# Mathematical talents search Um+
# Problems, season 2005 / 2006
# 4$^{th}$ grade

Problem 1. A cook has to boil an egg for exactly 15 minutes. He has no usual clock, but two sand-glasses, the first of which measures exactly 19 minutes and the second one - exactly 23 minutes. Help the cook to boil the egg.

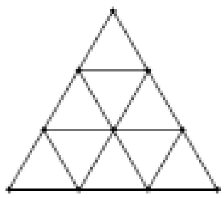
Problem 2. Solve the arithmetical puzzle **NOVINI + STATII = VESTNIK**, where different letters represent different digits and equal letters represent equal digits.

Problem 3. The lengths of the sides of a rectangle are different primes and its circumference equals 28 cm. A square is cut of from the rectangle so, that they have a common vertex. The length of the side of the square is a prime number, too. Find the circumference and the area of the remaining figure.

Problem 4. A student visited Rome and took a picture of the Coliseum. Then he visited Paris and took a picture of the Eiffel tower. After that he went to London and snapped Big Ben as well. When he came back home, he wrote on the flipside of each picture: “Paris”, “Not the Eiffel tower” and “Not the Coliseum”. After turning the pictures back to the right side, he noticed that only one of the captions **wa**s true. What was on every picture? Give your reasons why.

Problem 5. The difference between a four-digit and a three-digit number is 8002, while their sum is a five-digit number. Find these numbers.

Problem 6. Pepi made a triangular grid out of matches like shown in the picture below (all triangles are equilateral).



Pepi’s grid had 484 small triangles.

a) How many matches used Pepi?

b) If every matchbox contains 50 pieces, how many matchboxes used Pepi at least?

Problem 7. Tommy, Annika and Mr. Nilsson gave Pippi 14 candies. Tommy gave her two times less candies than Mr. Nilsson. Annika gave Pippi more candies than Tommy, but less than Mr. Nilsson. How many candies gave Pippi each of them?

Problem 8. The teacher gave the students in 4$^{a}$ class 779 congruent squares. The first student received a few squares and each next student received two more squares than the previous one. How many are the students in 4$^{a}$ class and how many squares received each of them? How many of the students can construct bigger squares (maybe more than one) using all of the received squares? Show how it can be done!

Problem 9. A rectangle with integer sides are can be divided into 12 congruent squares. The area of the rectangle, measured in square centimeters, equals to the least three-digit number, divisible by 9. Equilateral triangles are drawn on each side of the rectangle, outside of it . Find the circumference of the obtained figure.

Problem 10. On the both sides of a few cards Ivan wrote a digit so that each digit was written at least once and the difference between the digits written on each card was 1. He found out that he had written the maximum possible number of different cards and that any arrangement of all the cards in a row represented a different integer.

a) Find the smallest number with even digits, which Ivan can obtain using all cards;

b) How many different numbers, each of them with even digits, can be obtained by using some of the cards?
c) Which numbers are more – those with only even digits or those with only odd digits, that can be written using all cards?

# $5^{th}$ grade

<u>Problem 1</u>. We have two empty vessels with capacity 14 l and 11 l respectively and unlimited amount of water. Show how to measure exactly 1l of water.

<u>Problem 2</u>. Do there exist two-digit numbers $\overline{ab}$ and $\overline{cd}$ such that:

a) $LCM\left(\overline{ab},\overline{cd}\right)=\overline{abcd}$ ;

b) $LCM\left(\overline{ab},\overline{cd}\right)=a\cdot b\cdot c\cdot d$ ?

<u>Problem 3</u>. The rectangle in the picture is divided into 9 smaller rectangles with lines, parallel to its sides. The numbers written inside some of the rectangles, show their areas. Find the area of the rectangle indicated with a question mark.

| | | |
|---|---|---|
| ? | | 18 |
| | 32 | 8 |
| 18 | 24 | |

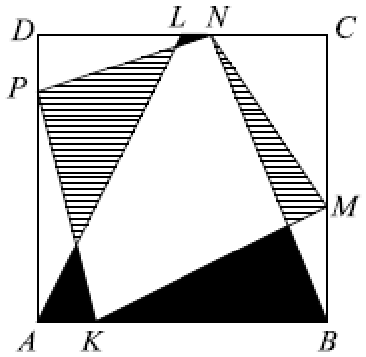
<u>Problem 4</u>. Ani, Biliana, Valia, Galia, Diana and Elena went on a trip. During the first day in the bus Ani sat next to Ognianova, Biliana – next to Nikolova, and Galia – next to Elena. On the next day Diana sat next to Koleva, Biliana and Ognianova listened to some music together and Marinova sat next to Elena. The trip finished by a march to peak Shipka. First on the peak climbed Biliana, after her together came Galia and Marinova, behind them was Diana and last came together Valia and Lilkova. It is known that Petrova and Lilkova are friends. Find the cognomen of each girl.

<u>Problem 5</u>. We gathered some apples from our apple-tree this autumn. The number of the apples is divisible by 13. Once we ate 6 of them, I noticed that the number of the remaining apples is a multiple of 2, 3, 4 and 5. How many apples did we gather from the apple-tree, if they are more than 500 and less than 700?

<u>Problem 6</u>. A rectangle with circumference of 14 dm is divided into 4 squares. Find, in square centimeters, the area of the rectangle.

<u>Problem 7</u>. Tommy and Annika bought presents for the Pippi's birthday and decided to pack them. Tommy packed one present, using 120 cm of band. Annika packed one present too, but she used 150 cm of band. If they have in common 34.5 m band, how many presents at most can they pack?

<u>Problem 8</u>. The square $ABCD$ has a side of 5 cm. The points $K, M, N$ and $P$ are on its sides (as shown in the picture) so that $AK=1\,cm, BM=2\,cm, CN=2\,cm$ and $DP=1\,cm$. The point $L$ is chosen on $DC$ so, that $DL=2\,cm$. Prove that the area of the marked part is equal to the area of the dark part.



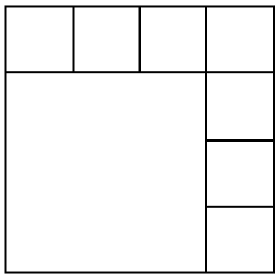

Problem 9. Replace the asterisks with digits so, that

a) The number $\overline{4*5*8*3*}$ be a multiple of 36;

b) The number $\overline{5*8*43**}$ be a multiple of 33.

Which numbers are more – these from a) or these from b)?

Problem 10. In how many different ways can the numbers from 1 to 8 be written in the squares in the picture (one number per square) so that there were no two adjacent squares containing numbers with difference greater or equal to 4? (Adjacent squares have more than one common point).



# 6th grade

Problem 1. A millionaire set an electrical lock on his safe. He arranged six lamps in a row, numbered consecutively and each lamp had its own switch. The first lamp could be switched on or off independently. For the rest of the switches there was a condition. Each of them activated (on and off) the corresponding lamp only when the closest previous lamp was switched on and all the rest lamps with smaller numbers were switched off. The safe only opens when all six lamps are switched on. If each switching (on or off) takes one second, how long it will take the millionaire to open the safe?

Problem 2. Find the smallest natural number, which divided by the sum of its digits gives:

a) 2005

b) 2006

Problem 3. Is there an integer number $k$, such that $2006^k + 2$ were:

a) a perfect square of an integer;

b) a sum of three squares of integers?

Problem 4. There are several cities in the shoreline of a big round lake. It is known that two cities are connected with a road only if the two respectively following them in counterclockwise direction are not. Is it possible that the cities were:

a) 4;

b) 2005?

Problem 5. Is it possible that the difference between the number $A$ and the number, written with the digits of $A$ in reverse order were the square of a natural number?

Problem 6. Before going to a party, the Cinderella's step-mother spilled on the floor beans, lentil and peas, one kilogram each, and had ordered Cinderella to pick them up. It is known that the ratio of the number of the grains in 1 kilogram of pea to the number of the grains in 1 kilogram lentil is 6:13 and the number of the bean grains is $20\frac{5}{6}\%$ of all grains. Find the numbers of all types of grain, if there were 2400 grains in total.

Problem 7. Prove that $\frac{1}{5}<\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}+\ldots+\frac{1}{98}-\frac{1}{99}+\frac{1}{100}<\frac{2}{5}$.

Problem 8. The base of the pyramid is a trapezium. It is known that the lengths in centimeters of the small base, the altitude and the big base of the trapezium are the three smallest odd primes. The volume of the pyramid is not less than 100 $cm^3$ and less than 140 $cm^3$. Find the length of the altitude of the pyramid in centimeters, if it is known that it is a prime number.

Problem 9. Can you find three different natural numbers, such that the sum of each two of them was power of:

a) 2;

b) 3?

Problem 10. How many six-digit numbers of the form $\overline{abcacb}$ are multiples of 23?

## $7^{th}$ grade

Problem 1. Show that each binary number, which comprises only 1's cannot be a power of an integer (square, cube, etc.).

Problem 2. The measurements of the dimensions of the rectangular parallelepiped in centimeters are three different primes. The sum of the lengths of all edges (again in centimeters) of the parallelepiped is a two-digit number, multiple of 8, less than 80, and its volume is a tree-digit number. Find the surface area and the volume of the parallelepiped.

Problem 3. Find all pairs of natural numbers, which are lengths of the sides of squares, such that:

a) The difference between their areas equals 2005;

b) The sum of their areas equals 2005.

Problem 4. There are several cities in the shoreline of a big round lake. It is known that two cities are connected with a road only if the two respectively following them in counterclockwise direction are not. Show that each city can be reached from any other one with no more than two remounts.

Problem 5. Find all pairs of integer numbers $p$ and $q$, only one of which is prime and such that $p^4+2005q^2=q^4+2005p^2$.

Problem 6. A rectangular coordinate system with unit of 1 cm is given and the points $A(0;5), B(3;2)$ and $C(5;6)$ are lying in this system. If the point $M\in BA$ and $\frac{AM}{MB}=\frac{1}{3}$, the point $N\in BC$ and $\frac{BN}{BC}=\frac{2}{3}$, find the area of the triangle $BMN$.

Problem 7. Which is the bigger number: $1{,}1^{100}$ or 1000? Justify your answer.

Problem 8. Let $a,b,c$ and $d$ be arbitrary numbers from the interval $[2005;2006]$. Prove that $-1\le ab-bc+cd-da\le 1$.

Problem 9. In $\Delta ABC, \sphericalangle BAC:\sphericalangle ABC:\sphericalangle ACB=10:3:5$. The points $P$ and $Q$ are chosen on the perpendicular bisector of $BC$ so that $AP=AQ=AC$, $Q$ and $A$ lie in the same semiplane in relation to $BC$. Prove that $\Delta ACP$ and $\Delta BCQ$ are equilateral.

Problem 10. Find all natural numbers, which are equal to the sum of three different divisors of theirs.