_i)_i \in \mathbf{C}^{\mathbf{N}};\ \|x\|_k = \sum_j |x_j|\, a_{j,k} < +\infty \text{ for all } k\}.$$

Equipped with $\{\|\ \|_k\}$, $\lambda(A)$ is a Fréchet space. It is nuclear if and only if for every k there exists p such that $(\frac{a_{j,k}}{a_{j,p}})_j \in l_1$.

For a given increasing unbounded sequence of positive real numbers $\alpha = \{\alpha_n\}_n$ the Köthe spaces $\Lambda_1(\alpha) = \lambda(\{e^{-\frac{1}{k}\alpha_n}\}_{k,n})$ and $\Lambda_\infty(\alpha) = \lambda(\{e^{k\alpha_n}\}_{k,n})$ are called finite and infinite type power series spaces, respectively. The space of analytic functions on a polydisc

$$\Delta^d(R) = \{(z_1, \ldots, z_d) \in \mathbf{C}^d;\ \max_{1 \leqq i \leqq d} (|z_i|) < R\}$$

with the compact open topology is isomorphic as Fréchet spaces to $\Lambda_1(n^{\frac{1}{d}})$ if $0 < R < \infty$ and to $\Lambda_\infty(n^{\frac{1}{d}})$ if $R = \infty$ ([24]). For a Fréchet space $\{E, \|\ \|_k\}_k$ we will call $U_k = \{x \in E;\ \|x\|_k \leqq 1\}$ the *neighborhood corresponding to* $\|\ \|_k$, $k = 1, 2, \ldots$ and denote by E_m the completion of the normed space $(E, \|\ \|_m)$ provided $\|\ \|_m$ is a norm. A Fréchet space $\{E, \|\ \|_k\}$ is said to have the property $(\underline{DN})$, (DN), (Ω), $(\bar{\Omega})$ in case the corresponding statements below hold:

$$(\underline{DN}):\ \exists n_0\ \forall k\, \exists k, c, d:\qquad \|\ \|_k^{1+d} \leqq c\, \|\ \|_{n_0} \|\ \|_k^d$$

$$(DN):\qquad \exists n_0\ \forall k\, \exists k, c:\qquad \|\ \|_k^2 \leqq C\, \|\ \|_{n_0} \|\ \|_k$$

$$(\Omega):\ \forall k\, \exists n\ \forall p\, \exists j, C:\ U_n \subset C r^j U_p + \frac{1}{r} U_k,\ \forall r > 0$$

$$(\bar{\Omega}):\qquad \forall k\, \exists n \forall p \exists C:\ U_n \subset C r U_p + \frac{1}{r} U_k,\ \forall r > 0.$$

The above conditions are independent to the particular fundamental system of seminorms and $(\underline{DN})$, (DN) are inherited by subspaces, (Ω), $(\bar{\Omega})$ by quotient spaces. These topological invariants were introduced by D. Vogt and M. J. Wagner ([33]) for the characterization of subspaces and quotient spaces of power series spaces (cf. [29]-[31]).

For two absolutely convex subsets U and V in a vector space E, such that U is absorbed by V, we define the n^{th} Kolmogorov diameter $d_n(U, V)$ as the infimum of all numbers $d > 0$ such that $U \subset dV + L$ for some, at most n-dimensional linear subspace L in E. The *diametral dimension* $\Delta(E)$ of a Fréchet space $\{E, \|\ \|_k\}_k$ is the set of all sequences (ξ_n) such that for every k there is a p with $\lim \xi_n d_n(U_p, U_k) = 0$.

A nuclear Fréchet space E satisfying the conditions ($\underline{DN}$) and (Ω) with an increasing sequence $\| \ \|_k$ of Hilbertian norms is said to be in *standard form* in case:

(1) $$\forall k \exists \lambda \in (0, 1), C>0 : \| \ \|_{k+1} \leqq C \| \ \|_k^{\lambda} \| \ \|_{k+2}^{1-\lambda}$$

(2) $$\forall k \exists j, C>0 : U_{k+1} \subset Cr^j U_p + \frac{1}{r} U_k, \quad \forall r>0.$$

For a nuclear Fréchet space E with properties ($\underline{DN}$) and (Ω) there is a unique (up to equivalence) sequence $\varepsilon(E)$, called the associated exponent sequence of E which has the property $\Lambda_1(\varepsilon(E)) \subset \Delta(E) \subset \Lambda_\infty(\varepsilon(E))'$. Therefore, if E is isomorphic to a power series space, then it is isomorphic to $\Lambda_1(\varepsilon(\mathrm{E}))$ or $\Lambda_\infty(\varepsilon(E))$, depending on the type of the power series space. If $\{E, \| \ \|_k\}$ is in standard form the associated exponent sequence is equivalent to $\{-\log d_n(U_p, U_k)\}_n$ for any p and k as long as $p>k$. For this and more information on associated exponent sequences we refer to [5].

We shall use the standard terminology and notation of complex analysis as in [10], [12]. Our main reference for the theory of Stein spaces is [9] but we will differ from the terminology of [9] at one point namely by a Stein space we mean a reduced Stein space in the sense of [9] which has a Hausdorff, separable topology.

Analytic functions on a Stein space equipped with the topology of uniform convergence on compact sets is a Fréchet space ([10, p. 158]). For a compact set K of X and a continuous function f on K, we will denote the supremum of $|f|$ on K by $\|f\|_K$. For a Stein space X the set of coherent analytic sheaves on X will be denoted by $\mathrm{Coh}(X)$. The sheaf of germs of analytic functions on X (the structure sheaf) will be denoted by $\mathcal{O}_X$. Let X be a Stein space and $\mathcal{F} \in \mathrm{Coh}(X)$. For a given holomorphically convex compact set K of X we can introduce a seminorm on the space of $\mathcal{F}$ over X, $\mathcal{F}(X)$, as follows. First, we choose a finite number of sections $\sigma_1, \ldots, \sigma_p \in \mathcal{F}(X)$ that generate $\mathcal{F}$ over a neighborhood of K'. Such sections exist in view of Cartan's theorem A. Then we set for

$$\sigma \in \mathcal{F}(X) : \|\sigma\|_K = \inf \max \|c_i\|_K,$$

where the infimum is taken over all $(c_i, \ldots, c_p)$ with the property that $\sigma = \Sigma c_i \sigma_i$ over a neighborhood of K. This definition depends upon the choice of $\sigma_1, \ldots, \sigma_p$ but another choice of generators will produce an equivalent seminorm ([12, p. 171]). We will put on $\mathcal{F}(X)$ the topology coming from these seminorms. With this topology $\mathcal{F}(X)$ is a nuclear Fréchet space ([10, p. 240]), (cf. [13]). We note that with respect to this topology, the maps induced from sheaf morphisms are automatically continuous (see [10, p. 237]).

§I. In this section we will investigate the structure of the nuclear Fréchet space $\mathcal{F}(X)$ for a coherent analytic sheaf $\mathcal{F}$, on a Stein space X. We will start with the property (Ω):

I.1. Proposition: *Let X be a Stein space and $\mathscr{F} \in \mathrm{Coh}(X)$. Then $\mathscr{F}(X)$ has the property* (Ω).

Proof: Let K be holomorphically convex compact subset of X and let $U \subset\subset X$ be an open neighborhood of K. The proposition will follow if we can show that for any holomorphically convex compact set $\tilde{K} \supset \bar{U}$, there exists $C>0$ and $j \in \mathrm{N}$ such that; for any $\sigma \in \mathscr{F}(X)$ with $\|\sigma\|_{\bar{U}} \leqq 1$ and $r>0$ there exists, $\sigma^r \in \mathscr{F}(X)$ satisfying;

(1) $$\|\sigma^r\|_{\tilde{K}} \frac{C}{r^j},$$

(2) $$\|\sigma^r - \sigma\|_K \leqq \frac{1}{r}.$$

To this end we fix a holomorphically convex compact set $\tilde{K} \supset U$. We choose a domain (Oka-Weil domain) $\mathscr{P}$ with $K \subset \mathscr{P} \subset \bar{\mathscr{P}} \subset U$ and a holomorphic mapping Φ from X into some C^N such that $\Phi|\mathscr{P}$ is a biholomorphic mapping of $\mathscr{P}$ onto a closed subvariety V of $\Delta(1) \subseteq \mathrm{C}^N$ ([10, p. 211]). Let $0<r_0<1$, $R>0$ be chosen so that $\Phi(K) \subset \Delta(r_0)$ and $\Phi(K) \subset \Delta(R)$.

In view of Cartan Thm. A. there exists a finite number of global sections of $\mathscr{F}$, say $\sigma_1, \ldots, \sigma_{p \in \mathscr{F}(X)}$ whose germs generate $\mathscr{F}_x$ at each point x of a relatively compact Stein neighborhood of K ([10, p. 224]).

Now let $\sigma \in \mathscr{F}(X)$, $\|\sigma\|_{\bar{U}} \leqq 1$, be given. Since for each $x \in \dot{U}$, $\mathscr{F}_x$ is generated by the germs of $\sigma_1, \ldots, \sigma_p$ at x, we can find p, analytic functions $f_1, \ldots, f_p \in \mathcal{O}(\mathscr{P})$ and a $C>0$ that does not depend upon σ, such that

$$\sigma = \sum_{i=1}^{p} f_i \sigma_i \text{ with } \max \|f_i\|_{\mathscr{P}} \leqq C$$

([10, p. 244]). Since the restriction operator from $\mathcal{O}(\Delta(1))$ to $\mathcal{O}(V)$ is onto ([10, p. 245]) for s with $r_0<s<1$, we can, in view of the open mapping theorem, find a constant, $C>0$ and p analytic functions $F_1, \ldots, F_p \in \mathcal{O}(\Delta(1))$, such that

(3) $$\max_i \|F_i\|_{\Delta(s)} \leqq C,$$

(4) $$F_i|_V = f_i \circ \Phi^{-1}, \quad i=1, \ldots, p,$$

where Φ^{-1} denotes the inverse of the biholomorphism $\Phi: \mathscr{P} \to V$.

Now let $r>0$ be given. By considering a suitable decomposition of the Taylor series expansion of F_i, $i=1, \ldots, p$ ([26, 3.3 Lemma]) we can find, $C>0$, $j \in \mathrm{N}$, and $G_i^r \in \mathcal{O}(\mathrm{C}^N)$, $i=1, \ldots, p$, such that

(5) $$\|G_i^r - F_i\|_{\Delta(r)} \leqq \frac{1}{r},$$

(6) $$\|G_i\|_{\Delta(R)} \leqq Cr^j, \quad i=1, \ldots, p.$$

Now set $\sigma^r = \Sigma_{i=1} (G_i^r \circ \Phi)_{\sigma_i}$. Then $\sigma^r \in \mathscr{F}(X)$ and in view of (3)-(6) it satisfies (1) and (2). The proof of the proposition is finished.

Remarks: (1) In the case when X is a Stein manifold I.1. Proposition is due to D. Vogt ([32, 2.4 Satz]). He first proves the assertion for the special case $X = \mathbf{C}^n$ and then uses the imbedding theorem for Stein manifolds to get the result. (2) In the case when $\mathscr{F}$ is the structure sheaf $\mathcal{O}$, I.1. Proposition is due to D. Vogt ([14, Theorem 2]), and M. Schottenloher ([25]).
(3) The proof of I.1. Proposition also demonstrates the amount of freedom in the choice of U_q when U_p is given as in the statement of the property (Ω). This is a typical situation that comes up very often when one investigates Ω-type conditions for analytic objects.

We now consider the property $(\underline{DN})$. Now every Fréchet space of the form $\mathscr{F}(X)$, (X a Stein, $\mathscr{F} \in \mathrm{Coh}(X)$) enjoys the property $(\underline{DN})$ since such a space need not even possess a continuous norm. For example, take the closed subvariety X of $\mathbf{C}^2$ that is made up of the union of an infinite number of vertical complex lines with a horizontal complex line, then although X is connected, $\mathcal{O}(X)$ does not have a continuous norm. We make the following definition.

I.2. Definition. Let X be a Stein space and $\mathscr{F} \in \mathrm{Coh}(X)$. Then $\mathscr{F}$ is said to by of type $(\underline{DN})$ in case the Fréchet space $\mathscr{F}(X)$ has the property $(\underline{DN})$.

Our next result gives a characterization of Stein spaces for which the structure sheaf is of type $(\underline{DN})$.

I.3. Proposition: For a Stein space X, the following are equivalent:

(1) $\mathcal{O}(X)$ has a continuous norm.

(2) X has finite number of irreducible components.

(3) $\mathcal{O}(X)$ is topologically and algebraically ismorphic to a closed subalgebra $(\mathcal{O}(\Delta^d(1)))^k$ for some d and k.

(4) $\mathcal{O}_X$ is of type $(\underline{DN})$.

Proof: Since the implications (3)⇒(4), (4)⇒(1) are clear we only need to prove the implications (1)⇒(2) and (2)⇒(3).

(1)⇒(2). We fix a compact set K such that $\| \ \|_K$ is a norm on $\mathcal{O}(X)$. With the anticipation of a contradiction we assume that there are infinite number of irreducible components $\{V_i\}_i$. There are only finitely many irreducible components that intersect K ([10, p. 155]) say $\{V_{i},\ldots, V_{i_s}\}$. We set $W = V_{i_1} \cup \ldots \cup V_{i_s}$. Then W is a proper closed subvariety of X and in view of Cartan Thm. A. there exists an analytic function $f \in \mathcal{O}(X)$ not identically zero such that $f|_W = 0$. In particular since $K \subset W$, $\|f\|_K = 0$, and $f \not\equiv 0$. This contradicts our choice of K.

(2)⇒(3). Let V be an irreducible component of X and suppose that the dimension of V is s. Fix a point $p \in X$. In a neighborhood around p, V is biholomorphically equivalent to a variety, which we will denote by V_p, in an open set containing O (the point identified with p) of $\mathbf{C}^N$ for some N. In view of the local parametrization theorem ([10, p. 98, 10. Theorem]), after a change of coordinates if necessary, there exists a polydisc around 0, $\Delta^N = \Delta^{N-s} \times \Delta^s$ such that the projection π, onto the last s coordinates is a proper mapping from $V_p \cap \Delta^N$ onto Δ^s and there exists a variety V'_p of Δ^s such that the singular elements of $V_p \cap \Delta^N$ are contained in $\pi^{-1}(V'_p)$. We can find a smaller polydisc around 0 in Δ^s, say B, such that the distinguished boundary, Γ, of B lies in $\Delta^s - V'_p$ ([19, p. 6]). Let us denote by $A(\pi^{-1}(\bar{B}) \cap V_p)$ the closed algebra of functions on $\pi^{-1}(\bar{B}) \cap V_p$ which can be approximated uniformly on $\pi^{-1}(\bar{B}) \cap V_p$ by functions analytic in some neighborhood of $\pi^{-1}(\bar{B}) \cap V_p$. Set $\hat{\Gamma} = \pi^{-1}(\Gamma) \cap V_p$. Then $\hat{\Gamma}$ is compact and is disjoint from the set of singular points of V. For each point $\xi \in (\pi^{-1}(\bar{B}) \cap V_p) \setminus \hat{\Gamma}$ we can find an analytic variety through ξ lying in $\pi^{-1}(\bar{B}) \cap V_p$ and containing ξ as a nonisolated point. This can be done by taking the inverse image under π of suitable variety in Δ^s and using the fact that as U varies over a neighborhood basis of ξ, $(U, \pi, \pi(U))$ is an analytic cover ([10, p. 103]). In view of the maximum modulus principle for varieties, arguing like ([11, Theorem 1]), we see that all the peak points of $A(\pi^{-1}(\bar{B}) \cap V_p)$ lie in $\hat{\Gamma}$. Since $\hat{\Gamma}$ is compact we conclude that $\hat{\Gamma}$ is a boundary of $A(\pi^{-1}(\bar{B}) \cap V_p)$.

Now given a compact set K of V the argument above and a compactness argument yields a compact subset of V disjoint from the singular points of V such that

$$(1) \qquad \sup_{z \in K} |f(z)| \leqq \sup_{\xi \in L} |f(\xi)|, \quad \forall f \in \mathcal{O}(V).$$

Let us denote the set of regular points of V by M. Then M is an open dense subset of V and is a connected complex manifold of dimension s. The restriction mapping from $\mathcal{O}(V)$ into $\mathcal{O}(M)$ is clearly a continuous and an injective algebra morphism. Moreover, in view of (1) and the fact that $\mathcal{O}(V)$ is a Montel space, R also has closed range.

Since $\mathcal{O}(M)$ is topologically and algebraically isomorphic to a closed subalgebra of $\mathcal{O}(\Delta^s(1))$ ([3, Lemma 4]) we see that $\mathcal{O}(V)$ is isomorphic as topological algebra to a closed subalgebra of $\mathcal{O}(\Delta^s(1))$.

Now let $X = V_1 \cup \ldots \cup V_k$ be the decomposition of X into its irreducible components. Let $d = \dim X$, and $s_i = \dim V_i$, $i = 1, \ldots, k$. Then $d = \max_{1 \leqq i \leqq k} s_i$. The mapping that sends $f \in \mathcal{O}(X)$ to $(f|_{V_1}, \ldots, f|_{V_k})$ is a continuous algebra morphism of $\mathcal{O}(X)$ onto a closed subalgebra of $\Pi_{i=1}^k \mathcal{O}(V_i)$. Since each $\mathcal{O}(\Delta^s(1))$, can be identified in a natural way with a closed subalgebra of $\mathcal{O}(\Delta^d(1))$, $i = 1, \ldots, k$, by the above argument we see that $\mathcal{O}(X)$ can be imbedded as a topological algebra into $(\mathcal{O}(\Delta^d(1)))^k$. This finishes the proof of I.3 Proposition.

Remarks: (1) The proof of the implication (2)⇒(3) shows that for a Stein space X with finite number of irreducible components one can take in the statements 3, $d = \dim X$ and $k =$ number of irreducible components.
(2) Although the spaces $\mathcal{O}(\Delta^d(1))^k$, $k > 1$, $d \in \mathbb{N}$, and $\mathcal{O}(\Delta^d(1))$ are not isomorphic as topological algebras, they are certainly isomorphic as Fréchet spaces.

We do not know of a general criterion that characterizes coherent analytic sheaves of type ($\underline{DN}$). Nevertheless there are some techniques that can be used to prove that certain sheaves are of type ($\underline{DN}$). One such technique stems from E. Bishops proof of the "Three-Domains Theorem" in several complex variables ([20, p. 130]) and was already used by D. Vogt in ([29, § 5]). Arguing along the same lines as ([29, § 5]) we see that a locally free sheaf on an irreducible Stein spaces is of type ($\underline{DN}$). Another technique is to use the results of V. P. Palamodov ([21], [22]). Following the reasoning given by B. Mitiagin and G. Henkin ([17, Proposition 6.1]) one can show that if $\mathscr{F}$ is a coherent analytic sheaf on a Stein manifold Ω and if D is relatively compact pseudoconvex domain in Ω then $\mathscr{F}|_D$ is of type ($\underline{DN}$). Another important class of scheaves of type ($\underline{DN}$) can be obtained by taking the coherent analytic subsheaves of $\mathscr{O}_X^p$, $p \in \mathbb{N}$, for an irreducible Stein space X. This follows immediately from the "closure of modules" theorem of Cartan which implies for an analytic subsheaf $\mathscr{F}$ of $\mathscr{O}_X^p$ that the Fréchet space $\mathscr{F}(X)$ is a closed subspace of $\mathscr{O}(X)^p$ ([10, p. 235]).

We now turn to the problem of determining the associated exponent sequence of $\mathscr{F}(X)$ for a given $\mathscr{F} \in \mathrm{Coh}(x)$ of type ($\underline{DN}$)on a Stein space X. We will first give a general result which is quite useful in the task of determining associated exponent sequences.

I.4. Lemma: *Let $\{X, |\ |_k\}$, $\{Y, \|\ \|_k\}$ be two nuclear Fréchet spaces with the properties ($\underline{DN}$) and (Ω). We also assume that they are in standard form. Denoting the neighborhood basis of zero associated to the norm system $\{|\ |_k\}$, ($\{\|\ \|_k\}$)* by *$\{U_k\}$, ($\{V_k\}$); we have:*

(A). *If there exists a linear mapping $T: X \to Y$, integers k_1, k_2, $k_1 < k_2$ and constants c_1, c_2 with*

$$(1)\qquad \begin{aligned} &\overline{T(U_{k_2})} \supset c_1 V_s (\text{in } Y_0)\\ &T(U_{k_1}) \subset c_2 V_0 \end{aligned}$$

then

$$\varepsilon(X) = 0(\varepsilon(Y)).$$

(B). *If there exist a linear mapping $S: X \to Y$ integers, s_1, s_2, k, $s_1 < s_2$, and constants c_3, c_4 with*

$$(2)\qquad \begin{aligned} &S(X) \cap V_{s_1} \subset c_3 S(U_0)\\ &S(U_k) \subset c_4 V_{s_2} \end{aligned}$$

then

$$\varepsilon(Y) = 0(\varepsilon(X)).$$

Proof: We start by proving (**A**). To this end we fix some n and suppose that $U_{k_2} \subset dU_{k_1} + L$ for some n dimensional subspace L of X. Then

$$T(U_{k_2}) \subset dT(U_{k_1}) + T(L).$$

Since T is linear, $T(L)$ is a subspace of Y with dimensional at most n. From (1) we have

$$T(U_{k_2}) \subset c_2 d V_0 + T(L).$$

It follows that $d_n(\overline{T(U_{k_2})}, V_0) \leqq c_2 d_n(U_{k_2}, U_{k_1})$ where the closure is taken in Y_0. Using (1) again, and taking the logarithm of both sides we have; for some constant $C>0$,

$$-\log d_n(U_{k_2}, U_{k_1}) \leqq -\log d_n(V_s, V_0) + C.$$

Since $\{-\log d_n(U_{k_2}, U_{k_1})\}$ and $\{-\log d_n(V_s, V_0)\}$ are equivalent to $\{\varepsilon(X)_n\}$ and $\{\varepsilon(Y)_n\}$ respectively, (see, § 1) we have that $\varepsilon(X) = O(\varepsilon(Y))$. Part (**B**) of the lemma is proved by arguing exactly like in 1.1. Proposition of ([4]) and so will be omitted.

We recall that for a given Stein space X and $\mathscr{F} \in \mathrm{Coh}(X)$, the set $S(\mathscr{F}) = \{x \in X : \mathscr{F}_x \neq 0\}$ is a closed subvariety of X ([9, p. 241]) called the support of $\mathscr{F}$.

I.5. Proposition: *Let X be Stein space and $\mathscr{F}$ a coherent analytic sheaf of type $(\underline{DN})$ with $S(\mathscr{F}) = X$. Then the associated exponent sequence of $\mathscr{F}(X)$ is $\{n^{\frac{1}{d}}\}_n$ where $d < \infty$, is the dimension of X.*

Proof: Let $X = \bigcup_i V_i$ be the decomposition of X into its irreducible components. Since $\mathscr{F}$ is of type $(\underline{DN})$, there exists a holomorphically convex compact set K of X such that $\| \ \|_K$ is a norm on $\mathscr{F}(X)$. Choose an open neighborhood O of K which is relatively compact in X. There are only finite number of irreducible components that has nonempty intersection, with $\bar{O}$, say, $V_i, \ldots, V_{i_k}$. Set $W = V_{i_1} \cup \ldots \cup V_{i_k}$. In view of Cartan's theorem B there exists a $f \in \mathcal{O}(X)$, $f \neq 0$ such that $f|_W = 0$.

We choose a point $z_0 \in X \setminus W$ such that $f(z_0) \neq 0$, and fix a global section $\sigma \in \mathscr{F}(X)$ with $\sigma_{z_0} \neq 0$. Such a $\sigma \in \mathscr{F}(X)$ exists in view of Cartan's theorem A, since $\mathscr{F}_{z_0} \neq 0$.

Now the section $f.\sigma$ vanishes on O and so $\|f.\sigma\|_K = 0$, but $(f\sigma)_{z_0} \neq 0$ since f_{z_0} is a unit and $\sigma_{z_0} \neq 0$. Hence it follows that the number of irreducible components of X is finite. Therefore the dimension of X is finite. In particular $\mathcal{O}(X)$ has the property $(\underline{DN})$ (I.3. Proposition).

We choose an exhaustion of X with holomorphically convex domains $\{D_n\}$ such that $\{\mathcal{O}(X); \| \ \|_{K_n}\}$ and $\{\mathscr{F}(X), \| \ \|_{K_n}\}$ are in standard form where $K_n = \bar{D}_n$, $n = 1, \ldots,$. We denote the neighborhoods corresponding to these norms by $\{U_i\}$ and $\{V_i\}$ respectively.

Now fix a K_n, $n > 2$, and choose a finite number of global sections, $\sigma_1, \ldots, \sigma_p$, that generate $\mathscr{F}_x$ for every x in a neighborhood of K_n. We consider the sheaf mapping $\tau : \mathcal{O}^p \to \mathscr{F}$, induced by these sections (see §0), and denote the corresponding operator from $\mathcal{O}(X)^p$ into $\mathscr{F}(X)$ by T. Since D_n is a Stein space, the operator induced by T on sections over D_n is onto. In view of the open mapping theorem, there exists a $C>0$, such that for every $\sigma\in\mathscr{F}(X)$ with $\|\sigma\|_{K_n}\leqq 1$ there exists $f_1,\dots,f_p\in\mathcal{O}(D_n)$ such that $\max_i\|f_i\|_{K_{n-1}}\leqq C$ and $\sigma=\Sigma_{i=1}^p f_i\sigma_i$ on D_n. In view of the fact that $\mathcal{O}(X)$ is dense in $\mathcal{O}(D_n)$, ([10, p. 241]), we can find a $C>0$ with the property that, $\forall\varepsilon>0$ and $\sigma\in\mathscr{F}(X)$ $\|\sigma\|_{K_n}\leqq 1$ there exists $f_1^\varepsilon,\dots, f_p^\varepsilon\in\mathcal{O}(X)$, $\max_i\|f_i\|_{K_{n-1}}\leqq C$ such that

$$\sum_{i=1}^{p}\|f_i^\varepsilon\sigma_i-\sigma\|_{K_{n-1}}\leqq\varepsilon.$$

This clearly implies that $\overline{T(U_{K_{n-1}})}\supset\frac{1}{C}V_{K_n}$, where the closure is taken in $\mathscr{F}(X_0)$. Since we plainly have $T(U_0)\subset\tilde{c}V_0$ for some $c>0$, I.4. Lemma (**A**) implies that

$$\varepsilon(\mathcal{O}(X)^p)=O(\varepsilon(\mathscr{F}(X))).$$

On the other hand, I.3. Proposition (3) combined with I.4. Lemma (**B**) gives: $\{n^{\frac{1}{d}}\}=O(\varepsilon(\mathcal{O}(X))^p)$. Therefore we have: $n^{\frac{1}{d}}=O(\varepsilon(\mathscr{F}(X)))$.

We now proceed to show the reverse domination. To this end we choose a regular point z_0 such that $\mathscr{F}$ in a neighborhood around z_0 is free and the dimension of X at z_0 is d. We recall that such points form a dense subset of X ([9, p. 92]). Hence there exists a neighborhood U of z_0, global analytic functions $f_1,\dots,f_d\in\mathcal{O}(X)$ and global sections $\sigma_1,\dots,\sigma_p\in\mathscr{F}(X)$ such that;

(1) $(\sigma_i)_x$ forms a basis for the free module $\mathscr{F}_x$ for every $x\in U$.

(2) The mapping $\alpha: X\to\mathbf{C}^d$, $\alpha=(f_1,\dots,f_d)$ maps U onto $\Delta^d(2)$ biholomorphically.

Now define $T:\mathcal{O}(\mathbf{C}^d)^p\to\mathscr{F}(X)$ by the rule

$$T(h_1,\dots, h_p)=\sum_{i=1}^{p}(h_i\circ\alpha).\sigma_i.$$

Clearly T is a continuous linear operator so the second condition of I.4. Lemma (**B**) is satisfied for any neighborhood of zero in $\mathscr{F}(X)$. Let $K=\alpha^{-1}(\Delta^d(1))\subset U$, where α^{-1} is the inverse function of α, thought as mapping from U onto $\Delta^d(2)$. Now in view of (1) there exists a $C>0$ such that

$$\|T(h_1,\dots, h_p)\|_K\geqq C\max_{1\leqq i\leqq p}\|h_i\circ\alpha\|_K=C\max_{1\leqq i\leqq p}\|h_i\|_{\Delta^d(1)}.$$

Therefore I.4. Lemma (B) implies that $\varepsilon(\mathscr{F}(X))=O(\varepsilon(\mathcal{O}(C^d)^p))$. Since $\mathcal{O}(\mathbf{C}^d)^p$ is isomorphic to $\mathcal{O}(\mathbf{C}^d)$ as Fréchet spaces we conclude that $\varepsilon(\mathscr{F}(X))=0(\{n^{\frac{1}{d}}\})$. This finishes the proof of I.5. Proposition.

R e m a r k : Let X be a Stein space X with $\dim X=d<\infty$, then for any $\mathscr{F}\in\mathrm{Coh}(X)$ whose support is X, the second part of the argument above combined with I.1. Proposition of ([4]) yields $\Delta(\mathscr{F}(X))\subset\Delta(\mathcal{O}(\mathbf{C}^d))$.

We are now in a position to apply the structure theory of nuclear Fréchet spaces to our setting.

I.6. Theorem: *Let X be a Stein space and $\mathscr{F} \in \mathrm{Coh}(X)$ be a coherent analytic sheaf of type $(\underline{DN})$ whose support is X. Then*
(1) *$\mathscr{F}(X)$ is isomorphic to a closed subspace of $\mathscr{O}(\Delta^d(1))$ and is a quotient space of $\mathscr{O}(\mathbf{C}^{d+1})$ where $d<\infty$, is the dimension of X.*
(2) *There exists a closed subspace of $\mathscr{F}(X)$ isomorphic to $\mathscr{O}(\mathbf{C}^d)$, where d is as in* (1).

Proof: (1) follows from I.1. Proposition, I.5. Proposition and 3.4 Satz of ([33]), and 3.2 Satz ([30]), and (2) follows from I.1. Proposition, I.5. Proposition and 2.2. Theorem of ([5]).

§II. In this section we will investigate the structure of global sections of coherent analytic sheaves defined on Stein spaces X for which $\mathscr{O}(X)$ is a finite type power series space.

II.1. Proposition: *Let X be a Stein space and $\mathscr{F} \in \mathrm{Coh}(X)$. If $\mathscr{O}(X)$ has the property $(\bar{\Omega})$ so does $\mathscr{F}(X)$.*

Proof: We argue as in I.1. Proposition. Suppose that a neighborhood of zero of $\mathscr{F}(X)$, say U, is given. Let K_1 be a holomorphically convex compact subset of X associated with U, and choose K_2, $K_1 \subset K_2$, coming from the property $(\bar{\Omega})$ for $\mathscr{O}(X)$. We choose a holomorphically convex relatively compact domain G containing K_2 and fix a finite number of global sections say, $\sigma_1, \ldots, \sigma_p \in \mathscr{F}(X)$ such that the mapping $T: \mathscr{O}(G)^p \to \mathscr{F}(G)$ defined via $T(c_1, \ldots, c_p) = \Sigma_{i=1}^p c_i \sigma_i$ is an epimorphism of Fréchet spaces. In view of open mapping theorem there exists a seminorm $|\ |_q$ of $\mathscr{F}(G)$ and a constant $C_1 > 0$ such that $\forall \sigma \in \mathscr{F}(G)$ with $|\sigma|_q \leqq 1$, $\exists\, c_i \in \mathscr{O}(G)$ $i=1, \ldots, p$, such that $\|c_i\|_{K_2} \leqq C_1$, $i=1, \ldots, p$, and $G = \Sigma_{i=1}^p c_i \sigma_i$. We set $W = \{\sigma \in \mathscr{F}(X) : |\sigma|_q \leqq 1\}$ and fix an ε, $0 < \varepsilon < C_1$ whose value is to be determined later. Now let a neighborhood of zero, V, of $\mathscr{F}(X)$ be given, and suppose that K_3 is the compact set associated with this neighborhood. Let $\eta > 0$ be a positive number, and σ be a given element of W. Then $\sigma = \Sigma_{i=1}^p \tilde{c}_i \sigma_i$ for some $\tilde{c}_i \in \mathscr{O}(G)$, $\|\tilde{c}_i\|_{K_2} \leqq C_1$, $i=1, \ldots, p$. Arguing as in the proof of I.5. Proposition, we choose $c_i \in \mathscr{O}(X)$, $i=1, \ldots, p$, such that $\max_{1 \leqq i \leqq p} \|c_i - \tilde{c}_i\|_{K_2} \leqq \varepsilon$ and set $\tau = \Sigma_{i=1}^p c_i \sigma_i$. By our choice of K_2 we can find $\alpha_i \in \mathscr{O}(X)$, $\beta_i \in \mathscr{O}(X)$ $i=1, \ldots, p$ and a constant $C_2 = C_2(K_3) > 0$ such that $c_i = \alpha_i + \beta_i$, $i=1, \ldots, t$ with;

$$\|\alpha_i\|_{K_1} \leqq \frac{C_3}{\eta}, \quad \|\beta_i\|_{K_3} \leqq C_3 \eta, \quad i=1, \ldots, t. \tag{1}$$

We set $A = \Sigma_{i=1}^p \beta_i \sigma_i$ and $B = \sigma - \tau + \Sigma_{i=1}^p \alpha_i \sigma_i$. Then $A, B \in \mathscr{F}(X)$ and $A + B = \sigma$. In view of (1) and the choice of τ there exists constants C_3 and C_4 which depends upon K_1 and K_3 such that

$$\|B\|_{K_1} \leqq \left(\varepsilon + \frac{C_3}{\eta}\right) C_4 p \text{ and } \|A\|_{K_3} \leqq C_4 p \eta.$$

Now for a given $r>0$ we choose $\varepsilon \leqq \dfrac{1}{2rpC_4}$ and $\eta = 2pC_3C_4r$. With this choice of ε and η, we obtain,

$$\|B\|_{K_1} \leqq \frac{1}{r} \text{ and } \|A\|_{K_3} \leqq C_5 r \text{ for some } C_5>0.$$

Hence it follows that $\mathscr{F}(X)$ has the property $(\bar{\Omega})$. This completes the proof of II.1. Proposition.

II.2. Theorem: *Let X be a Stein space with the property that $\mathcal{O}(X)$ is isomorphic to a finite type power series space. Then for any coherent analytic sheaf $\mathscr{F}$ of type $(\underline{DN})$ with support X, the Fréchet spaces $\mathscr{F}(X)$ and $\mathcal{O}(\Delta^d(1))$ are isomorphic, where $d<\infty$ is the dimension of X.*

P r o o f : Since $\mathcal{O}(X)$ is isomorphic to a finite type power series space, it has the property (Ω). In view of II.1. Proposition, the nuclear Fréchet space $\mathscr{F}(X)$ has the properties $(\underline{DN})$ and $(\bar{\Omega})$. So $\mathscr{F}(X)$ is isomorphic to a finite type power series space ([30]). In view of I.5. Proposition the associated exponent sequence of $\mathscr{F}(X)$ is $\{n^{\frac{1}{d}}\}$. Therefore $\mathscr{F}(X)$ and $\mathcal{O}(\Delta^d(1))$ are isomorphic. This finishes the proof of II.2. Theorem.

II.3. Corollary: *Let X be a Stein space with the propety that $\mathcal{O}(X)$ is isomorphic to a finite type power series space. Then for any closed submodule M of $\mathcal{O}(X)^p$, $p \in \mathbb{N}$, M and $\mathcal{O}(\Delta^d(1))$, $d=\dim X$, are isomorphic as Fréchet spaces.*

P r o o f : There exists a coherent analytic subsheaf $\mathscr{M}$ of $\mathcal{O}^p$ such that $\mathscr{M}(X)=M$ ([6]). Since $\mathscr{M}$ is of type $(\underline{DN})$ the corollary follows from II.2. Theorem.

We recall that a continuous mapping between two topological spaces is called p r o p e r in case the inverse image of every compact set is compact. Investigations of the existence of proper holomorphic mappings from Stein spaces into special Stein spaces like balls or polydiscs is an active field of research in complex analysis.

II.4. Corollary: *Let X be a Stein space and $\mathscr{F}$ be a coherent analytic sheaf of type $(\underline{DN})$, on X with $S(\mathscr{F})=X$. If there exists a proper holomorphic mappings φ of X into a Stein space D and if $\mathcal{O}(D)$ is isomorphic to a finite type power series space then $\mathscr{F}(X)$ is isomorphic to $\mathcal{O}(\Delta^d(1))$, $d=\dim X$.*

P r o o f : The image $V=\varphi(X)$ is an analytic subvariety of D and the direct image sheaf $\mathscr{F}_*$ is a coherent analytic sheaf on V ([10, p. 162], [9, p. 207]). Since $\mathcal{O}(V)$ is a quotient space of $\mathcal{O}(D)$ it has the property $(\bar{\Omega})$. In view of, II.1. Proposition, $\mathscr{F}_*(V)$ has the property $(\bar{\Omega})$. But $\mathscr{F}(X)$ and $\mathscr{F}_*(V)$ are isomorphic as Fréchet spaces, hence $\mathscr{F}(X)$ also has the property $(\bar{\Omega})$. So the corollary follows from the proof of II.2. Theorem.

R e m a r k : The class of Stein spaces X for which $\mathcal{O}(X)$ is isomorphic to a finite type power series space can be characterized as those Stein spaces which admits a bounded pluri-subharmonic exhaustion function ([34], [2]). Among Stein spaces belonging to this class are; bounded convex domains in $\mathbf{C}^n$, smoothly bounded pseudoconvex domains in Stein manifolds, bounded complete Reinhardt domains (cf. [17], [2], [1]).

§III. In this section we consider coherent sheaves on Stein spaces X for which $\mathcal{O}(X)$ is isomorphic to an infinite type power series space. A natural example of such a Stein space is $\mathbf{C}^d$, $d \in \mathbf{N}$. Since any coherent analytic sheaf on a given Stein manifold can be realized as a coherent analytic sheaf on some $\mathbf{C}^N$, we cannot hope to obtain a result analogous to II.2. Theorem in this setting. We will restrict our attention to two particular classes of sheaves, namely subsheaves of the structure sheaf and locally free sheaves. We recall that the germs of holomorphic sections in a holomorphic vector bundle on a Stein space X is locally free ([9, p. 90]).

III.1. Theorem: *Let X be a Stein space with the property that $\mathcal{O}(X)$ is isomorphic to a power series space of infinite type. Let $\mathcal{F} \in \mathrm{Coh}(X)$ be either a locally free sheaf or a subsheaf of $\mathcal{O}^p$ for some p. Then the Fréchet spaces $\mathcal{F}(X)$ and $\mathcal{O}(\mathbf{C}^d)$ are isomorphic, where $d < \infty$ is the dimension of X.*

Proof: Let $\mathcal{F}$ be given, as in the statement of the theorem. We will show that $\mathcal{F}(X)$ has the property (DN). We first take up the locally free case. Since $\mathcal{O}(X)$ has the property (DN), there is a holomorphically convex compact set $K_0 \subset X$ corresponding to the dominating norm in the condition (DN). Choose a holomorphically convex domain $\Omega_0 \subset\subset X$, and set $S_0 = \bar{\Omega}_0$. Suppose now we are given a compact set $S_1 \supset S_0$. We choose a holomorphically convex domain $S_1 \subset \Omega_1 \subset\subset X$ and let $K_1 = \bar{\Omega}$. Since $\mathcal{O}(X)$ has the property (DN), there exists a compact set K_2, and $C > 0$, such that $\| \ \|_{K_1} \leqq C \| \ \|_{K_2}^{\frac{1}{2}} \| \ \|_{K_0}^{\frac{1}{2}}$. Now choose holomorphically convex domains Ω and Ω_2 such that $K_2 \subset \Omega_2 \subset\subset \Omega \subset\subset X$. We choose, as in II.1. Proposition, a sheaf morphism $\rho: \mathcal{O}^p \to \mathcal{F}$ over Ω. Since $\mathcal{F}_x$ is a free $\mathcal{O}_x$-module for every $x \in X$, the associated sheaf morphism from $\mathrm{Hom}(\mathcal{F}, \mathcal{O}^p)$ into $\mathrm{Hom}(\mathcal{F}, \mathcal{F})$ is onto ([10, p. 257], [9, p. 91]). Hence there exists a sheaf morphism $\tau: \mathcal{F} \to \mathcal{O}^p$ over Ω, such that $\rho \circ \tau$ is the identity. Passing to sections and keeping in mind that Ω_i's are Stein spaces we obtain the following commutative diagram:

$$\begin{array}{ccccc}
\mathcal{F}(\Omega) & \xrightarrow{\tau_*} & \mathcal{O}^p(\Omega) & \xrightarrow{\rho_*} & \mathcal{F}(\Omega) \\
\downarrow & & \downarrow & & \downarrow \\
\mathcal{F}(\Omega_2) & \xrightarrow{\tau_*} & \mathcal{O}^p(\Omega_2) & \xrightarrow{\rho_*} & \mathcal{F}(\Omega_2) \\
\downarrow & & \downarrow & & \downarrow \\
\mathcal{F}(\Omega_1) & \xrightarrow{\tau_*} & \mathcal{O}^p(\Omega_1) & \xrightarrow{\rho_*} & \mathcal{F}(\Omega_0) \\
\downarrow & & \downarrow & & \downarrow \\
\mathcal{F}(\Omega_0) & \xrightarrow{\tau_*} & \mathcal{O}^p(\Omega_0) & \xrightarrow{\rho_*} & \mathcal{F}(\Omega_0),
\end{array}$$

where the columns mappings are restrictions. It follows that C_1, C_2, C_3 are constants such that;

$$(1)\qquad \begin{aligned} &\|\rho_*(\xi)\|_{S_1} \leqq C_1 \|\xi\|_{K_1}, \ \forall \xi \in \mathcal{O}^p(\Omega) \\ &\|\tau_*(\sigma)\|_{K_0} \leqq C_2 \|\sigma\|_{S_0}, \ \forall \sigma \in \mathcal{F}(\Omega) \\ &\|\tau_*(\sigma)\|_{K_2} \leqq C_3 \|\sigma\|_{S_2}, \ \forall \sigma \in \mathcal{F}(\Omega). \end{aligned}$$

In view of (1) we have

$$\|\sigma\|_{S_1} = \|\rho_* \circ \tau_*(\sigma)\|_{S_1} \leqq C \|\tau_*(\sigma)\|_{K_1} \leqq C \|\tau_*(\sigma)\|_{K_0}^{\frac{1}{2}} \|\tau_*(\sigma)\|_{K_2}^{\frac{1}{2}} \leqq C \|\sigma\|_{S_0}^{\frac{1}{2}} \|\sigma\|_{S_2}^{\frac{1}{2}}$$

for every $\sigma \in \mathcal{F}(X)$.

Therefore $\mathcal{F}(X)$ has the property (DN). On the other hand, if $\mathcal{F}$ is a coherent subsheaf of $\mathcal{O}^p$, then the Fréchet space $\mathcal{F}(X)$ is a closed subspace of $\mathcal{O}(X)^p$ ([10, p. 235]), so clearly $\mathcal{F}(X)$ has the property (DN). So in both cases $\mathcal{F}(X)$ has the property (DN). Also $\mathcal{F}(X)$ has the property (Ω) (I.1. Proportion) and the associated exponent sequence of $\mathcal{F}(X)$ is $\{n^{\frac{1}{d}}\}$ (I.5. Proposition). So $\mathcal{F}(X)$ is isomorphic to $\mathcal{O}(\mathbf{C}^d)$ in view of 2.2 Theorem of ([4]). This concludes the proof of III.1. Theorem.

Arguing as in the proof of (II.3), but employing III.1. instead of II.2. we obtain the following analog of II.3. Corollary.

III.2. Corollary: *Let X be a Stein space with the property that $\mathcal{O}(X)$ is isomorphic to an infinite type power series space. Then for any closed submodule M of $\mathcal{O}(X)^p$, $p \in N$, M and $\mathcal{O}(\mathbf{C}^d)$,* $d = \dim X$, *are isomorphic as Fréchet spaces.*

Remark: The class of Stein space X for which $\mathcal{O}(X)$ is isomorphic to an infinite type power series space can be characterized as those Stein spaces on which every bounded plurisubharmonic function is constant ([4]). Among Stein space belonging to this class are: Parabolic open Riemann surfaces and algebraic subvarieties of $\mathbf{C}^N$ ([8]). In particular we have:

III.3. Corollary: *Let $X \subset \mathbf{C}^N$ be an algebraic variety. Then for any holomorphic vector bundle on X, the Fréchet space of all global holomorphic sections of this bundle is isomorphic to $\mathcal{O}(\mathbf{C}^d)$, $d = \dim X$.*

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