

MATHEMATICS COMPETITION HITAR PETAR (SLY PETER)

Name of the Competition: Hitar Peter (Sly Peter): a Bulgarian folk hero, known for his shrewd intellect and thrift.

Area: Mathematics

Style of the Competition:

- Hitar Petar is an inclusive, presence competition with a multiple choice and a classical component.
- Students have to answer 15 multiple choice questions (5 easier, 5 medium and 5 difficult) and provide a written solution to one classical problem: “The Problem of Hitar Peter” The multiple-choice questions are worth 2,4 or 6 points while a wrong answer is penalized with -1 point. The Problem of Hitar Peter is worth 20 points.
- Students have 90 minutes for all problems and may not use calculators.
- The top six places in each grade receive custom-made prizes, made specifically for the competition. The House of Humor and Satire provides a prize for the best solution to the Problem of Hitar Petar in each grade. Additionally, the mayor of Gabrovo gives a sizeable monetary prize to the student from Gabrovo with the highest score.

Target Group: Open to all students with good to excellent mathematical abilities

Age of Participants: 11-14 years of age

School level of Participants: Middle school: 4th through 7th grade

Number of Participants in the Last 4 Years:

Year	Total	4 th Gr.	5 th Gr.	6 th Gr.	7 th Gr.
2004	457	144	148	104	61
2005	527	118	165	158	86
2006	440	92	161	132	55
2007	384	103	108	95	78

History of the Competition: The Mathematical Competition Hitar Petar was first held in 1995 by the Gabrovo Branch of the Union of Bulgarian Mathematicians, under the initiative of the Branch President, Prof. Stoyan Kapralov. Since then, organizers of the competition are the Gabrovo Branch of UBM, in cooperation with The House of Humor and Satire (Gabrovo’s world-renowned humor museum) and the Regional Branch of the Ministry of Education and Science. The competition is held on or near April Fools’ Day: April 1st.

Financial Basis of the competition: The competition is supported by the registration fee that each participant pays and has also been supported by the Union of Bulgarian Mathematicians and private sponsors.

Competition Problems: The competition problems are designed by a team from the Union of Bulgarian Mathematicians, Gabrovo Branch, comprising of math teachers and university professors from Gabrovo.

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Problems from the competition in 2007 with answers

Problems for 4th grade

1. Calculate: $3125 - 3125 : (303.5 - 5.302)$

- A) 3000 B) 2500 C) 600 D) 3100 E) 0

2. The sum of the digits of a three-digit number is 14, and the digit of its tens is two times bigger than the digit of its hundreds. How many such numbers are there ?

- A) 2 B) 3 C) 4 D) 20 E) 40

3. Calculate the sum of the numbers a and b , if

$$20072007 - a = 20071234 \quad \text{and} \quad b = 2007 + 0 : (2007 - 276.5)$$

- A) 3407 B) 3400 C) 2780 D) 2634 E) 0

4. Find a number, which is 9 times smaller than the difference between the biggest three-digit number with a digit of its tens 8 and the least such number with a digit of its ones 7.

- A) 98 B) 19 C) 9 D) 99 E) 109

5. Ivan is in the same class as Maria. Ivan's male classmates are 5 more than his female classmates. What's the difference in the number of Maria's female classmates and male classmates?

- A) 4 B) 5 C) 6 D) 7 E) 8

6. If we reduce the width of a rectangle with 3 cm, and its length with 12 cm, there will be a square with a perimeter of 24 cm. What's the difference in square cm between the area of the rectangle and the area of the square?

- A) 36 B) 54 C) 126 D) 162 E) 18

7. There are 40 flags in a box – 10 blue, 10 red, 10 green and 10 yellow. What's the least number of flags we have to take from the box without looking, to be sure we have 10 which are the same colour?

- A) 10 B) 21 C) 36 D) 37 E) 40

8. The cost of a bus ticket is 60 stotinki. Peter gave two coins and got a change. How many different ways of getting a change in two coins which worth the same are there? (There are coins which worth 1, 2, 5, 10, 20, 50 and 100 stotinki)

- A) 2 B) 4 C) 5 D) 9 E) 10

9. **Note:** This problem is concerned with the Bulgarian alphabet and is non-translatable to English.

10. For the birthdays of Eve, Nelly, Poly and Kate their favorites cakes were made – chocolate cake, biscuit cake, fruit cake and vanilla cake. Kate doesn't like biscuit cake and doesn't eat fruit. Eve loves fruit cake. On Nelly's birthday they ate chocolate cake. What kind is Poly's cake?

- A) chocolate B) biscuit C) fruit D) vanilla E) you can't tell from this information

11. A tourist walked 9 days in the Balkan Mountains and walked 120 km with an even speed. For 3 days he walked half of the way, walking 4 hours a day. How many hours a day must he walk in the second half of the way?

- A) 1 B) 2 C) 3 D) 4 E) 5

12. There are 100 candies ordered in a line 1 m away from each other. Peter is 1 meter away from the first candy and there is a basket next to him. How many meters will he walk, if he takes a candy and brings it back to the basket, then goes for another candy and so on, until he collect all the candies?

- A) 10000 B) 5050 C) 10100 D) 20000 E) 200

13. Find the sum of all natural numbers with a digit of their tens is 3 times bigger than the digit of their ones and their sum with the number, written with the same digits but in inverted order, is two-digit number.

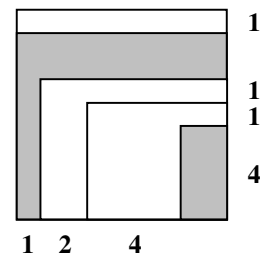
- A) 93 B) 132 C) 186 D) 264 E) 318

14. The arithmetical operation \star is such as for every two numbers a and b , $a \star b$ is $1/3$ of their sum. Calculate $((3 \star 6) \star 9) \star 8$

- A) 3 B) 4 C) 5 D) 6 E) 9

15. A square with a side 9 cm is split as shown in the picture. What's the sum of the areas of the darkened parts (in square cm)?

- A) 26 B) 28 C) 30 D) 31 E) 32



The Problem of Hitar Peter for 4th grade.

Place the numbers 2, 10, 12, 13, 20, 21, 23, 30, 31 and 32 in the table, so that the sum of the numbers in each row, column and diagonal is equivalent (magical square). How many possible ways of doing it are there?

0	11	22	33
		1	
	3		

Problems for 5th grade

1. What is the value of $\frac{11}{12} + \left(\frac{5}{6} - \frac{3}{4}\right)$?

- A) $\frac{5}{6}$ B) 1 C) $\frac{6}{5}$ D) $\frac{19}{4}$ E) $\frac{25}{12}$

2. What is the value of $\left(3\frac{3}{4} + 4\frac{1}{2}\right) \cdot 1\frac{1}{2}$?

- A) $\frac{76}{8}$ B) $\frac{85}{8}$ C) $\frac{93}{8}$ D) $\frac{99}{8}$ E) 12

3. What is the value of $\frac{27}{4} : 9 + 5 : \frac{18}{7}$?

- A) $\frac{13}{18}$ B) $\frac{47}{18}$ C) $\frac{61}{36}$ D) $\frac{97}{36}$ E) $\frac{113}{36}$

4. Three diggers dig three holes in three hours. How many holes can dig two diggers in two hours?

- A) 1 B) $1\frac{1}{3}$ C) 2 D) $2\frac{2}{3}$ E) 3

5. N is a positive integer. The product of $N+1$ and $N-1$ is 2208. What is the value of N ?

- A) 41 B) 43 C) 45 D) 47 E) 49

6. Several people have to sit on several identical benches. If they sit so that there are 6 people on each bench, then on the last one there will be only three people. If they sit so that there are 5 people on each bench, four of them will have no place to sit. How many benches are there?

- A) 7 B) 8 C) 9 D) 10 E) 11

7. A shop receives 478 levs from selling two types of cake. The first type costs 12 levs and the second – 17 levs. What is largest possible number of expensive cakes sold?

- A) 10 B) 14 C) 18 D) 22 E) 26

8. In a box there are 10 pairs of brown gloves and 10 pairs of red gloves. What is the smallest possible number of gloves we have to pull out of the box (without looking inside) so that we can be sure we have a pair of gloves of the same color?

- A) 2 B) 3 C) 11 D) 21 E) 40

9. A ship traveled 24 km with speed 16km/h and returned to the same point with speed 24km/h. What was his average speed?

A) 19,2 km/h B) 20 km/h C) 22,3 km/h D) 22,5 km/h E) 24 km/h

10. The road from Gabrovo to Kazanlak consists only of ascent and descent (there are not level sections). A bicyclist ascends with 20 km/h and descends with 30 km/h. From Gabrovo to Kazanlak he traveled for 3 hours and returned for 3 hours and 20 minutes. How many kilometers is the length of the road from Gabrovo to Kazanlak?

A) 58 B) 62 C) 66 D) 72 E) 76

11. A bicyclist is traveling on a flat road with speed 15km/h, when he ascends – with 12km/h, and when he descends – with 20km/h. He traveled from A to B for 2 hours. What is the length in kilometers of the distance from A to B?

A) 12 B) 15 C) 20 D) 24 E) undetermined

12. If now it's 9 a.m., how many minutes will it take the long arrow to overtake the short arrow?

A) 45 B) $49\frac{1}{11}$ C) $51\frac{2}{11}$ D) $48\frac{1}{11}$ E) $47\frac{2}{11}$

13. In a battle of 100 cats, 60 of them lost an ear, 70 lost a whisker, 85 – a tooth and 90 – a nail. What is the numbers of cats that lost an ear, a whisker, a tooth and a nail at the same time?

A) 5 B) 10 C) 15 D) 20 E) 25

14. How many two-digit positive integers are there which are equal to three times the sum of their digits?

A) 0 B) 1 C) 2 D) 3 E) 4

15. Two people have to pass 12 kilometers using one bicycle, but they cannot ride it at the same time both. Each one of them travels on foot with speed 5 km/h and rides a bicycle with speed 20 km/h. What is the smallest amount of time for which they can pass the distance?

A) 75 min B) 90 min C) 2 h D) 144 min E) 160 min

The Problem of Hitar Peter for 5th grade

Six musicians wearing numbers from 1 to 6 take part in a competition. Each member of the jury classifies the participants – the best receives 1 point, the second – 2 points and so on. In this way we get three lists. The final classifying is based on the sum of these points from each of the lists. Those who have the smallest sum of points win a prize. For example, if the lists are 4, 3, 6, 2, 5, 1; 2, 6, 4, 3, 5, 1; 2, 4, 1, 3, 6, 5, then the musicians with numbers 2 and 4 who have six points will be awarded. What is the largest possible sum that could the awarded have?

Problems for 6th grade

1. What is the ratio of the numbers $1\frac{1}{2007}$ and $1\frac{28}{223}$?

- A) 1:1 B) 1:28 C) 8:1 D) 9:1 E) 8:9

2. What is the value of $A = 7,2 : (-0,9) + \left(-\frac{1}{2}\right)^3 - 2 : \left(-\frac{1}{4}\right)$ e:

- A) $-\frac{1}{6}$ B) $-\frac{1}{8}$ C) $\frac{1}{6}$ D) $\frac{1}{8}$ E) $\frac{1}{2}$

3. If $3x - \left(-11 - \left(-4 + \frac{1}{3}x\right)\right) = 2007$, than x equals:

- A) 0 B) 600 C) 1000 D) 1200 E) 2007

4. How many rectangles are there?

- A) 6 B) 7 B) 14 D) 16 E) 18

5. How many of the fractions $\frac{4}{21}$, $\frac{5}{36}$, $\frac{10}{63}$, $\frac{13}{84}$ и $\frac{19}{126}$ are between $\frac{1}{7}$ and $\frac{1}{6}$?

- A) 1 B) 2 C) 3 D) 4 E) 5

6. What is the next year in which March 31 will be in Saturday again?

- A) 2010 B) 2011 C) 2012 D) 2013 E) 2014

7. Hitar Petar's wife is 20% shorter than him and their son is 25% taller than his mother. How many percent is the son taller than his father?

- A) 0 B) 5 C) 10 D) 15 E) 45

8. A dairy farm produces yellow cheese shaped like a rectangular parallelepiped that weigh 800 grams and smaller ones which measurements are 4 times smaller than normal. What is the weight of the smaller ones?

- A) 12,5 B) 25 C) 50 D) 100 E) 200

9. For how many integers n the value of $\frac{18}{2n+1}$ is an integer?

- A) 3 B) 4 C) 6 D) 8 E) 12

10. In some of the cells of the table is written their perimeter is centimeters. How many centimeters is the perimeter of the whole table?

- A) 26 B) 27 C) 52 D) 54 E) 56

	8		
			14
12			
		20	

11. In the equation $9.(G+A+B+R+O+V+O)=2007$ the six different letters are six consecutive numbers. Which number is the letter O?

- A) 0 B) 28 C) 32 D) 34 E) 36

12. How many irreducible proper fractions have a denominator 2007?

- A) 1003 B) 1109 C) 1330 D) 1332 E) 1338

13. The number of students in a class is less than 30. How many students of this class attend additional math lessons if they are 60% of the boys and 45% of the girls?

- A) 10 B) 11 C) 12 D) 13 E) 14

14. There is a triangle $A_1A_2A_3$. The points $A_4, A_5, A_6, \dots, A_{2007}$ divide in two equal parts the lines $A_1A_3, A_2A_4, A_3A_5, \dots, A_{2004}A_{2006}$, respectively. The area of the rectangle $A_{2005}A_{2006}A_{2007}$ is 1 cm^2 . How many square cm is the area of the triangle $A_1A_2A_3$?

- A) 2^{2004} B) 2^{2005} C) 2^{2006} D) 2^{2007} E) 2^{2008}

15. The participants in a culinary show have to cut a piece of cheese shaped like a cube into 64 identical cubes. What is the smallest possible number of cuttings if after each cutting the pieces can be rearranged?

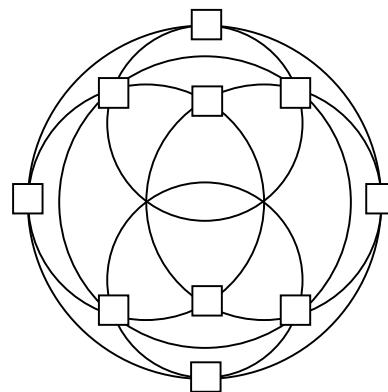
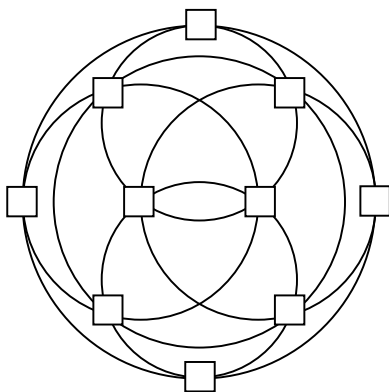
- A) 5 B) 6 C) 7 D) 8 E) 9

The Problem of Hitar Peter for 6th grade.

There is a question concerning each of the figures:

Can you put the integers from 1 to 10 into the squares so that the sum of the numbers on each of the six circumferences is the same as the others?

Provide reasons for your answer.



Problems for 7th grade

1. How many zeroes does the number 2000^{2007} end with?

- A) 2007 B) 5003 C) 8028 D) 4014 E) 6021

2. Which are the three numbers that cannot be sides of a triangle?

- A) 3, 4, 5 B) 2, 3, 4 C) 2, 3, 5 D) 3, 3, 4 E) 5, 7, 9

3. The trapezium $ABCD$ has bases AB and CD and the diagonal AC bisects the angle BAD . Which of the following statements is true?

- A) $AD > DC$ B) $AD = DC$ C) $AD < DC$ D) $AD + AC < DC$ E) $AD + DC < AC$

4. The length of a rectangle has been increased with 25%. How many percent the width has to be decreased with so that the area of the rectangle stays the same?

- A) 10% B) 15% C) 20% D) 25% E) 30%

5. Hitar Petar has sheep, hares and hens. Together these animals have 76 legs and 23 heads. How many hens are there?

- A) 6 B) 8 C) 10 D) 15 E) 19

6. How many positive integers divide the number 2007?

- A) 2 B) 3 C) 4 D) 5 E) 6

7. Which of the fractions is the smallest?

- A) $\frac{2006}{2007}$ B) $\frac{2005}{2006}$ C) $\frac{2004}{2005}$ D) $\frac{2003}{2004}$ E) $\frac{2007}{2006}$

8. How many numbers satisfy the equation:

$$(2x-3)^3 - 26(2x-1) = 2x(2x-1)^2 - 7(2x-5)(2x+5) ?$$

- A) 0 B) 1 C) 2 D) 3 E) more

9. How many positive integers satisfy the inequality:

$$(2x-3)(4x^2+6x+9) > 8x^3+2x-59 ?$$

- A) 13 B) 14 C) 15 D) 16 E) 17

10. Hitar Petar's grandchildren have to put in order his wood. The first of them can do it alone for 90 minutes, the second – for 1 hour, and the third – for 3 hours. If they work together, how many minutes will it take them to do the job?

- A) 25 B) 30 C) 35 D) 40 E) 45

Answers

	1	2	3	4	5		6	7	8	9	10		11	12	13	14	15
4th grade	B	B	C	A	D		B	D	D	B	B		B	C	A	B	E
5th grade	B	D	D	B	D		A	E	D	A	E		B	B	A	B	B
6th grade	E	B	B	E	C		C	A	A	C	D		D	D	C	A	B
7th grade	E	C	B	B	B		E	D	A	C	B		D	D	B	B	C

Answers of the problems of Hitar Petar

4th grade:

0	11	22	33		0	11	22	33
31	32	1	2		23	32	1	10
23	20	13	10		31	20	13	2
12	3	30	21		12	3	30	21

5th grade: 10 points

6th grade: impossible for both figures

7th grade:

2	2	2	2	1	2	2	1	1	2	1	2	1	1	1	1	2	2		
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