Winter Mathematical Competitions, grade 5-8

Area: Mathematics

Style of Competition: Nation wide; Presence; for individuals

Competition rules: 4 problems with requirement the students to present arguments (proofs) in written form. Work time – 4 hours, maximum score – 40 points (10 for every problem).

The competition is held in the end of January or beginning February in Burgas, Varna, Pleven or Ruse.

Target Group: For students of high abilities;

Age of Participants: 9 – 15 years old;

School level of Participants: Primary& Secondary schools: 4 – 8 grades;

Number of Participants in the Last 3 Years: 500 – 600 annually.

History of Competition:

The principal organizers are Union of Bulgarian Mathematicians, Ministry of Education.

Financial Basis of the competition: Self-supporting event and sponsored by Ministry of Education.

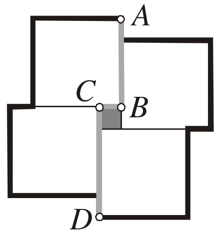
Competition Problems:

WINTER MATHEMATICAL COMPETITIONS – BURGAS, 2007.
4^{th} GRADE

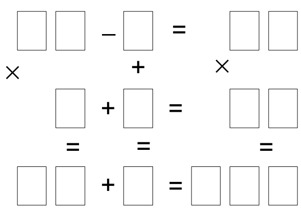
4.1. Twenty fives are written one after another 5 5 5 … 5 5. Write between some of the digits sign „+", to obtain a sum 1000. (The digits without a sign between them we regard as one number.)

4.2. The numbers 2, 7, 12, 17, 22, 27, ..., 2002, 2007 are written following the rule: every number is obtained by adding 5 to the previous one. How many numbers are written?

4.3. The figure on the picture is composed of four big squares with equal sides and one small square. The side of the big square is 4 times bigger than the side of the small square. The length of the broken line *ABCD* is 30 cm. Find the area and circumference of the figure.



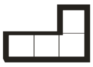
4.4. Write on every piece of cardboard one digit in such way that all the six equalities to be realized.

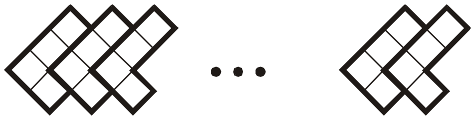


5^{th} GRADE

5.1. Solve the puzzle A . DDA = DUUM : A, where the different letters correspond to different digits.

5.2. In the circus there were three clown costumes of shirt, trousers and shoes. First costume was blue, second – red, and the third – green. Clowns AN, BAN and CAN mix up the costumes and appear on the arena dressed in this way: The shirt and shoes of AN were in the same color. Nothing dressed by CAN was red. The shoes of BAN were green, and shirt and trousers in the other two colors. What is the color of shirt, trousers and shoes of each of them?

5.3. A tile Γ-tetramino  is composed of four squares with side length 3 cm. Such tiles are arranged one to another as it is shown:



a) If the number of tiles is 2007, find the area and circumference of the obtained figure.
b) If the circumference of the figure is 7002 cm, find its area.

5.4. The lap "Traffic lights" in the race is 9 km long. On the second kilometer the traffic light is 3 minutes green, 3 minutes red and so on. On the forth kilometer the traffic light is 2 minutes green, 1 minute red and so on. On the sixth kilometer the traffic light is 4,5 minutes green, 5,5 minutes red and so on. Each car started when all three traffic lights are red and is not permitted to stop or to change its speed till the end. The "Eco" car had started at 10:45 and covers the lap according the rules, for minimum possible time. Find the finish time and velocity (km/h) of "Eco".

6^{th} GRADE

6.1. In a parking lot the red cars are 25% of all cars. After one hour there were 3 cars more in the parking lot and the red cars are 12% of all cars. What is the minimum possible number of the cars in the beginning and how many of them have been red?

6.2. Find all pairs of natural numbers *m* and *n*, such that $1!+2!+3!+\cdots+n!=m^2$.
(*n*! denotes the product of the natural numbers from 1 to *n*: $n!=1.2.3.\cdots.(n-1).n$)

6.3. In triangle *ABC*, *P* is the midpoint of the side *BC*, *T* is a point on the side *AC* and $AT=4TC$. The segments *AP* and *BT* intersect in the point *M*. Find what part of the area of quadrangle *TMPC* is the area of triangle *CPM*.

6.4. The number 4608 is written on the blackboard. Every minute the number is multiplied or divided (only if possible without reminder) with 2 or with 3. The result is written on the blackboard and the previous number is erased. Is it possible exactly after 33 hours 27 minutes on the blackboard to be written the number 27? How many minutes at least are necessary the number 27 to appear on the blackboard?

7^{th} GRADE

7.1. At 9 o'clock from port *A* to port *B*, started a motor-launch, upstream the river, (the stream velocity is 3 km/h). Two hours and twenty minutes after the start the engine of the motor-launch breaks down, and the crew needed 1 h 20 min to repair it. After the break the engine dissipates power and the velocity of launch reduced with 25%. The launch arrived in *B* 2 hours 48 minutes after the restart. Find the proper velocity of the launch before the break and the distance between A and B, if the distance covered before the break is with 7 km more than the distance covered after the break.

7.2. Given a square $ABCD$ with side a. Points M and N lie on the sides BC and CD respectively, and $\angle MAN = 45°$. Find the perimeter of triangle MNC.

7.3. Find the least natural number k, for which the equation

$$x_1^2 + x_2^2 + \ldots + x_k^2 = 2007$$

Has a solution in the set of natural numbers.

7.4. The set E consists of 37 natural numbers none of which is divisible by 10. Prove that there are 5 numbers in E, such that for any pair of them the digits of tens are different and the digits of units are different.

8th GRADE

8.1 Given the equation $|2x-1|-1 = x^2 - a$.

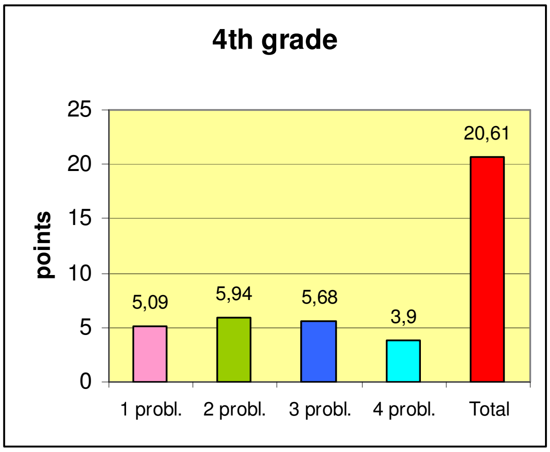
a) Solve the equation when $a = 2$;
b) Find the values of the parameter a, for which the equation has two integer roots.

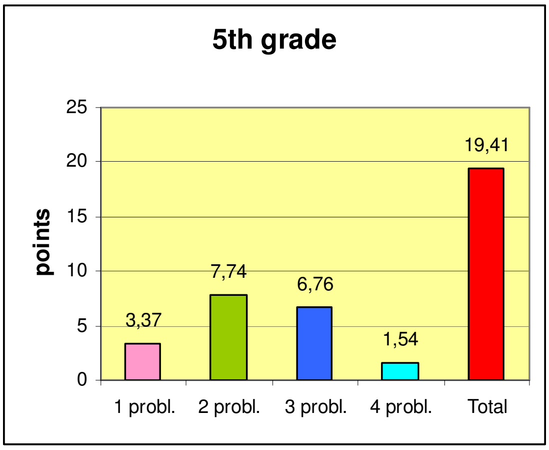
8.2. Let $x(4-3x) + y(4-3y) = 3xy$. Prove that $0 \le x + y \le \frac{16}{9}$.

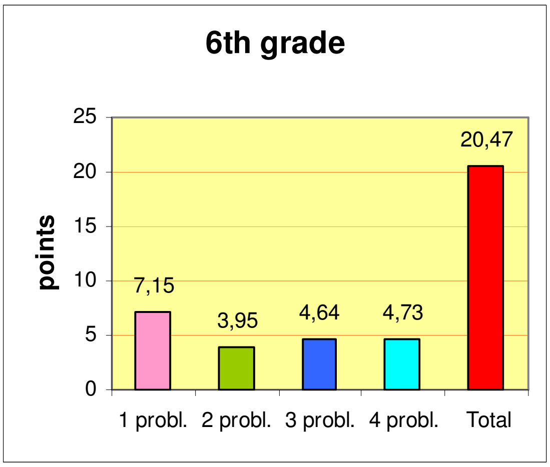
8.3. Given triangle ABC with $\angle ACB = 2\angle ABC$. The point M on the side AC is such that $CM = BC$. Find the measures of the angles of triangle ABC, if $BM = AC$.

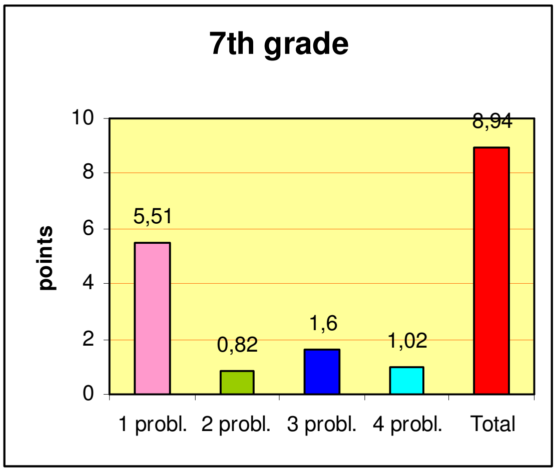
8.4. In a square table 2007 x 2007 are nonnegative integers, in such way, that if the number in a cell is 0, then the sum of the numbers in the row and the column, which intersect in this cell, is not less than 2007. Prove that the sum of all the numbers in the table is not less than 2 014 025.

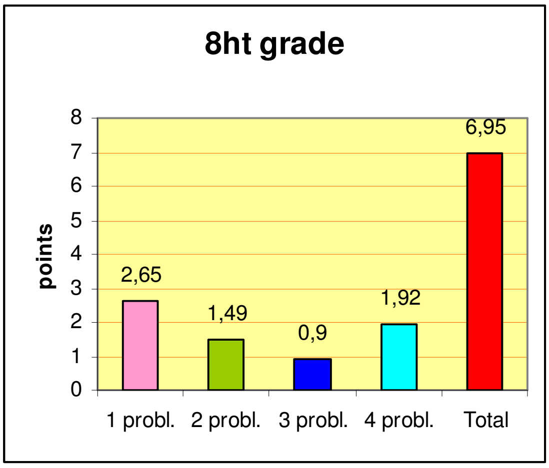
Results Scored:

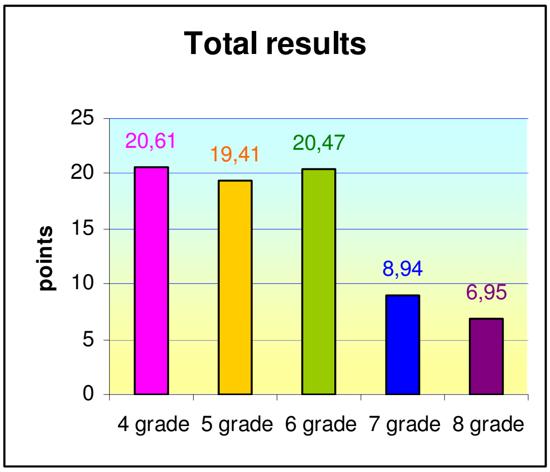












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