Mathematics Tournament "Acad. Kiril Popov"

Area: Mathematics

Style of Competition: "Nation wide" combined:

- Part I "For individuals":

6 problems "Multiple-choice";
3 problems "Open answer";
1 problem "Classical style";
Duration – 75 min, maximal score – 50 points.

- Part II "Team competition": 4 problems, including 2 problems for research. The team (up to 4 members) submits joint solution of each problem.

Duration – 135 min, maximal score – 50 points.

- Ranking is independent for each part and grade.
- The tournament is held in the first half of May in Shumen.

Target Group: For students of average abilities;

Age of Participants: 9 – 15 years old;

School level of Participants: Primary& Secondary schools: 3 – 8 grades;

Number of Participants in the Last 3 Years: 500 – 600 annually.

History of Competition: First tournament was held in 1986 in Shumen. In 1995 – 1998 the tournament with new regulations took place in Veliki Preslav, and since 1999 is held again in Shumen. The competition problems are published in journal "Mathematics and Informatics" and in 2005 was published a book "Mathematics Tournament Acad. Kiril Popov". Host of competition is Mathematical High school "Nancho Popovich" Shumen. The principal organizers are Union of Bulgarian Mathematicians – Shumen section, Regional Inspectorate for Education in Shumen.

Financial Basis of the competition: Shumen municipality**;** Self-supporting event.

Competition Problems:

12th competition Shumen, May 5 2007

PART I, INDIVIDUAL COMPETITION

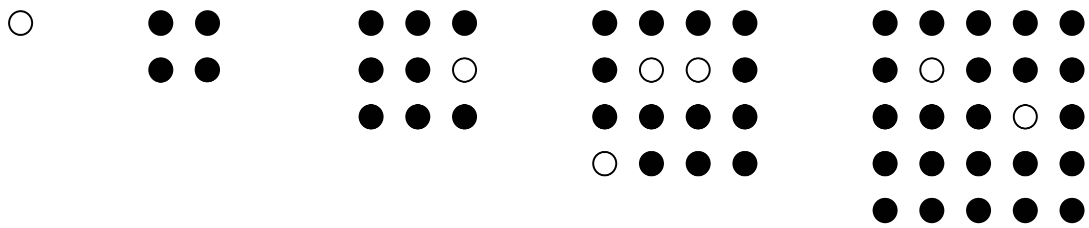
2ND GRADE

In problems 1. – 6. mark the correct answer.

1. Calculate: **3.7 + 18 – 15:3 =**

а) 22; б) 34; в) 44; г) 23;

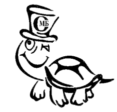
2. Vesko has white and black pieces, which he arranged in the follovong way. How many are the black pieces?



а) 47; б) 55; в) 49; г) 48;

3. For 7 hats the tortoise Nety paid 63 leva. How much willcost 3 hats?

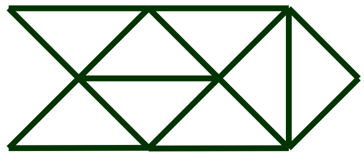
а) 12; б) 21; в) 27; г) 9;



4. The circumference of rectangular is 12 cm. One of the sides was enlarged with 3 cm, and the other was decreased with 1 cm. The circumference of the new rectangular is:
а) 16; б) 14; в) 10; г) 12;

5. The sum of two consecutive even numbers is the least two digit even number with equal digits. What number will be obtained, if add to the sum the less of the two numbers?
а) 22; б) 12; в) 34; г) 32;

6. How many triangles are there on the figure?
а) 12; б) 10; в) 9; г) 11;



In problems 7, 8 and 9 write your answer.

7. Which number we have to put in the squares, to obtain correct equality?

$$21 - \square + 11 + 41 = \square + 65 - 34$$

Answer. ....

8. In Luck's box there are 2 green, 2 pink and 2 yellow chewing gums. How many chewing gums we have to take out at least (without looking) so that among them to have at least 1 green, 1 pink and 1 yellow chewing gum?
Answer. ....

9. Today is **05.05.2007.** If we sum the digits of the date we will obtain 19. Write the three dates in this year which sum of the digits is 28.
Answer. ....

Write your solution of problem 10.

10. The giant Momo has 4 magic swards – fiery, water, earthly and heavenly. The magic of the swards is in their lengths. The fiery and water swards together are long as the heavenly sward. The water sward is long as fire and earthly swards together. The earthly sward is half of the heavenly which is 12 m long. Determine the lengths of the rest swards.

3RD GRADE

In problems 1. – 6. mark the correct answer.

1. Newborn chicken weighs 134 g. After a year it will be 5 times heavier. With how many grams the chicken will be heavier after a year?
а) 536; б) 670; в) 546; г) 804.

2. Three numbers are written on the blackboard. The first is 27. The second is with 14 bigger, and the third is with 18 less than the second. What number should be added to the sum of the three numbers to obtain 100?

а) 8; б) 9; в)18; г) 23.

3. Students are arranged in two columns. In the column of Tania there are eight children in front her and five behind her. In the next column Ivan is on the seventh place and is just in the middle. How many are the children?

а) 25 б) 26 в) 27 в) 28

4. Daddy Bear eat up third part of a pie. Mummy Bear eat up the half of the rest. What part of the pie had remained?

a) one half; b) one third; c) one forth; d) nothing.

5. Albena, Angel, Boriana and Bogdan have altogether 180 levs. Albena and Angel have the same amount; Boriana and Bogdan also have the same amount. How many levs have Albena and Boriana together?

а) 45; б) 100; в) 90; г) impossible to say.

6. How many are the two digit numbers, less than 30, which are divisible by the sum of their digits?

а) 3; б) 6; в) 7; г) 10.

In problems 7, 8 and 9 write your answer.

7. Pigeons and sparrows are alighting on a tree – altogether 47. Suddenly come 12 pigeons and 15 sparrows more. Then the pigeons and sparrows become equal in number. How many pigeons were on the tree initialy?

Answer. ....

8. Irena, Kati, Ana, Lily and Elena live in a house of two stories. Two of them live on the fist floor, and the rest – on the second. Lily lives on different floor from Kati and Elena. Ana lives on different floor from Irena and Kati. Who of them live on the first floor?

Answer. ....

9. Three chewing gums and six croissants cost 2 leva and 88 stotinki, and seven chewing gums and four croissants cost 2 leva and 52 stotinki. What is the price of one chewing gum and one croissant together?

Answer. ....

Write your solution of problem 10.

10. Plamen collected on the seaside 50 shells for five days. Every day he found three shells more than the previous day. How many shells he had fond on the fifth day?

4TH GRADE

In problems 1. – 6. mark the correct answer.

1. What is the longest?

а) 1000000 mm; б) 10000 cm; в) 100 dm; г)1 m.

2. Let A be the sum of the odd numbers bigger than 54 and less than 68, and B – the sum of the even numbers bigger than 54 and less than 68. Than A – B is equal to

a) 6; b) 12; c) 21; d) 51.

3. Ana and Biliana run in race. Biliana outrun 20 children, including Ana. Ana come to an end after 5 children, including Biliana. Three children come to an end between Ana and Biliana. Find the number of participants.

а) 30; б) 27; в) 25; г) 22.

4. Summing the digits of 2007 you obtain 9. For how many years in 21 century (2001 – 3000) you will obtain sum of the digits 9?

а) 10; б) 12; в) 32; г) 36.

5. In a farmyard there are donkeys, horses, cows and chickens. Altogether there are 60 legs, 16 wings and 6 horns. The horses are as many as the donkeys. How many are the donkeys?

а) 2; б) 3; в) 4; г) 5.

6. Find the sum of the first 30 numbers in the sequence:

1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, …?

а) 139; б) 140; в) 148; г) 156.

In problems 7, 8 and 9 write your answer.

7. Three exercise books and two pencils cost 61 stotinki, and two exercise books and seven pencils cost 86 stotinki. What is the price of one exercise book and one pencil together?

Answer. ....

8. In a caravan with 28 camels each one carries one, two or three bags. Altogether there are 50 bags. The camels carrying one bag are as many as the camels carrying two or three bags. How many camels carry three bags?

Answer. ....

9. Let A, B, C and D are digits (not necessarily different) such that AA . BB = CCGG. Find the sum A +B.

Answer. ....

Write your solution of problem 10.

10. A company of tourists arrive late in a chalet. In the kitchen there were 17 ham sandwiches and 31 cheese sandwiches. Seven of them eat one ham sandwich and one cheese sandwich, five – two cheese sandwiches, and four tourists do not eat anything. Every one of the rest eat one sandwich. Find the number of tourists, if it’s known that all the sandwiches have been eaten?

5^{TH} GRADE

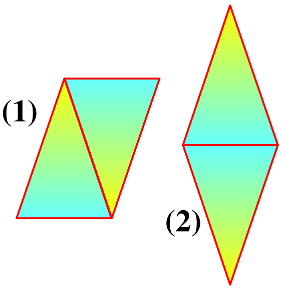
In problems 1.–6. mark the correct answer.

1. If divide the number 98765432 by 8 which nonzero digit will be missing in the quotient?
а) 2 б) 4 в) 8 г) 9

2. The sum of two decimal fractions is 193.8, and one of them is $\frac{9}{10}$ of the other. The bigger fraction is:
а) 91,8 б) 92,6 в) 102 г) 101,4

3. The last digit of the number
1.2.3.4 + 2.3.4.5 + 3.4.5.6 + 4.5.6.7+ 5.6.7.8 + ... +2002.2003.2004.2005 + 2003.2004.2005.2006 is
а) 4 б) 8
в) 2 г) 0

4. The figures **(1)** and **(2)** are constructed of two congruent isosceles triangles. The perimeter of one triangle is 3 cm less than the perimeter of **(1)** and 7 cm less than perimeter of **(2).** How many centimeters is the perimeter of one triangle?



а) 13 б) 18
в) 12 г) impossible to determine.

5. Deo's TV has channels with numbers from 0 to 87. If Deo starts from channel 16 to push the button to increase the number oh the channel and push it 518 times on which channel he will stop?

а) 6 б) 7 в) 8 г) 9

6. In a vessel there are 26 l water and in other – 7 l. Each of them was filled up with the same quantity of water, such that the water in one of the vessels becomes three times more than in the other. How many liters water was used to fill up each of the vessels?

а) 2 б) 2,5 в) 3 г)7,5

In problems 7, 8 and 9 write your answer.

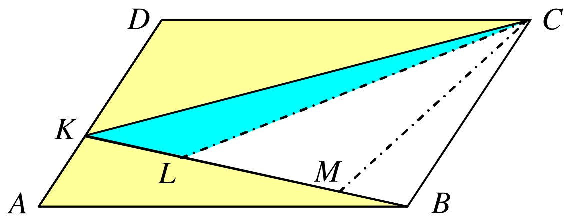
7. In our residental distrct there are 20 houses with garrage, 35 family houses and 40 houses with garden. I and my two friends don't live in family houses, but have garrage and garden. Out of all family houses 5 have garrage and garden and 11 only garden. Moreover there are 16 houses witout garage and garden, and 5 of them are not familly houses. How many houses are thre in the district?

Answer. ....

8. Several equal in number companies of children visited one and the same number tourist destinations. Each child visiting destination received a hat. It is known that the number of visited destinations by every child is bigger than the number of children in a company, but less than the number of the companies. How many destinations visited each company it the children received altogether 1547 hats?

Answer. ....

9. On the figure *ABCD* is parallelogram and $BK = 24$ cm. Find the length (in cm) of *ML*, if $S_{KLC} = 0,25(S_{ABK} + S_{DCK})$ and $12S_{MBC} = S_{ABCD}$.



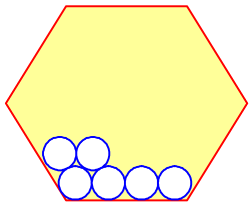
Answer. ....

Write your solution of problem 10.

10. One day Sasho feels dully and he write three different digits *a*, *b* and *c*, and after that the six numbers with these digits. Surprisingly Sasho discovered that $\overline{abc}$ is divided by 2, $\overline{bac}$ is divided by 3, $\overline{acb}$ is divided by 4, $\overline{bca}$ is divided by 5, $\overline{cab}$ is divided by 6, and dividing $\overline{cba}$ by 7 we have remainder 5. Find the number $\overline{abc}$.

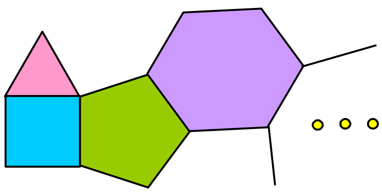
6TH GRADE

In problems 1.–6. mark the correct answer.



1. How many coins can be arranged in the classifier? (The classifier has the shape of regular hexagon.)

а) 16 б) 30 в) 37 г) more than 40



2. Regular polygons, each with side length 2 are adjoined one to another. Every next polygon has one side more than the previous, and the last one has 10 sides. Find the perimeter of the obtained figure.

а) 104 б) 100 в) 74 г) 64

3. One can change 22 yens for 14 dinars, 12 vons for 21 dinars, 10 vons for 3 euro, 5 pounds for 2 euro. How many yens you can change for 24 pounds?

а) 24 б) 44 в) 88 г) impossible to determine.

4. The average age of the 11 football players of the team "Lion" is 22 years. The captain of team was kicked out for fault. The average age of the 10 rest players become 21 years. How old is the captain of the team?

а) 22 б)24 в) 26 г) 30

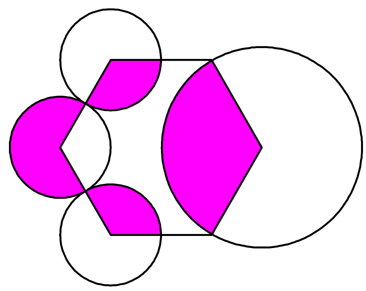
5. Jenny runs 5 km with speed 10 km/h and 10 km with speed 5 km/h. The mean velocity of Jenny's race is:

а) 6 б) 6,5 в) 7 г) 7,5

6. The beetle Bubby moves with speed 1 cm/s. She starts northwards; cover the distance of 1 cm and turns left, i.e. westwards, cover the distance of 2 cm and again turns left, i.e. southwards, cover the distance of 3 cm and so on. Each time Bubby covers a distance with 1 cm more then previous. What will be the direction of Bubby after 1 minute?

а) North б) west в) south г) east

In problems 7, 8 and 9 write your answer.



7. The centers of three little coins and one big coin are vertices of a regular hexagon. Find the ratio of the colored part to the non colored part of the four coins. (The sum of the angles of a hexagon is 360°)

Answer. ....

8. Find the sum of the first 51 natural numbers, coprime with 583.

Answer. ....

9. Let a, b and c be three nonzero digits and have different reminders dividing by 3. Among the six three digit numbers written with them, at least four are even and one is perfect square. Find this perfect square.

Answer. ....

Write your solution of problem 10.

10. In the kindergarten "Winy the Pooh" hold an inquiry. To the question "What you prefer – juice or sandwich?" most of them answered – juice, less – sandwich and one find it difficult to give an answer. Among the sandwich eaters 70 % prefer hamburgers and 30 % – cheeseburgers. Among the juice drinkers 56.25 % prefer orange juice, 37.5 % – apple juice and one find it difficult to give an answer. How many children are interviewed in the kindergarten "Winy the Pooh"?

7^{TH} GRADE

In problems 1.–6. mark the correct answer.

1. The sum of all the digits in the sequence $1; 2; ...; 100$ is:

а) 901; б) 900; в) 445; г) 551.

2. IF $\overline{abcd}+\overline{bcd}+\overline{cd}=2007,$ then how many of the digits a, b, c and d are perfect squares?

а) 0; б) 1; в) 2; г) 3.

3. Given ΔABC, such that $\angle BAC = \angle ABC + 30°$. If he point D lies on the side BC and $AC = CD$, then the measure of $\angle DAB$ is:

а) 12°; б) 20°; в) 15°; г) 30°.

4. If d is the greatest common divisor of numbers 20072007 and 200720072007 then:

а) $d = 9$; б) $9 < d < 2007$; в) $d = 2007$; г) $2007 < d$.

5. The biggest three digit number with exactly three positive divisors is:

а) 841; б) 961; в) 989; г) 931.

6. For every positive number x denote by $[x]$ the biggest integer less or equal to x (for instance $[39,2] = 39$; $[15] = 15$; $\left[\frac{5}{7}\right] = 0$). The number of the solutions of the equation $[x] = \frac{3x-1}{7}$ is:

а) 0; б) 1; в) 2; г) 3.

In problems 7, 8 and 9 write your answer.

7. $2007! = 1.2.3...2006.2007$. The biggest integer k, for which $2007!$ is divisible by 2007^k is:

Answer. ....

8. Given a parallelogram $ABCD$. Let M and N be the midpoints of AD and BC respectively. The bisector of $\angle BAC$ intersects MN in the point L. let CH $(H \in AB)$ be the altitude of the parallelogram through the vertex C. If $CH = ML = h$ and $AD = LN$, then the area of the parallelogram $ABCD$ is:

Answer. ....

9. Given a segment AB. The point C is between A and B ,such that $AC = 4$ and $BC = 7$. In one and the same halhplane with respect to AB are constructed the squares $ACMN$ и $CBPQ$. AP intersects CQ in the point L. The area of the quadrilateral $ALMN$ is:

Answer. ....

Write your solution of problem 10.

10. Find all pairs of positive integers $(a;b)$, such that $7^a = 3 \cdot 2^b + 1$.

8TH GRADE

In problems 1.–6. mark the correct answer.

1. Which three of the graphs of functions $y_1 = \frac{1}{2}x - 2$, $y_2 = 2x - 5$, $y_3 = \frac{1}{3}x - \frac{2}{3}$ и $y_4 = \frac{1}{6}x - \frac{4}{3}$ intersect in one point?

а) y_1, y_2 and y_3; б) y_1, y_2 and y_4; в) y_1, y_3 and y_4; г) y_2, y_3 and y_4.

2. If $a < b < c$, then the least value of $y = |x-a| + |x-b| + |x-c|$ is:

а) 0; б) $a+c$; в) a; г) $c-a$.

3. Think of a four digit natural number with pair wise different digits. With each of the numbers 3456, 4567 and 5409 my number has exactly three common digits, but on different positions. What is correct?

а) The digit 9 is in the number;
б) The first digit of the number is 6;
в) There is no digit 0 in the number;
г) The last digit of the number is 5.

4. Let a, b and c are natural numbers, such that $a^2 + b^2 = c^2$. If $a > 5$ is prime number, than not always is correct that:

а) $2b = a^2 - 1$; б) $a^2 - b = c$; в) $60|(bc)$; г) $5|c$.

5. Given ΔABC. The points P and Q, respectively on the sides BC and AC, are such that $BC = 3BP$ and $2AC = 3AQ$. The median $CM\,(M \in AB)$ of ΔABC intersects PQ in point N. The ratio $CN : NM$ is equal to:

а) $2:3$; б) $3:4$; в) $4:5$; г) $3:2$.

6. In the equation $\overline{abcd} + 2007 = \overline{efgh}$ each letter a, b, c, d, e, f, g, h denotes different digit. If V is the biggest possible value of $\overline{efgh}$ then:

а) $9000 < V$ б) $8000 < V \le 9000$; в) $7000 < V \le 8000$ г) $V < 6000$.

In problems 7, 8 and 9 write your answer.

7. The sum of all the solutions of the equation $|x^2 - 2| = |2x - 1|$ is:

Answer. ....

8. Given a trapezoid $ABCD$ $(AB \| CD,\ AB > CD)$. Let M and N be the midpoints of AD and BC respectively. The bisector of $\angle BAC$ intersects MN in the point L. let CH $(H \in AB)$ be the altitude of the trapezoid through the vertex C. If $CH = ML = h$ and $AD = LN$, then the area of the trapezium $ABCD$ is:

Answer. ....

9. In the equation $\overline{aaa} \cdot \overline{abbc} = \overline{ddeeed}$, equal letters denote equal digits and different letters denote different digits. The number of solutions of the equation is:

Answer. ....

Write your solution of problem 10.

10. Let N be the midpoints of the side AC of the triangle ABC and $\angle ABN = 15°$. The point M is interior for the triangle and $\angle AMC = 90°$, $\angle MCA = 75°$. If $AM = 10$, find the distance between the circumcentres of ΔABM and ΔACM.

PART II, TEAM COMPETITION

Work time 135 min.

2ND GRADE

Problem 1. **(Clock)** Zdravko has an electronic clock and often looked at it. He found that in [00:00] only the segments in the middle do not shine (fig. 1). Tell Zdravko how many minutes the segments shown on fig. 2 will shine together from [02:00] till [03:00].

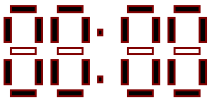


Fig. 1

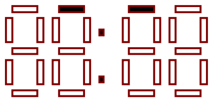
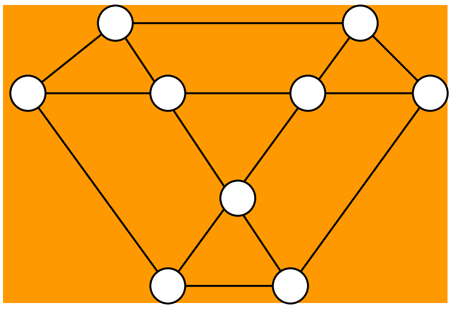


Fig. 2

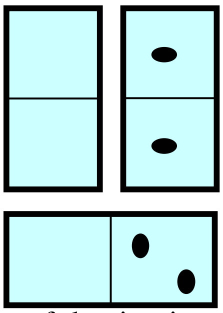
Problem 2. **(Candies)** Carlson has tree boxes with candies in which there are 21 candies. In the second box there are 4 times more candies than in the first. In the third the candies are more than in the fist and less than in the second. How many candies is there in each box?

Problem 3. **(Nine numbers)** Nine circles on the figure are vertices of four small triangles and three big triangles. Write in these circles the numbers from 1 to 9 in such way that the sum of the numbers in the vertices of the seven triangles to be one and the same.

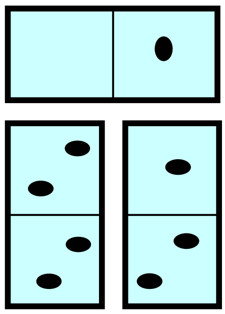


Problem 4. **(Domino)** In the game "Domino" there are 28 tiles. Three of them are arranged as it shown on the picture and an interesting addition:

01
01
———
02



01
21
———
22



Arrange 5 figures from 3 tiles of domino in such way that the sum on the lowest row to be 33, and draw them.

3D GRADE

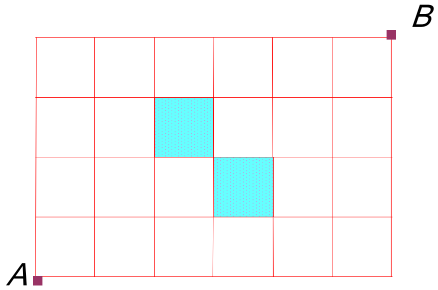
Problem 1. **(Candies)** Angel, Bob and Vesko eat together 7 candies, and everyone eat at least one candy. Angel eats most candies and Vesko least of all. How many candies eat Bob? Give reasons for your answer!

Problem 2. **(Collection)** May and Ray collect stamps. May has two times more stamps than Ray. If May gives 60 stamps to Ray, then they will have equal number stamps. How many stamps have May initially?

Problem 3. **(Wizards)** In the magic school "Hogwarts" is held a council of Wizard-teachers. One noticed that the age of any wizard is a three digit number, with product of digits 8. How many members there are in the council, if it's known that there are no wizards of one and same age?

Problem 4. **(Routes)** On the picture is shown scheme of a garden with lanes. The colored squares are under repair and one cannot walk on its sides (including the vertices).

How many are the routes from A to B, if one can walk only up and left?



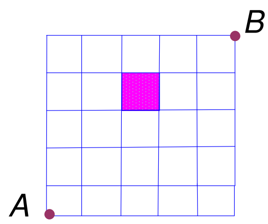
4TH GRADE

Problem 1. **(Sum of digits*)*** The sum of all digits used to write the natural numbers from 11 to 15 is 20. Find the sum of all digits used to write the natural numbers from 1 to 111.

Problem 2. **(Partition)** Five brothers come into money. According to the will the oldest inherited the half and 1 lev more. The second inherited the half of the rest and 2 levs more, the third – the half of the rest and 3 levs more, and the forth – the half of the rest and 4 levs more. The fift brother inherited 500 levs. What was the amount of the inheritance?

Problem 3. **(2007)** Find all representations of the number 2007 as a sum of no more than 20 consecutive natural numbers.

Problem 4. **(Routes)** On the picture is shown scheme of a garden with lanes. The colored square is under repair and one cannot walk on its sides (including the vertices). How many are the shortest routes from A to B?



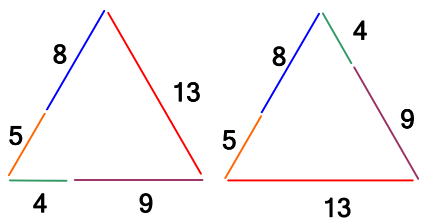
5TH GRADE

Problem 1. **(Aquarium)** You have all the materials necessary to built "aquarium" without cutting – sheet of paper, glue and ruler. The aquarium has to store at least 1565 ml, but not more than 1705 ml of water. Construct the model and describe how you made it.

Problem 2. **(Partition)** Five brothers come into money. According to the will the oldest inherited the half and 1 lev more. The second inherited the half of the rest and 2 levs more, the third – the half of the rest and 3 levs more, and the forth – the half of the rest and 4 levs more. The fift brother inherited 500 levs. What was the amount of the inheritance?

Problem 3. **(Operation (Process))** Given a square 3x3 in each little square of which are written zeros. The following operation is carried out: add 1 in every little square of every square 2x2. After several such operations the numbers in the central square and in corner squares are erased. The remaining numbers are 9, 10, 12 and 13. Find the number in the central square before the erasing.

Problem 4. **(Matches)** One can construct equilateral triangle using matches with length 4 cm, 5 cm, 8 cm, 9 cm and 13 cm, as it is shown on the figure. We should not distinguish

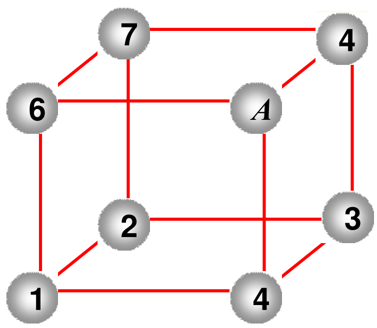


the triangles with one and the same side length, when only the positions of matches are changed. You have seven matches with length 1 cm, 2 cm, 3 cm, 4 cm, 5 cm, 6 cm and 7 cm. How many different equilateral triangles you can construct using these matches (or part of them) without braking, overlapping or sticking?

6^{TH} GRADE

***Problem 1.* (Triangular numbers)** The numbers 1, 3, 6, 10, 15,... are known as "triangular numbers" and each of them is obtained by the formula $\frac{n(n+1)}{2}$, where n is natural number. Find the value of n, for which is obtained:
a) The least three digit triangular number;
б) The greatest triangular number, less than 2007?

***Problem 2.* (In the vertices)** In each vertex of the cube is written a number. For one move you can add 1 to both numbers on one (arbitrary) edge. After several moves all the numbers become equal. Find the values of A, for which this is possible.



***Problem 3.* (Digits)** Find the sum of the digits of the number $\underbrace{11\;\ldots\;11}_{9\text{ digits}}\cdot\underbrace{11\;\ldots\;11}_{18\text{ digits}}$.

***Problem 4.* (Arrangements)** a) Put 6 points on 4 segments, in such way, that on every segment lie 3 points.
b) Draw 5 segments with equal length, in such way, that on every segment lie 4 points from altogether 10 points.
c) Draw 6 segments, in such way, that on every segment lie 3 points from altogether 10 points.
d) Draw 6 segments, in such way, that on every segment lie 4 points from altogether 12 points.
д) Arrange 24 chairs in 6 rows, in every row 5 chairs.

7^{TH} GRADE

***Problem* 1.** The numbers a and b, are such that the equation $\left(2x-a^2-b^2\right)^2+\left(x-ab\right)^2+\left(x-1\right)^2=0$ has a solution. Find a and b and solve the equation.

***Problem* 2.** Given a triangle ABC, M is the midpoint of the side BC and $\angle BAM=15°$. If the altitude of ΔABC, through the vertex C is equal to h and the area of ΔABC is equal to h^2, find the measures of the angles of ΔABC.

***Problem* 3.** The segment $AL\left(L\in BC\right)$ is bisector in the right triangle ABC. The point M on the hypotenuse AB is such that $\angle ALM=90°$. Prove that $AM<BM+AC$.

***Problem* 4.** Are there natural numbers x, y and z such that $x^2+y^3+z^3=2^{2007}$? Give reasons for your answer!

8^{TH} GRADE

***Problem* 1.** The real numbers a and b, are such that the equation $\left(a^2+3b^2\right)x^2-\left(4a+6b\right)x+7=0$ has a solution 2007. Find a and b and solve the equation.

***Problem* 2.** a) Prove that for every natural number m there exists natural number n such that the number n^2+9n-6 is divisible by 2^m;

b) Find all the pairs of natural numbers x and y, such that $x^2+9x-6=2^y$.

***Problem* 3.** Let M and N be arbitrary points on the side AB of the triangle ABC. If the points G, G_1 and G_2 are centroids of the triangles ABC, ANC and BMC respectively, prove that G, G_1 and G_2 are collinear.

***Problem* 4.** Find all the pairs of natural numbers x and y, such that $\sqrt{x}+\sqrt{y}=\sqrt{2007}.$

Results Scored:

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