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Transformation of Diffusion Fields Created by Two Wiener Processes

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1. Problem statement and notation

Let $R_+ = [0,\infty)$, $(\Omega, \mathcal{A}, P)$ be a complete probability space, $s,t \in R_+$, $z = (s,t)$, $\mathcal{F} = \{\mathcal{F}_z,\ z \geq 0\}$ a flow of σ-algebras from this probability space, $w = \{w(z), z \geq 0\}$ be an $\mathcal{F}$-adapted Wiener field,

$$(1)\qquad \begin{aligned} &t_1 = s,\ t_2 = t,\ \partial_k = \partial/\partial t_k,\ k = 1,2,\\ &\partial_3 = \partial/\partial x,\ x \in R = (-\infty,\infty). \end{aligned}$$

Denote by $C(p,q,r)$ ($\hat{C}(p,q,r)$) the space of real functions $f(s,t,x)$ defined on $R_+^2 \times R$ and possessing the continuous derivatives $\partial_1^i f$, $\partial_2^j f$, $\partial_3^k f$ ($\partial_1^i\partial_2^j\partial_3^k f$, respectively), for every i,j,k satisfying the inequalities $0 \leq i \leq p$, $0 \leq j \leq q$, $0 \leq k \leq r$. The spaces $C(p,q)$ and $\hat{C}(p,q)$ of the real functions on R_+^2, as well as $C(p)$, $\hat{C}(p)$ on R_+ are determined in the same way.

Within the above notation:

$$(2)\qquad l_k = \partial_k + f_k\partial_3 + \frac{1}{2}(g_k)^2\partial_3^2,\quad k = 1,2,$$

where $f_k \in C(1,1,2) \cap \hat{C}(1,1,0)$, $g_k \in C(1,1,3) \cap \hat{C}(1,1,1)$, we also introduce a random $\mathcal{F}$-adapted value ξ_0 and two independent standard separable Wiener processes $w_k(t_k)$, $k = 1,2$, also $\mathcal{F}$-adapted.

Consider the problem of existence and construction of the diffusion field $\xi(z)$ determined by the system of Ito stochastic differential equations:

$$(3)\qquad d_k\xi = f_k(z,\xi)\,dt_k + g_k(z,\xi)\,dw_k(t_k),\ \xi(z_0) = \xi_0,\quad k = 1,2.$$

For the solution of the given problem we use the method of transformation of diffusion fields, which arises from the generalization of our method of diffusion processes transformation. The basic formulations of this paper have been previously reported without proofs [1]. Most of the notations adopted here are the same as in [2].

2. Problem (3) solvability condition

The Cauchy problem (3) is solvable if $\xi(z)$ is expressed in terms of Wiener processes by means of the path-independent line integral along the curve connecting the points z_0 and z.

As demonstrated in [3], the following conditions are necessary and sufficient for the Cauchy problem solution:

$$g_1 \partial_3 g_2 = g_2 \partial_3 g_1, \quad l_2 f_1 = l_1 f_2 \tag{4}$$
$$l_k g_r = g_r d_3 f_k, \quad k, r = 1, 2.$$

The last equality in (4) is supposed valid for $k, r = 1, 2$, $k \neq r$, i.e. actually it comprises two equalities. If, instead of these two, the following one is considered

$$g_1(\partial_2 g_1 + f_2 \partial_3 g_1) + g_2^2 \partial_3 f_1 \;=\; g_2(\partial_1 g_2 + f_1 \partial_3 g_2) + g_1^2 \partial_3 f_2,$$

then the Kolmogorov set of equations

$$l_k u = 0, u(z_0, x) = X_0(x), \quad k = 1, 2, \tag{5}$$

provided that the first two conditions in (4) hold, is solvable (for the class of limited functions). Further, [3] offers "all kinds of examples" of solvable systems (3) and (5).

However, the solvability conditions (4) of the Cauchy problem (3) may be simplified if we explicitly express f_k in terms of g_k. Then all the examples and particular cases mentioned in [3] would arise from this formula. All abovementioned may be presented in the form of a theorem.

Theorem 1 *For the solution of problem (3) it is necessary and sufficient that with $g_1 \in C(1,1,3) \cap \hat{C}(1,1,1)$, $g_1 > 0$, there exist functions $C_0 \in \hat{C}(1,1)$, $\mathcal{K}_j(t) \in C(1)$, $\mathcal{K}_j > 0$, $j = 1, 2$, by means of which the functions f_1, f_2, g_2 could be expressed by the following equalities:*

$$g_2 = g_1 \sqrt{\delta}, \quad \delta = [\mathcal{K}_1(s)\,\mathcal{K}_2(t)]^2, \tag{6}$$

$$f_k = g_{3-k} \left\{ 2^{-1} \delta^{1_k} \partial_3 g_{3-k} - \int_{x_0}^{x} \partial_k [1/g_{3-k}(z, y)]\, dy + C_k \right\}, \tag{7}$$

$$C_k \equiv \mathcal{K}_{3-k}^{1_k} \partial_k C_0, \quad k = 1, 2, \tag{8}$$

where δ and C_k are given according to the adapted notation, $1_k = (-1)^k$.

P r o o f. *Necessity.* Assume that the problem is solvable, then the last equality in (4) gives:

$$\partial_3 f_k = (g_{3-k})^{-1} l_k g_{3-k},$$
$$\partial_3^2 f_k = (g_{3-k})^{-1} \left[f_k \partial_3^2 g_{3-k} + \partial_3 \partial_k g_{3-k} + g_k(\partial_3 g_k) \partial_3^2 g_{3-k} + 2^{-1} g_k^2 \partial_3^3 g_{3-k} \right], \quad k = 1, 2.$$

Introducing these expressions into the second equality in (4), and taking into account the first one, we obtain:

$$\text{(9)}\quad \begin{aligned} \partial_2 f_1 + (f_2/g_2)\partial_1 g_2 + 2^{-1} g_2\partial_3\partial_1 g_2 &= \partial_1 f_2 + (\partial_2 g_1)(f_1/g_1) + 2^{-1} g_1\partial_2\partial_3 g_1, \\ g_1[\partial_2(f_1/g_1) - 2^{-1}\partial_2\partial_3 g_1] &= g_2[\partial_1(f_2/g_2) - 2^{-1}\partial_1\partial_3 g_2]. \end{aligned}$$

Furthermore, from the last equality in (4) we derive:

$$(g_{3-k})^2\,\partial_x \frac{f_k}{g_{3-k}} = \partial_k g_{3-k} + 2^{-1}(g_k)^2\partial_x^2 g_{3-k},$$

i.e., we have obtained equality (7), where $\delta = (g_2/g_1)^2$, with C_1, C_2, δ being functions not depending on x.

After corresponding transformations of (9), taking into account (7), we obtain: $\partial_{12} = \partial_1\partial_2$,

$$\text{(10)}\quad \begin{aligned} &g_1\left\{\sqrt{\delta}\int_{x_0}^{x}\partial_{12}(1/g_2)dy + (\partial_2\sqrt{\delta})\int_{x_0}^{x}\partial_1(1/g_2)dy - \partial_2(C_1\sqrt{\delta})\right\} = \\ &= g_2\left\{\frac{1}{\sqrt{\delta}}\int_{x_0}^{x}\partial_{12}(1/g_1)dy + \partial_1(\frac{1}{\sqrt{\delta}})\int_{x_0}^{x}\partial_2(1/g_1)dy - \partial_1(\frac{C_2}{\sqrt{\delta}})\right\}, \end{aligned}$$

$$\begin{aligned} &\sqrt{\delta}\partial_1(\frac{C_2}{\sqrt{\delta}}) - \partial_2(C_1\sqrt{\delta}) + \sqrt{\delta}\int_{x_0}^{x}\partial_{12}(\frac{1}{g_2})dy + \partial_2\sqrt{\delta}\int_{x_0}^{x}\partial_1(\frac{1}{g_2})dy - \\ &- \int_{x_0}^{x}\partial_{12}(\frac{1}{g_1})dy + (2\delta)^{-1}(\partial_1\delta)\int_{x_0}^{x}\partial_2(\frac{1}{g_1})dy = 0. \end{aligned}$$

At $x = x_0$, the condition follows:

$$\text{(11)}\quad \sqrt{\delta}\partial_1(C_2/\sqrt{\delta}) = \partial_2(C_1\sqrt{\delta}).$$

Assume that the derivative of the left-hand part of equality (10) vanishes, then a chain of transformations will give:

$$(\sqrt{\delta}/g_1)\partial_{12}(1/\sqrt{\delta}) + (\partial_2\sqrt{\delta})g_1^{-1}\partial_1(1/\sqrt{\delta}) = 0,$$

$$\partial_{12}\ln\sqrt{\delta} = 0,$$

i.e. equality (6) is proved: $\sqrt{\delta} = \mathcal{K}_1(s)\mathcal{K}_2(t)$.

Since $\sqrt{\delta} = g_2/g_1 = \mathcal{K}_1\mathcal{K}_2$, provided that $C_0 \in \hat{C}(2,2)$, we derive (8) from equality (11).

Sufficiency. Assume that there are functions $C_0 \in \hat{C}(1,1)$, $\mathcal{K}_1, \mathcal{K}_2 \in C(1)$ having continuous first derivatives, $\mathcal{K}_r > 0$, by which the functions C_k, g_2, f_k are expressed in (8), (6) and (7) respectively. Under this condition, it is necessary to prove that correlations (4) are identities. The first equality in (4) is patent. Now calculate the right-hand part of the last equality in (4) for $k = 1$, $r = 2$, having for (7) and (6):

$$g_1 > 0, \quad g_2\partial_3 f_1 = (g_2)^2[(2\delta)^{-1}\partial_3^2 g_2 - \partial_1(1/g_2)] + \\ + g_2(\partial_3 g_2)[(2\delta)^{-1}\partial_3 g_2 - \int_{x_0}^{x} [\partial_1(1/g_2)]dy + C_1].$$

This expression is also equal to $l_1 g_2$, therefore the last equality in (4) is proved for $k = 1$, $r = 2$. By analogy, it is proved that $l_2 g_1 = g_1 \partial_3 f_2$, $g_1 \in C(1,1,3) \cap \hat{C}(1,1,1)$.

Now consider the second equality in (4). First prove equality (9). After a number of transformations its left-hand part will take the form:

$$g_1[\partial_2(\frac{f_1}{g_1}) - \frac{1}{2}\partial_{31}g_1] = \mathcal{K}_1 g_1 \{\partial_{12}C_0 - \\ - \mathcal{K}_2 \int_{x_0}^{x} \partial_{12}(\frac{1}{g_1\mathcal{K}_1\mathcal{K}_2})dy - \mathcal{K}_2^{-1}(\partial_2\mathcal{K}_2)\int_{x_0}^{x} \partial_1(\frac{1}{\mathcal{K}_1 g_1})dy\right\}.$$

The right-hand part in (9) obtains the form:

$$g_2[\partial_1(\frac{f_2}{g_2}) - \frac{1}{2}\partial_{31}g_2] = g_1\{\mathcal{K}_1\partial_{12}C_0 + \mathcal{K}_1^{-1}(\partial_1\mathcal{K}_1)\int_{x_0}^{x} \partial_2(\frac{1}{g_1})dy - \int_{x_0}^{x} \partial_{12}(\frac{1}{g_1})dy\},$$

hence we can see that it coincides with the left-hand part, and thus equality (9) is proved.

To prove the second equality in (4), using (9) we apply the following transformations:

$$\begin{aligned} l_2 f_1 &= [\partial_2 + f_2\partial_3 + \frac{1}{2}(g_2^2)\partial_3^2] f_1 = \\ &= \partial_2 f_1 + f_2(g_2)^{-1} l_1 g_2 + \frac{1}{2} g_2[f_1\partial_3^2 g_2 + \partial_{31} g_2 + g_1(\partial_3 g_1)\partial_3^2 g_2 + \frac{1}{2}(g_1)^2\partial_3^3 g_2] = \\ \text{(12)} \quad &= \partial_1 f_2 + (f_1/g_1)\partial_2 g_1 + \frac{1}{2} g_1 \partial_{23} g_1 + (f_2/g_2)[f_1\partial_3 g_2 + \frac{1}{2}(g_1)^2\partial_3^2 g_2] + \\ &+ \frac{1}{2} g_2 f_1 \partial_3^2 g_2 + g_1(\partial_3 g_1)(\partial_3^2 g_2)(g_2/2) + \frac{1}{4} g_1^2 g_2 \partial_3^3 g_2 = \partial_1 f_2 + f_1\partial_3 f_2 + \alpha, \end{aligned}$$

$$\begin{aligned} \alpha &= \frac{1}{2} g_1 \partial_{32} g_1 + (2g_2)^{-1} f_2 (g_1\partial_3)^2 g_2 + \frac{1}{2} g_1 g_2 (\partial_3 g_1)\partial_3^2 g_2 + \frac{1}{4} g_2 (g_1)^2 \partial_3^3 g_2 = \\ &= \frac{1}{2} g_1^2 [\partial_{32} g_1 + g_2(\partial_3 g_1)\partial_3^2 g_2 + \frac{1}{2} g_1 g_2 \partial_3^3 g_2 + f_2 \partial_3^2 g_1] g_1^{-1} = \frac{1}{2}(g_1\partial_3)^2 f_2. \end{aligned}$$

Now it is obvious that (12) is equivalent to the second equality in (4) and hence the theorem is proved.

3. Wienerization of a Special Diffusion Field

Consider a simultaneous stochastic system (3). As it arises from the already proved Theorem 1, the functions f_k, g_k, $k = 1,2$ provide the existence of certain functions $\mathcal{K}_k(t_k)$, $k = 1,2$ and C_0 of the corresponding classes, by means of which f_1, f_2, g_2 could be expressed in terms of g_1 according to the formulas:

$$g_2 = \mathcal{K}_1\mathcal{K}_2 g_1,$$

$$(13) \qquad \begin{aligned} f_1(z,x) &= g_1(z,x)\{\mathcal{K}_1(s)\partial_s C_0(z) + 2^{-1}\partial_3 g_1(z,x) - \\ &- \mathcal{K}_1(s)\mathcal{K}_2(t)\int_{x_0}^{x}\partial_s[1/g_2(z,y)]dy\}, \\ f_2(z,x) &= g_2(z,x)\{[1/\mathcal{K}_2(t)]\partial_2 C_0(z) + 2^{-1}\partial_3 g_2(z,x) - \\ &- [\mathcal{K}_1(s)\mathcal{K}_2(t)]^{-1}\int_{x_0}^{x}\partial_t[1/g_1(z,y)]dy\}. \end{aligned}$$

In order to construct a diffusion field $\xi(z)$ which is the solution of the set of equations (3), we shall apply the field $\eta(\tilde{z})$ obtained from the formulas:

$$(14) \qquad \begin{aligned} \tau_i &= \phi_i(t_i), \quad i = 1,2, \quad \tilde{z} = (\tau_1,\tau_2), \\ \eta(\tilde{z}) &= \psi(\psi_1(\tau_1), \psi_2(\tau_2), \xi(\psi_1(\tau_1), \psi_2(\tau_2))), \end{aligned}$$

where ϕ_i are continuously differentiable functions for $t_i \in R_+$, $\phi_i'(t_i) > 0$, $\phi_i(R_+) = R_+$, $i = 1,2$, while $\psi \in C(1,3)$.

Introduce certain Wiener processes

$$(15) \qquad \tilde{w}_i = \{\tilde{w}_i(\tau_i), \tau_i \in R_+\}, \quad i = 1,2,$$

and adopt the notation ψ_0, ψ_1, ψ_2 for the functions inverse to ψ, ϕ_1, ϕ_2 respectively. Thus, for every $x \in R$, $t_1, t_2 \in R_+$, $i = 1,2$:

$$\begin{aligned} \psi_0(z, \psi(z,x)) &= x = \psi(z, \psi_0(z,x)), \\ \psi_i(\phi_i(t_i)) &= t_i = \phi_i(\psi_i(t_i)). \end{aligned}$$

The problem is equivalent to constructing the functions ϕ_1, ϕ_2, ψ, such that for $\eta_0 = \psi(0,0,\xi_0)$ there exist independent Wiener processes (15), for which

$$(16) \qquad \begin{aligned} &\tilde{d}_k\eta = \sqrt{2}\tilde{d}_k\tilde{w}_k(\tau_k), \quad \tilde{d}\eta(\tilde{z}) = \sqrt{2}[\tilde{d}_1\tilde{w}_1(\tau_1) + \tilde{d}_2\tilde{w}_2(\tau_2)], \\ &\eta(\tilde{z}) = \eta_0 + \sqrt{2}[\tilde{w}_1(\tau_1) + \tilde{w}_2(\tau_2)], \quad k = 1,2 \end{aligned}$$

hold.

With these functions having been constructed, the solution of system (3) takes the form:

$$\xi(z) = \psi_0(z, \eta(\bar{z})) = \psi_0(z, \eta_0 + \sqrt{2}[\tilde{w}_1(\phi_1(s)) + \tilde{w}_2(\phi_2(t))]). \tag{17}$$

Theorem 2. *Provided that solvability conditions are observed, then system (3) is equivalent to system (16), and the transformation functions satisfy:*

$$\left.\begin{array}{c} \phi_1(s) = \int_0^s [\mathcal{K}_1(u)]^{-2}\, du, \\ \phi_2(t) = \int_0^t [\mathcal{K}_2(u)]^2\, du, \\ \psi(z, x) = \sqrt{2}\{[1/\mathcal{K}_1(s)] \int_{x_0}^x [1/g_1(z, y)] dy - C_0(z) + C(0, 0)\}, \end{array}\right\} \tag{18}$$

if we assume that $\psi(z, R) = R$.

Proof. To apply the result of [4], introduce the notation:

$$\alpha_k = (1/\sqrt{2}) g_k, \quad \beta_k = g_k \int_{x_0}^x \frac{dy}{g_k(z, y)},$$

$$\gamma_k = 2 f_k + 2 g_k \int_{x_0}^x [\partial_k (1/g_k)]\, dy - \partial_3[(1/2)(g_k)^2],$$

$$\Delta_k = \begin{vmatrix} \alpha_k & \beta_k & \gamma_k \\ \partial_3 \alpha_k & \partial_3 \beta_k & \partial_3 \gamma_k \\ \partial_3^2 \alpha_k & \partial_3^2 \beta_k & \partial_3^2 \gamma_k \end{vmatrix}, \quad k = 1, 2.$$

As $\gamma_1 = \alpha_1[2\sqrt{2}\mathcal{K}_1(\partial_1 C_0)] + 2\beta_1 \partial_1 \ln \mathcal{K}_1$, it is clear that $\Delta_1 = 0$. Hence, in order to bring the first equation in (3) to the form of the first equation in (16), we may construct the functions $\phi_1(t_1)$, $\psi(z, x)$ using the formulas from [4]. For this purpose we shall introduce additional notation:

$$W_k = \begin{vmatrix} \alpha_k & \gamma_k \\ \partial_3 \alpha_k & \partial_3 \gamma_k \end{vmatrix}, \quad P_k = \begin{vmatrix} \beta_k & \gamma_k \\ \partial_3 \beta_k & \partial_3 \gamma_k \end{vmatrix}, \quad k = 1, 2.$$

As $W_1 = \sqrt{2} g_1 \partial_1 \ln \mathcal{K}_1$, $(\alpha_1)^{-1} W_1 = \partial_1 \ln(\mathcal{K}_1^2)$, then

$$\phi(s) = \int_0^s \exp[-\int_0^{\tilde{u}} \frac{1}{\alpha_1(u, x)} W_1(u, x) du] d\tilde{u} = [\mathcal{K}_1(0)]^2 \int_0^s [\mathcal{K}_1(u)]^{-2} du.$$

From $P_1/\alpha_1 = -2\sqrt{2}\mathcal{K}_1 \partial_1 C_0$, we obtain that

$$\psi(s, x) = \sqrt{\phi'(s)} \frac{\beta_1}{\alpha_1} + \frac{1}{2} \int_0^s \frac{P_1(u, x)}{\alpha_1(u, x)} \exp[-\frac{1}{2} \int_0^u \frac{W_1(\tilde{u}, x)}{\alpha_1(\tilde{u}, x)} d\tilde{u}] du =$$

$$= \sqrt{2}\mathcal{K}_1(0)[\frac{1}{\mathcal{K}_1(s)} \int_{x_0}^x \frac{1}{g_1(z, y)} dy - C_0(z) + C_0(0, t)].$$

The functions $\phi(s)$ and $\psi(s,x)$ may be divided by constant factors: the former by $[\mathcal{K}_1(0)]^2$, the latter by $\mathcal{K}_1(0)$. Moreover, a constant summand $\sqrt{2}[C_0(0,0) - C_0(0,t)]$ may be added to the function ψ. All these transformations will not affect the basic properties of the transformation functions. As a result, we obtain the first and the third formula in (18).

It remains to prove that using the functions $\phi_2(t)$ and $\psi(z,x)$ in (18), the second equation of the system (3) takes the form of the second equation of the system (16), i.e., $k=2$.

For this purpose we shall find the derivatives of the functions $\phi_2(t)$, $\psi(z,x)$ at a fixed τ_1:

$$\phi_2'(t) = [\mathcal{K}_2(t)]^2,$$

(19)
$$\partial_2\psi = \sqrt{2}\left\{\frac{1}{\mathcal{K}_1(s)}\int_{x_0}^{x}[\partial_2\frac{1}{g_1(z,y)}]dy - \partial_2 C_0\right\},$$

$$\partial_3\psi = \frac{\sqrt{2}}{\mathcal{K}_1(s)}\cdot\frac{1}{g_1}, \partial_3^2\psi = \frac{-\sqrt{2}\partial_3 g_1}{\mathcal{K}_1(s)(g_1)^2}.$$

Consider now the system of Kolmogorov equations corresponding to the simultaneous stochastic system (3):

(20)
$$l_k f = 0, \quad k = 1,2,$$

where $f = f(t_1,t_2,x,s_1,s_2,y)$ is the density of transitional probabilities of the field $\xi(z)$, for $t_i \le s_i$. Within the adopted notation:

$$\tilde{x} = \psi(z,x),\ \tilde{y} = \psi(s_1,s_2,y),\ \delta_i = \phi_i(s_i),\ i = 1,2,$$
$$\tilde{f}(\tau_1,\tau_2,\tilde{x},\delta_1,\delta_2,\tilde{y}) = f(t_1,t_2,x,s_1,s_2,y)[\partial_3\psi(s_1,s_2,y)]^{-1}.$$

Now we establish from (19) that in terms of $\tilde{f}$ the first equation in (20) is of the form $\tilde{\partial}_1\tilde{f} + \tilde{\partial}_x^2\tilde{f} = 0$.

Finally, we take the left-hand part of the second equation in (20):

$$\begin{aligned} l_2 f &= (\tilde{\partial}_2\tilde{f})\mathcal{K}_2^2 + (\tilde{\partial}_x\tilde{f})\sqrt{2}[\frac{1}{\mathcal{K}_1}\int_{x_0}^{x}(\partial_2\frac{1}{g_1})dy - \partial_2 C_0] - \\ &- g_2[\frac{1}{\mathcal{K}_1\mathcal{K}_2}\int_{x_0}^{x}(\partial_2\frac{1}{g_1})dy - \frac{1}{2}\partial_3 g_2 - \frac{1}{\mathcal{K}_2}\partial_2 C_0](\tilde{\partial}_x\tilde{f})(\sqrt{2}/(g_1\mathcal{K}_1)) + \\ &+ 2^{-1}(g_2)^2[2(\tilde{\partial}_x^2\tilde{f})(\mathcal{K}_1 g_1)^{-2} + (\tilde{\partial}_x\tilde{f})(-\sqrt{2})\mathcal{K}_1^{-1}g_1^{-2}\partial_3 g_1] = \\ &= [\mathcal{K}_2(t)]^2(\tilde{\partial}_2\tilde{f} + \tilde{\partial}_x^2\tilde{f}) = 0, \end{aligned}$$

as the coefficient before $\tilde{\partial}_x\tilde{f}$ has proved to be edual to

$$\begin{aligned} \epsilon &= \frac{\sqrt{2}}{\mathcal{K}_1}\int_{x_0}^{x}\partial_2(1/g_1)dy - \sqrt{2}\partial_2 C_0 + \\ &+ \sqrt{2}\mathcal{K}_2\left[\frac{1}{2}\partial_3 g_2 + \frac{1}{\mathcal{K}_2}\partial_2 C_0 - \frac{1}{\mathcal{K}_1\mathcal{K}_2}\int_{x_0}^{x}\partial_2(1/g_1)dy\right] - \\ &- \frac{1}{2}\mathcal{K}_1\mathcal{K}_2^2\sqrt{2}\partial_3 g_1 = 0. \end{aligned}$$

Thus, the system of Kolmogorov equations for the transitional density $\tilde{f}$ of the field $\eta(\tilde{z})$ has the form:

$$\frac{\partial \tilde{f}}{\partial \tau_1} + \frac{\partial^2 \tilde{f}}{\partial \tilde{x}^2} = 0, \quad \frac{\partial \tilde{f}}{\partial \tau_2} + \frac{\partial^2 \tilde{f}}{\partial \tilde{x}^2} = 0. \tag{21}$$

This means that there exist Wiener processes $\tilde{w}_k(\tau_k)$, such that Ito equations for the field $\eta(\tilde{z})$ are of the form (16). Therefore, the theorem is proved.

It should be noted that Theorem 2 enables us to construct the probabilities distribution of the field $\xi(z)$, since we may consider that for $\eta(\tilde{z})$ such distribution is known.

Further, it is apparent that the theorem incorporates Assumptions 1 and 2 of the article [3], where the possibility of the above transformation was merely stated, but the transformation function itself (ψ) was not constructed. Thus we may conclude that the method reported in [4] can also be applied to solving certain parabolic Kolmogorov systems.

Moreover, if we start proving Theorem 2 by constructing the functions $\phi_2(t)$ and $\psi(z, x)$, then for $\Delta_2 = 0$ we would obtain the same transformation (18), $s, t \in R_+$, $x \in R$.

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