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Isomorphism Properties of Some Sheaves of Generalized Functions on Manifolds

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Let $\mathcal{D}_M$ and $\mathcal{G}_M$ denote the presheaves of Schwartz distributions and, respectively, Colombeau generalized functions, each extended on a smooth n-manifold M by a known construction—as collections of 'compatible' ordinary distributions defined on the charts of some C^∞-atlas on M. It has been shown that $\mathcal{D}_M$ and $\mathcal{G}_M$ satisfy the requirements to be sheaves with values in the category of topological vector spaces and, respectively, of $\mathbf{C}$-algebras. Here we study some joint isomorphism properties of these sheaves of generalized functions considered on $\mathbf{R}^n$ or on different manifolds M, the isomorphism being specified so as to be in consistency with the C^∞-structure on the bases.

The paper deals with some properties of (local) isomorphism of the presheaves of certain generalized functions on smooth manifolds, such as the distributions of Schwartz and the multiplicative generalized functions of Colombeau. The unifying approach to these generalized functions considered on a manifold M (and a natural extension of the ordinary functions defined on M) is that the sections are introduced by a known explicit construction — as collections of 'compatible' generalized functions, each given on the charts of some C^∞-atlas on M. It has been shown in [1, 4] that each of the presheaves in question satisfies the requirements to be a sheaf with values in a relevant category.

Further, one is naturally led to studying some joint properties of these sheaves of generalized functions and, especially, their morphisms when defined on different base spaces. To do this, one could consider general presheaves on topological spaces taking values in arbitrary *concrete* category $\mathbf{K}$ (i.e. of sets with additional structures on them). When the base space is moreover a smooth manifold M, the sheaf morphisms can be specified so as to be in consistency with the C^∞-structure on M.

Then, it turns out that each of the sheaves of generalized functions on M is locally isomorphic in $\mathbf{K}$ (in the specified sense and particular choice of $\mathbf{K}$) with the corresponding sheaf of generalized functions on $\mathbf{R}^n$. Moreover, a functorial property concerning the sheaves of each type on different manifold bases can be proved that, whenever two manifolds are globally diffeomorphic, then there is an isomorphism in $\mathbf{K}$ between the sheaves on them.

1. The sheaf of Schwartz distributions on a manifold

We start by recalling in brief the definition of distributions on a smooth n-manifold, which is based on the next lemma by L. Hörmander [5 : § 6.3]. Henceforth Ω will stand for a non-empty open set in $\mathbf{R}^n$. If M is n-manifold and $\mathcal{A} = \{M_i, \kappa_i\}_{i \in I}$ a given C^∞-atlas on it, we set : $\widetilde{M}_i = \kappa_i(M_i) \subseteq \Omega$, $M_{ij} = M_i \cap M_j$ and $\kappa_{ij} := \kappa_i \circ \kappa_j^{-1} : \kappa_j(M_{ij}) \to \kappa_i(M_{ij})$. For any open set $U \subseteq M$, we denote by $U_i = U \cap M_i$ and $\widetilde{U}_i = \kappa_i(U_i)$ ($\subseteq \widetilde{M}_i$).

Lemma 1.1. *Let U_1, U_2 be open subsets of Ω and $\kappa : U_1 \to U_2$ be a C^∞-map with surjective derivative. Then there exists a unique continuous linear map of the distribution spaces $\kappa^* : \mathcal{D}(U_2) \to \mathcal{D}(U_1) : T \mapsto \kappa^*(T)$ (pull-back of T) coinciding with the usual composition of functions when $T \in C^0(U_2)$, and such that $(\kappa \circ \kappa_1)^* = \kappa_1^* \circ \kappa^*$ for any other map κ_1 of the same type.*

Definition 1.2 . Let for each coordinate chart $\kappa_i \in \mathcal{A}$ on a manifold M we be given an ordinary distribution $T_i \in \mathcal{D}(\widetilde{M}_i)$, so that for any other chart κ_j its pull-back by κ_{ij} satisfies : $T_j = \kappa_{ij}^*(T_i)$ on $\kappa_j(M_{ij}) \subseteq \Omega$. Then we call the collection $\{T_i\}_{i \in I}$ a distribution T on M and denote the set of distributions on M by $\mathcal{D}(M)$.

Remark 1.3. This extends the alternative definition of a C^r-function f on M as a 'compatible' collection of functions $f_i \in C^r(\widetilde{M}_i)$ ($i \in I$). Moreover, one easily checks that 1.2 gives rise to an equivalence relation with respect to the C^∞-compatible atlases on M, and thus a distribution T is uniquely determined when defined on a *given* atlas.

Definition 1.4 . For any open $U \subseteq M$, let $\mathcal{D}_M(U)$ denote the set of distributions on the submanifold U of M (hence the index 'M'). Define on each $\mathcal{D}_M(U)$ component-wise linear operations as well as a topology generated by the family of projection maps $\{\pi^i_U : \mathcal{D}_M(U) \to \mathcal{D}(\widetilde{U}_i) : T \mapsto T_i\}_{i\in I}$.

Now it is straightforward to check that, for each open $U \subseteq M$, the set $\mathcal{D}_M(U)$ of distributions on U is a Hausdorff topological $\mathbf{C}$-vector space. Then a canonical definition of the presheaf of distributions on M and its sheaf properties are given by the following.

Definition - Proposition 1.5 [4]. *Associate to any open $U \subseteq M$ the space $\mathcal{D}_M(U)$, and to each pair of open sets $V \subset U$ in M the restriction morphism: $\mathcal{R}_{UV} : \mathcal{D}_M(U) \to \mathcal{D}_M(V) : T \mapsto T\mid_V := \left\{R_{\widetilde{U}_i\widetilde{V}_i}(T_i)\right\}_{i\in I}$. Here $R_{\widetilde{U}\widetilde{V}} : \mathcal{D}(\widetilde{U}) \to \mathcal{D}(\widetilde{V}) : T \mapsto T\mid_{\widetilde{V}}$ is the restriction morphism of the sheaf $\mathcal{D}$ on Ω, defined by $< T\mid_{\widetilde{V}}, f > := < T, f_{\widetilde{U}} >$ with $f \in C_0^\infty(\widetilde{V})$ and $f_{\widetilde{U}}$ 'extended by 0' on $\widetilde{U}\backslash\widetilde{V}$. Then, $\mathcal{D}_M$ proves to be a sheaf of Hausdorff topological $\mathbf{C}$-vector spaces.*

2. The generalized functions of Colombeau on a manifold

The class $\mathcal{G}(\Omega)$ of generalized functions on Ω has been introduced in the 'non-linear theory' of J.-F. Colombeau (cf. [3]). In many respects they seem to be a most relevant multiplicative system of generalized functions: $\mathcal{G}(\Omega)$ is a $\mathbf{C}$-algebra, containing (a copy of) the distribution space $\mathcal{D}(\Omega)$ as a $\mathbf{C}$-vector subspace. A recent introduction to these 'new generalized functions', including also their extension to a smooth manifold, is [1] from which we shortly recall some basic facts.

Notation 2.1 . If q is a nonnegative integer, denote by $A_q(\mathbf{R}) := \{\varphi \in C_0^\infty(\mathbf{R}) : \varphi$ is even, constant in a 0-neighbourhood and such that $\int_0^\infty t^{\,j/m}\varphi(t)\,dt = \delta_{0q}$ for $1 \le j \le q,\ 1 \le m \le q\}$. This also extends to the n-dimensional case, denoted briefly by $A_q(n)$. Let $\varphi_\varepsilon := \varepsilon^{-n}\varphi(\varepsilon^{-1}x)$ for any $\varphi \in A_q(n)$. Then we denote by $\mathcal{E}(\Omega;\mathbf{C})$ the set of all functions $f = f(\varphi, x) : A_0(n) \times \Omega \to \mathbf{C}$ that are indefinitely differentiable with respect to x, for each φ from the 'index set' $A_0(n)$. Note that $\mathcal{E}(\Omega;\mathbf{C})$ is a $\mathbf{C}$-algebra with the point-wise function operations.

Definition 2.2 . Each generalized function of Colombeau is element of the quotient algebra $\mathcal{G}(\Omega;\mathbf{C}) := \mathcal{E}_M(\Omega;\mathbf{C})/\mathcal{N}(\Omega;\mathbf{C})$. Here the subalgebra $\mathcal{E}_M(\Omega;\mathbf{C})$ of 'moderate' functions and the ideal $\mathcal{N}(\Omega;\mathbf{C})$ are, respectively, the sets of all functions $f \in \mathcal{E}(\Omega;\mathbf{C})$ such that for each compact $K \subset\subset \Omega$ and each

$m, p \in \mathbf{N}$ there is $q \in \mathbf{N}$ so that for each $\varphi \in A_q(n)$ there are $c > 0$, $\eta > 0$ satisfying

$$sup_{x \in K} \|f^{(m)}(\varphi_\varepsilon, x)\| \leq c\varepsilon^{-q}$$

and, respectively,

$$sup_{x \in K} \|f^{(m)}(\varphi_\varepsilon, x)\| \leq c\varepsilon^{p}, \text{ if } \varepsilon \in (0, \eta).$$

The algebra $\mathcal{G}(\Omega; \mathbf{C})$, or briefly $\mathcal{G}(\Omega)$ includes canonically in itself both all infinitely-differentiable functions on Ω, and the distributions $T \in \mathcal{D}(\Omega)$ (by means of a convolution with φ_ε, $\varphi \in A_0(n)$) so that $\mathcal{D}(\Omega)$ is embedded as a $\mathbf{C}$-vector subspace. Furthermore, one can define for any $G \in \mathcal{G}(\Omega)$ both a canonical restriction $G \mid_V$ to any open set $V \subseteq \Omega$, and a 'pull-back' map $\kappa^*(G)$ when κ is a diffeomorphism of open subsets of Ω, so that the map κ^* induces homomorphism of $\mathbf{C}$-algebras. Moreover, it was proved in [1 : § 9.1] the following.

Lemma 2.3. *Let $U_{1,2}$ be open subsets of Ω and the map $\kappa : U_1 \to U_2$ be a C^∞-diffeomorphism. Then $\kappa^* : G \in \mathcal{G}(U_2) \mapsto \kappa^*(G) \in \mathcal{G}(U_1)$ is an isomorphism of $\mathbf{C}$-algebras. Moreover, it holds: (a) $\kappa^*(C^\infty(U_2)) = C^\infty(U_1)$, (b) $(\kappa \circ \kappa_1)^* = \kappa_1^* \circ \kappa^*$, where $\kappa_1 : U_2 \to U_3$ is another diffeomorphic map.*

Note that, in spite of the good algebraic properties of $\mathcal{G}(\Omega)$, there are difficulties with introducing a vector topology for this space (Cf. [2 : § 1.7]). Thus, considering the generalized function in $\mathcal{G}(\Omega)$, we shall restrict ourselves to the category ${}_{\mathbf{C}}\mathbf{Alg}$ of $\mathbf{C}$-algebras.

2.4. Now a generalized function G on a C^∞-manifold M can be defined, exactly as in 1.2 concerning distributions, as a collection $\{G_i \in \mathcal{G}(\widetilde{M_i}\}_{i \in I}$ such that $G_j = \kappa_{ij}^*(G_i)$ for any two indices $i, j \in I$. The set of generalized functions on M is denoted by $\mathcal{G}(M)$. Then given an atlas on M, any collection of generalized functions satisfying the above requirements defines a unique $G \in \mathcal{G}(M)$, and this is independent of the atlas chosen. This latter claim was shown in [1 : § 9.2], together with following.

Proposition 2.5 . *Associating to each open set U in a given manifold M the $\mathbf{C}$-algebra $\mathcal{G}_M(U)$, and to each pair of open sets $V \subset U$ in M the restriction morphism $\mathcal{R}_{UV}^M : \mathcal{G}_M(U) \to \mathcal{G}_M(V) : G \mapsto G \mid_V := \{R_{\widetilde{U}_i \widetilde{V}_i}(G_i) = G_i \mid_{\widetilde{V}_i}\}_{i \in I}$, we thus obtain a sheaf $\mathcal{G}_M$ with a base space M and values in ${}_{\mathbf{C}}\mathbf{Alg}$.*

3. Local isomorphism of the sheaves of generalized functions

We would like now to clear the connection between the above sheaves of generalized functions on $\mathbf{R}^n$ and on different manifold bases M. Note that they (as well as the infinitely-differentiable functions on M representing a sheaf $\mathcal{E}_M$ of topological $\mathbf{C}$-algebras) are sheaves with values in a particular concrete category $\mathbf{K}$. We will therefore consider the sheaf morphisms in that general setting, recalling first how the morphisms of presheaves over different base spaces are defined, given a map between the bases (cf. [6 : Ch.3]).

Definition 3.1 . Let $\mathcal{F}_X, \mathcal{H}_Y$ be $\mathbf{K}$-valued presheaves and $\kappa : X \to Y$ be a continuous map. A κ-morphism $\psi : \mathcal{H}_Y \to \mathcal{F}_X$ is a family of morphisms in $\mathbf{K}$ $\psi_U : \mathcal{H}_Y(U) \to \mathcal{F}_X(U)$ for all open $U \subseteq Y$, so that $R^{\mathcal{F}}_{U'V'} \circ \psi_U = \psi_V \circ R^{\mathcal{H}}_{UV}$ for any open $V \subset U$. If κ is a homeomorphism and each ψ_U is isomorphism in $\mathbf{K}$, then we refer to ψ as a κ-isomorphism and write: $\mathcal{F} \overset{\kappa}{\approx} \mathcal{H}$. When κ is a local homeomorphism, $\mathcal{F}_X, \mathcal{H}_Y$ are said to be locally κ-isomorphic if for each $x \in X$ there are open $U \ni x$ and open V in Y, so that the restrictions satisfy: $\mathcal{F}_X |_U \overset{\kappa}{\approx} \mathcal{H}_Y |_V$.

When X is a smooth n-manifold M and Y is an open set $\Omega \subseteq \mathbf{R}^n$, we can specify further the latter definition so as to put it in consistency with the C^∞-structure on M.

Definition 3.2 . Let M be a manifold with an atlas $\mathcal{A}$, and $\mathcal{F}_M$, $\mathcal{H}$ ($\equiv \mathcal{H}_\Omega$) be presheaves on M, Ω respectively. We shall say that $\mathcal{F}_M$ is locally $\mathcal{A}$-isomorphic to (or, modelled on) $\mathcal{H}$ if, for any chart $(M_i, \kappa_i) \in \mathcal{A}$, it holds: $\mathcal{F}_M |_{M_i} \overset{\kappa_i}{\approx} \mathcal{H} |_{\widetilde{M}_i}$.

Now, we can show that the sheaves of two types above—Schwartz distributions and Colombeau generalized functions—canonically obey this latter definition.

Notation 3.3 . Below $\mathcal{F}$ will denote any presheaf of generalized functions ($\mathcal{D}$ or $\mathcal{G}$) on Ω, considered as a presheaf with values in some concrete category $\mathbf{K}$. Accordingly, $\mathcal{F}_M$ will denote a presheaf of generalized functions with a base space M whose sections are defined from those of $\mathcal{F}$ as in 1.2 and 2.4. More exactly, given a sheaf $\mathcal{F}$ of generalized functions and an atlas $\mathcal{A} = \{M_i, \kappa_i\}_{i \in I}$ on M, we associate to any open U the set $\mathcal{F}_M(U) \in Ob\,\mathbf{K}$ with elements F, each given by

(1) $$F = \{ F_i \in \mathcal{F}(\widetilde{U}_i) : F_i = \kappa^*_{ij}(F_j) \quad \text{for any } i, j \in I \} ;$$

and, for any open $V \subset U \subseteq M$, define a restriction morphism

(2) $$\mathcal{R}^M_{UV} : \mathcal{F}_M(U) \to \mathcal{F}_M(V) : F \mapsto F |_V := \left\{ R_{\widetilde{U}_i \widetilde{V}_i}(F_i) = F_i |_{\widetilde{V}_i} \right\}_{i \in I} .$$

Premise 3.4. On the strength of the corresponding claims for the Schwartz distributions and Colombeau generalized functions we shall take for granted that $\mathcal{F}_M$ is a sheaf with values in **K**, exactly as is $\mathcal{F}$ (cf. 1.5, 2.5), so that:

(a) Whenever $\kappa : U \to V$ is a diffeomorphism of open sets in Ω, then the pull-back map $\kappa^* : \mathcal{F}(V) \to \mathcal{F}(U) : F \mapsto \kappa^*(F)$ is an isomorphism in **K** (cf. 1.1, 2.3).

(b) For any open $V \subseteq M$, the diagonal of maps $\pi_V : \mathcal{F}_N(V) \to \prod_{j\in I} \mathcal{F}(\widetilde{V}_j)$: $F \mapsto \{F_j\}_{j\in I}$ is a *monomorphism* in **K** (cf. 1.3, 2.4), i.e. injective map which is moreover a **C**-linear continuous map (when $\mathcal{F} = \mathcal{D}$) or respectively, a **C**-algebra homomorphism ($\mathcal{F} = \mathcal{G}$).

Then it holds the following.

Proposition 3.5. *If M is a given manifold with a C^∞-atlas $\mathcal{A}$, then the sheaves $\mathcal{F}_M$ and $\mathcal{F}$ are locally $\mathcal{A}$-isomorphic in* **K**.

Proof. We are to show that, for each chart $(M_i, \kappa_i) \in \mathcal{A}$, there exists an invertible κ_i-morphism $\psi^i : \mathcal{F}|_{\widetilde{M}_i} \to \mathcal{F}_M|_{M_i}$, or equivalently, for each open $U \subseteq M_i$ and $\widetilde{U}_i = \kappa_i(U) \subseteq \widetilde{M}_i$, there is a family of κ_i-isomorphisms in **K** $\psi^i_{\widetilde{U}} : \mathcal{F}(\widetilde{U}_i) \to \mathcal{F}_M(U)$.

Actually, define the canonical map $\psi^i_{\widetilde{U}} : \mathcal{F}(\widetilde{U}) \to \mathcal{F}_M(U) : F_i \mapsto F = \kappa_i^*(F_i)$. This is an isomorphism in **K** by 3.4(a) since the abbreviated map $ab\ \kappa_i : U_i \to \widetilde{U}_i$ is diffeomorphic. Further, one can check that, for any open $V \subset U \subseteq M_i$ and $i \in I$,

$$\mathcal{R}^M_{UV} \circ \psi^i_{\widetilde{U}} = \psi^i_{\widetilde{V}} \circ R_{\widetilde{U}_i\widetilde{V}_i}. \tag{3}$$

Indeed, in view of (1), (2), one readily obtains for the r.h.s. of (3): $\psi^i_{\widetilde{U}} \circ R_{\widetilde{U}_i\widetilde{V}_i}(F_i) = \psi^i_{\widetilde{U}}(F_i|_{\widetilde{V}_i}) = F|_{\kappa_i^{-1}(\widetilde{V}_i)} = F|_V \in \mathcal{F}_M(V)$, which obviously coincides with the l.h.s. there. Hence $\psi^i_{\widetilde{U}}$ is shown to be a κ_i-morphism, which completes the proof.

Moreover, concerning the sheaves of generalized functions, it holds the following functorial property of the local isomorphism in consideration, sending any global diffeomorphic map between the manifold bases to a sheaf isomorphism 'in a whole'.

Proposition 3.6. *Any global diffeomorphic map of smooth n-manifolds $f : M \to N$ induces an f-isomorphism in* **K** *between the sheaves $\mathcal{F}_M$, $\mathcal{F}_N$ of generalized functions on them.*

Proof. Let $\mathcal{A} = \{M_i, \kappa_i\}_{i\in I}$, $\Xi = \{\chi_j, N_j\}_{j\in I}$ be given C^∞-atlases on M, N respectively. The map f can be written in local coordinates as $f_{ji} = \chi_j \circ$

$f \circ \kappa_i^{-1}$, acting on $\kappa_i(M_i \cap f^{-1}(N_j))$. Construct now a morphism $\psi : \mathcal{F}_N \to \mathcal{F}_M$ by a family of maps for all open U in M and $V = f(U) \subseteq N$, each defined by the composition of maps: $\psi_V := \overline{\pi}_U \circ \omega \circ \pi_V$. Here $\pi_V = \prod_{j\in I} \pi_V^j : \mathcal{F}_N(V) \to \prod_j \mathcal{F}(\widetilde{V}_j)$ is a diagonal (product) of the canonical projection maps:

$$\pi_V^j : \mathcal{F}_N(V) \to \mathcal{F}(\widetilde{V}_j) : F \mapsto F_i \equiv F \circ \kappa_i^{-1}. \tag{4}$$

Likewise, $\overline{\pi}_U := \prod_{i\in I} \left(\pi_U^i\right)^{-1} : \prod_i \mathcal{F}(\widetilde{U}_i) \to \mathcal{F}_M(U)$ $\left(= {\pi_U}^{-1}\right)$ is a codiagonal of the injection maps $\left(\pi_U^i\right)^{-1} : F_i \mapsto F = \kappa_i^*(F_i)$. The coordinate maps of ω are defined by the pull-backs, as in (1): if $\mathcal{H} \in \mathcal{F}_N(V)$, $F \in \mathcal{F}_M(U)$ are given by $\{H_j\}_{j\in I}$, $\{F_i\}_{i\in I}$ respectively, then $\omega_{ji} : \mathcal{F}(\widetilde{V}_j) \to \mathcal{F}(\widetilde{U}_i) : H_j \mapsto F_i = f_{ji}^*(H_j)$. Thus, we can write

$$\psi_V = \prod_{i,j\in I} \left(\left(\pi_U^i\right)^{-1} \circ f^*_{\widetilde{U}_i \widetilde{V}_j} \circ \pi_V^j \right). \tag{5}$$

Now we prove that, for any open $U' \subset U$, $V' = f(U') \subset V$,

$$\psi_V = \left(\mathcal{R}^M_{UU'}\right)^{-1} \circ \psi_{V'} \circ \mathcal{R}^N_{VV'}, \tag{6}$$

which is equivalent to the requirement for consistency of ψ with the restriction morphisms of the sheaves $\mathcal{F}_M$, $\mathcal{F}_N$. Indeed, taking into account (2) and (4), these latter morphisms can be written as

$$\mathcal{R}^N_{VV'} = \prod_{j\in I} \left((\pi^j_{V'})^{-1} \circ R_{\widetilde{V}_j \widetilde{V}'_j} \circ \pi^j_V \right), \quad \mathcal{R}^M_{UU'} = \prod_{i\in I} \left((\pi^i_{U'})^{-1} \circ R_{\widetilde{U}_i \widetilde{U}'_i} \circ \pi^i_U \right). \tag{7}$$

Replacing now (5) and (7), we obtain for the r.h.s. of (6):

$$\prod_{j,i\in I} \left((\pi^i_U)^{-1} \circ (R_{\widetilde{U}_i \widetilde{U}'_i})^{-1} \circ \pi^i_{U'} \right) \circ \left((\pi^i_{U'})^{-1} \circ f^*_{\widetilde{U}'_i \widetilde{V}'_j} \circ \pi^j_{V'} \right) \circ \left((\pi^j_{V'})^{-1} \circ R_{\widetilde{V}_j \widetilde{V}'_j} \circ \pi^j_V \right) =$$

$$= \prod_{j,i\in I} \left((\pi^i_U)^{-1} \circ (R_{\widetilde{U}_i \widetilde{U}'_i})^{-1} \circ f^*_{\widetilde{U}'_i \widetilde{V}'_j} \circ R_{\widetilde{V}_j \widetilde{V}'_j} \circ \pi^j_V \right) = \prod_{j,i\in I} \left((\pi^i_U)^{-1} \circ f^*_{\widetilde{U}_i \widetilde{V}_j} \circ \pi^j_V \right) = \psi_V.$$

Hence, the map ψ is consistent with the restriction morphisms of the sheaves $\mathcal{F}_M, \mathcal{F}_N$, thus representing an f-morphism between them.

Finally, we prove that any map ψ_V is an isomorphism in $\mathbf{K}$. Indeed, every map π_V is isomorphism onto the image $\pi_V(\mathcal{F}_N(V))$, according to 3.4 (b). (Note that this is valid both in any balanced category, such as ${}_{\mathbf{C}}\mathbf{Alg}$, as well as in $\mathbf{VTop}$ whenever π_V is a closed embedding map, cf. [4]). The same holds for each $\overline{\pi}_U$ when restricted on $\overline{\pi}_U^{-1}(\mathcal{F}_M(U))$. Now, since ω is defined on $\pi_V(\mathcal{F}_N(V))$, the claim follows on showing that ω is invertible morphism in **K**.

Actually, the elements of any collection $\{H_j\}_{j\in I} \in \pi_V(\mathcal{F}_N(V))$ satisfy $H_k = \chi^*_{jk}(H_j)$, for all $j,k \in I$. Then, in view of (1), the components $\{F_i\}_i$ of the image by ω satisfy

$$\begin{aligned}\kappa^*_{ki}(F_k) &= \kappa_{ki}\left(f^*_{jk}(H_j)\right) = \kappa_{ki}\left(f^*_{jk}\left(\chi^*_{kj}(H_k)\right)\right) = \\ &= \left(\chi_{kj}\circ f_{jk}\circ \kappa_{ki}\right)^*(H_k) = f^*_{ki}(H_k) = F_i.\end{aligned}$$

Therefore, as implication of 3.4(b), the collection $\{F_i\}_{i\in I} = \omega(\{H_j\}_{j\in I})$ represents a unique $F \in \mathcal{F}_M(U)$, the two sets of elements (on U, V) being in 1–1 correspondence. Moreover, any component map ω_{ij} is isomorphism in **K** by 3.4(a), and so is therefore the map ω. (This is again valid in the categories we have in view, viz. whose morphisms are either **C**-linear maps or **C**-algebra homomorphisms, and are continuous maps.)

The proof is complete.

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"New Analytic Solution of the Trinomial Algebraic Equation $z^n + pz + q = 0$ by Means of the Goursat Hypergeometric Function, I"

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in Vol. 8, 1994, fasc. 4, pp. 375-396.

	Printed	**To read**
p.377		
l.1 from below	ϵ_r^n	ϵ_k^r
p.284		
l.4 from above	a_n	a_r

"New Analytic Solution of the Trinomial Algebraic Equation $z^n + pz + q = 0$ by Means of the Goursat Hypergeometric Function, II"

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	Printed	**To read**
p.135		
l.1 from above	$\zeta(\tau)$	$\zeta(t)$
p.143		
l.15 from above	$t \leq 1$	$t \leq -1$
p.147		
l.12 from below	$p > 0$	$p < 0$
p.147		
l.11 from below	imaginary	real
p.148		
l.9 from above	$p < 0$	$p > 0$
p.148		
l.10 from above	real	imaginary