National Mathematics Competition for Students of Foreign Language Schools

Area: Mathematics

Style of Competition: Nation wide, for individuals, 3 problems "Classical style".

Target Group: For students from foreign language schools.

Age of Participants: 15 – 18 years old;

School level of Participants: High schools: 8 – 12 grades;

Number of Participants in the Last 3 Years: 350 annually.

History of Competition: Initiated in 1998 by experts in mathematics to involve students from foreign language schools in mathematics competitions.
The principal organizers of the first and subsequent editions of the event are Union of Bulgarian Mathematicians, Regional Inspectorate for Education in Lovech and Ministry of Education and Science.

Financial Basis of the competition: Supported by Ministry of Education and Science and competition fees.

Competition Problems:

10th competition Lovech, April 2007

8TH GRADE

1. Find the values of the parameter *a*, for which the equations
$(1-x)^2-(a-2)^2-a(a-x-1)=3-(a-x)(a+x)$ and $3ax-1=x-\left|4x(3a+x)-(3a+2x)^2\right|$
are equivalent.

2. Let M and N be the midpoints of the sides BC and AB of triangle ABC respectively, and point D is on the side BC. Prove that if $P=AD\cap MN$, then $AP\perp BP$, if and only if AD is the bisector of the angle CAB.

3. Let n be a natural number and $d_1<d_2<d_3<d_4<...$ be the natural divisors of n. Find all n, such that $n=d_1^2+d_2^2+d_3^2+d_4^2$.

9TH GRADE

1. Let $A=\dfrac{\dfrac{\sqrt{b^2-2b+1}}{b}+b\sqrt{b^2-2b+1}+2-\dfrac{2}{b}}{\sqrt{b-2+\dfrac{1}{b}}}$:

a) Simplify *A*.

b) Calculate A if $b=\frac{1}{9}\left(\frac{1}{x_1^2}+\frac{1}{x_2^2}-x_1^3-x_2^3\right)$, where x_1 and x_2 are the real roots of the equation $3x^2-9x+2=0$.

2. Given square *ABCD* and equilateral triangle *ABK*, where *K* is inside *ABCD*. The lines *BK* and *AD* intersect in the point *P*. Prove that the segment, connecting the midpoints of segments *KD* and *CP* is equal to half of the side of square.

3. Find all three digit natural numbers, such that the digit of the hundreds is three times less than the digit of units and sum of this number and the number formed by changing the places of the digit of units and the digit of tens is divisible by 72.

10TH GRADE

1. Solve the equation

a) $\sqrt{x+4}+\sqrt{x-4}=\sqrt{2x}$; b) $\left(\sqrt{x+3}+\sqrt{x+2}\right)\left(\sqrt{x+8}-\sqrt{x+2}-1\right)=1$.

2. Let *O* be the circumcenter of the triangle *ABC* and *M* is the midpoint of the side *AB*. The circumcircle of the triangle *AMO* intersects the side *AC* in point *K*. If $AK=3, MK=4$ and $\sphericalangle AOM=45^0$, find:

a) the lengths of the sides *AC* and *BC*; б) the length of the side *AB*.

3. Prove that if the numbers a,b,c are such that $a+b+c=5$ and $ab+bc+ca=8$, then a,b and c are numbers of the interval $\left[1,2\frac{1}{3}\right]$.

11TH GRADE

1. Given a quadrangle *ABCD*, such that $AB=5\sqrt{2}$, $BC=5-\sqrt{3}$, $DC=2$, $AD=4$ and $AC=2\sqrt{7}$.

a) Find the measures of the angles of the quadrangle.

b) Find the length of the segment, connecting the midpoints of the diagonals of the quadrangle.

2. All possible five digit numbers, using only digits 1, 2 or 3 are written.

a) How many are these numbers?

b) What is the probability to choose one of these numbers which sum of the digits is not less than 12?

3. In the sequence $a_1, a_2, a_3, \ldots$ each term after the first is equal to the sum of the digits of the previous number multiplied by 243. Prove that:

a) If $a_1=2007$, then $a_3=a_4=a_5=\ldots$

б) For any a_1, since some term further on all the numbers of the sequence are equal to 4374.

12TH GRADE

1. Let $f(x) = 16x^2 - 8mx + m^2 - 4m + 11$, where m is real parameter.

a) Find the values of m, for which the equation $f(x) = 0$ has exactly one real root in the interval (0; 1).

b) Find the values of m, for which the minimum value of $f(x)$ in the interval [0; 1] is positive.

2. Let BB_1 and CC_1 be the altitudes through the vertices B and C in the acute triangle ABC. Let M be the incenter of the triangle AB_1C_1 and T is the point of tangency of the incircle of triangle ABC with the side AB. Prove that $MT = r$, where r is the radius of the incircle of triangle ABC.

3. In the triangular pyramid $ABCQ$ the edge AQ is perpendicular to base and AQ=1, AB=1, $AC = BC = \frac{\sqrt{2}}{2}$. The plane α, through a point M on the edge AB, is perpendicular to AB.

a) Express the area of the intersection of α with the pyramid as function of $AM = x$.

б) Find the values of x, for which the intersection is a circumscribed polygon.

Results Scored:

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