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PREFACE

The Workshop is organized by the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences. The purpose of the event is to present the current state of the art in group theory and ring theory and their applications. In particular, we emphasize on combinatorial group theory, profinite groups, finite simple groups, combinatorial and computational ring theory, noncommutative ring theory and theory of PI-algebras, commutative and noncommutative invariant theory, automorphisms of polynomial and other free algebras, representation theory of groups, Lie algebras and Lie superalgebras, Galois theory. The applications are oriented but not limited to scientific computations, coding theory, statistics, finance.

The scientific program of the Workshop includes more than 50 invited and ordinary talks presented by mathematicians from 17 countries in Europe, Asia, North and South America. Many of the participants are established mathematicians and recognized leaders in their fields. We are very glad that also young people from several countries participate at the meeting with their own scientific contributions which is a good promise for the future of Algebra.

Sofia, July 2015

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MAIN TALKS
Classical invariant theory and its applications

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Keywords: Classical invariant theory, locally nilpotent derivations, special functions, polynomial identities, graph invariants

2010 Mathematics Subject Classification: 13A50, 11B68, 05A19, 05C30

We consider an application of classical invariant theory to two combinatorial problems.

Let \( \{P_n(x)\} \), \( n = \deg P_n(x) = 0, 1, 2, \ldots \), be a system of polynomials over \( \mathbb{Q} \). We are interested in finding polynomial identities for the system of polynomials, i.e., identities of the form

\[
F(P_0(x), P_1(x), \ldots, P_n(x)) = 0,
\]

where \( F \) is some polynomial in \( n + 1 \) variables. Using methods of classical invariant theory a general approach to finding of identities for some well-known families of polynomials (Bernoulli, Euler, Hermite, Fibonacci, Lucas, Kravchuk polynomials) is proposed.

Also, we find the generating function for the numbers of simple graphs with \( n \) vertices and \( k \) edges and calculate the algebras of invariants of simple graphs for small \( n \).
Construction of infinite finitely presented nilsemigroup* 

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Keywords: Prime spectrum, Krull dimension, growth.

2010 Mathematics Subject Classification: 20M05, 20M07, 20F32.

We present the solution of the problem of Shevrin and Sapir. In [1] we construct an infinite finitely presented nilsemigroup with identity $x^9 = 0$. The new method of construction is based on aperiodic tilings and Goodman-Strauss-type theorems on uniformly elliptic spaces. The space is called uniformly elliptic iff there is a universal constant $\lambda > 0$ such that any two points $A$ and $B$ on distance $D$ can be joined by a family of geodesic lines generating a disc of wideness $\lambda \cdot D$. Any defining relation in the semigroup corresponds to a local equivalence of two paths on the constructed space.

References


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Set-theoretic solutions of the Yang–Baxter equation, braces and braided groups

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**Keywords:** Yang-Baxter, quantum groups; multipermutation solution; matched pair of groups; braided group; permutation group.

**2010 Mathematics Subject Classification:** 81R50, 16W35, 16W22, 20B35.

Set-theoretic solutions of the Yang–Baxter equation (YBE) form a meeting-ground of mathematical physics, algebra and combinatorics. Such a solution consists of a set $X$ and a bijective map $r : X \times X \to X \times X$ which satisfies the braid relations. It is known that set-theoretic solutions of YBE are closely related to braided groups and the theory of matched pairs of groups. Recently several authors investigate symmetric sets $(X, r)$, (nondegenerate involutive solutions of YBE) via the theory of braces. We study the intimate relations between symmetric groups and general braces. As an application we find some close relations between the properties of a symmetric set $(X, r)$ and the associated symmetric groups and left braces: the Yang-Baxter group $G = G(X, r)$, and the permutation group $\mathcal{G} = \mathcal{G}(X, r)$.

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Anomalies on codimension growth of algebras

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Keywords: Polynomial identity, codimension, growth.

2010 Mathematics Subject Classification: 16R10, 16P90.

In the last century A. Regev conjectured that if $A$ is an associative PI-algebra, its codimension sequence is asymptotically equal to $Cn^t d^n$, where $C$ is a constant, $d$ is an integer and $t$ is a half integer. I shall discuss the positive results obtained in the associative case ([1], [2], [5], [6]) and I shall provide some counterexamples for nonassociative algebras ([3], [4], [7]).

References


Twisted localization of weight modules

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Keywords: Lie algebra, indecomposable representations, quiver, weight modules, twisted differential operators.

2010 Mathematics Subject Classification: 17B10.

The twisted localization functor is a localization-type functor for non-commutative rings. This functor plays crucial role in the classification of the simple objects of various categories of weight modules, i.e., modules that decompose as direct sums of weight spaces. In this talk we will discuss various applications of the twisted localization for finite-dimensional Lie algebras and algebras of differential operators. Most of the talk will be based on a joint work with Vera Serganova.
Words and strong completeness of profinite groups: beyond finite generation

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Keywords: Profinite groups, verbal width, group words.
2010 Mathematics Subject Classification: 20E18, 20F69.

Let $G$ be a profinite group and $w$ be a group word. There has been a lot of progress towards understanding the verbal subgroup $w(G)$ for many interesting words $w$ (for example commutators and powers) when $G$ is topologically finitely generated. In particular in the examples above $w(G)$ is closed in $G$ and $G$ is strongly complete (i.e., each subgroup of finite index is open in $G$). None of this remains true when $G$ is not finitely generated. Nevertheless in some situations we can prove suitable analogues even when $G$ is not finitely generated. In this talk I will discuss some natural questions and partial results in this more general setting.
Fundamental algebras with polynomial identities

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Keywords: PI-algebras, invariants of matrices, Kemer polynomials.

2010 Mathematics Subject Classification: 16R10, 16R30.

I will discuss some geometric aspects of the theory of Kemer for finite dimensional algebras. In particular I will show that the semisimple part of a basic or fundamental algebra is determined by its polynomial identities. This requires to give, to the space of Kemer polynomials, the intrinsic structure of module over the commutative algebra of semisimple representations of the free algebra, into the semisimple part of the given algebra. This is the coordinate ring of a subvariety of the variety whose algebra are suitable matrix invariants.
On the amount of central polynomials

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Keywords: PI-algebras, central polynomials, Young tableaux.
2010 Mathematics Subject Classification: 16R10, 15A75.

We study the quantity of the central polynomials of the infinite dimensional Grassmann algebra, and of the algebra of matrices.
Remarks on differential polynomial rings, tensor products and growth of algebras

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Keywords: Differential polynomial rings, nil ring, the Jacobson radical, growth of algebras, the Gelfand-Kirillov dimension, Golod-Shafarevich algebras.

2010 Mathematics Subject Classification: 16S32, 16N20, 16N40, 16D25, 16W25, 16P40, 16S15, 16W50.

We describe a new way of constructing examples of differential polynomial rings by embedding them into bigger rings, which we call platinium rings. As an example, we show that there is a ring $R$ and a derivation $D$ on $R$ such that $R$ is not nil and the differential polynomial ring $R[x; D]$ is Jacobson radical. This is in contrast with Amitsur’s theorem from 1956, which says that if a polynomial ring $R[x]$ is Jacobson radical then $R$ is a nil ring. In the second half of the talk, we mention other results on nil rings related to differential polynomial rings, groups and tensor products. We will also look at a construction of algebras with various growth functions satisfying arbitrary prescribed homogeneous relations under some restrictions on the number of relations of each degree.
Graded codimensions of Lie superalgebras

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Keywords: Graded polynomial identities, Lie superalgebras, codimensions, exponential growth, PI-exponent.

2010 Mathematics Subject Classification: 17B01, 16P90, 16R10.

We consider finite dimensional Lie superalgebras over a field of characteristic zero and study their $\mathbb{Z}_2$-graded identities. We pay main attention to the numerical invariants of identities, in particular, to the graded codimensions and their asymptotic behaviour.

It is well-known that in case $\dim L < \infty$ both graded and ordinary codimensions are exponentially bounded ([1]). One of the more important questions of the theory of numerical invariants of polynomial identities is: does the (graded) PI-exponent exist?

There are many papers where the existence of the PI-exponent is proved for different classes of algebras. For example, if $A$ is an associative PI-algebra or a finite dimensional Lie, Jordan or alternative algebra then its PI-exponent exists and is a non-negative integer (see [2], [3], [4], [5]). The existence of the PI-exponent for any finite dimensional simple algebra was proved in [6]. It is not difficult to show that if the PI-exponent of $A$ exists then it is less than or equal to $d$ provided that $d = \dim A < \infty$ (see for example [1]).

In many important classes of algebras over an algebraically closed field (associative, Lie, Jordan, alternative) the equality $\exp(A) = \dim A$ is equivalent to the simplicity of $A$ ([2], [3], [5]). Recently it was shown in [6] that $\exp(L) <$

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dim $L$ for any finite dimensional simple Lie superalgebra $L$ of the type $b(t)$, $t \geq 3$, (in the notation of [7]). The existence of the PI-exponent and the similar inequality $\exp(L) < \dim L$ for $b(2)$ was also proved in [6] although $b(2)$ is not a simple superalgebra.

Graded codimensions of Lie superalgebras were studied much less. In the talk we prove the existence of the graded PI-exponent for any finite dimensional simple Lie superalgebra. Moreover, for the series $b(t)$, $t \geq 2$, we show that $\exp^{gr}(b(t))$ does not exceed $t^2 - 1 + t\sqrt{t^2 - 1}$. For $b(2)$ this is the precise value, i.e., $\exp^{gr}(b(2)) = 3 + 2\sqrt{3}$. Since the ordinary PI-exponent is less than or equal to the graded PI-exponent it follows that the PI-exponent of any simple Lie superalgebra $b(t)$, $t \geq 3$, is bounded by the value $t^2 - 1 + t\sqrt{t^2 - 1}$ which is strictly less than $\dim b(t) = 2t^2 - 1$.

All details concerning numerical invariants of polynomial identities one can find in [8].

References


Specht’s problem
and pro-$p$ identities of pro-$p$ groups

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Keywords: Pro-$p$ group, polynomial identity.
2010 Mathematics Subject Classification: 16R50, 20E18.

We will discuss connections between some versions of the Specht Problem and existence of pro-$p$ identities in linear pro-$p$ groups.
Infinite dimensional representations of Lie groups and Lie algebras

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Keywords: Smooth representation, algebraic dual, $H$-finite vectors.
2010 Mathematics Subject Classification: 22E45, 17B10.

Let $G$ be a finite dimensional reductive algebraic group over the reals. Let $(\pi, V)$ be a smooth representation of $G$ in a complete locally convex topological vector space over the complex numbers. Let $V$ also denote the corresponding module over the Lie algebra $\mathfrak{g}$ of $V$, and let $V^*$ be the algebraic dual module. Let $H$ be a closed connected subgroup of $G$ and let $\Gamma_H(V^*)$ denote the $H$-finite vectors in $V^*$. Although $\Gamma_H(V^*)$ is no longer a module over the group $G$, it is nevertheless a compatible $(\mathfrak{g}, H)$-module.

By abstraction, we can consider the category of all compatible $(\mathfrak{g}, H)$-modules. Harish-Chandra and Langlands considered the case when $H$ is (the identity component of) a maximal compact subgroup of $G$. Kostant considered the case when $H$ is a maximal nilpotent subgroup of $G$.

Penkov, Serganova, and Zuckerman applied both geometric and algebraic methods to the theory of $(\mathfrak{g}, H)$-modules when $H$ is the identity component of an arbitrarily chosen real algebraic subgroup of $G$. In particular, they established the existence of $H$-admissible simple $(\mathfrak{g}, H)$-modules $V$ satisfying a natural assumption on the so called Fernando-Kac subalgebra associated to $V$. Currently, Penkov, Serganova, and Zuckerman are pursuing their program for the case $H$ of type $A_1$. This case may be a model for the general case of $H$ real reductive.
TALKS
On G-graded verbally prime algebras

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Keywords: Polynomial identity, graded algebra.
2010 Mathematics Subject Classification: 16R10, 16W50.

One of the outcomes of Kemer’s representability theorem for PI (associative) algebras over an algebraically closed field of characteristic zero $F$ is the classification of the verbally prime algebras (precisely, these are $M_n(F)$, $n \geq 1$, or $E(A)$, the Grassmann envelope of any finite dimensional $\mathbb{Z}_2$-graded simple algebra $A$ over $F$). The condition of being verbally prime may be stated either in terms of products of $T$-ideals or in terms of products of multilinear polynomials with disjoint sets of variables.

Kemer’s representability theorem was extended to the context of $G$-graded algebras where $G$ is a finite group and it is natural to pose the analogous question in that context: “classify the $G$-graded verbally prime algebras”. One may think that the answer is the natural one, namely the finite dimensional $G$-graded simple algebras over $F$ or the Grassmann envelope of finite dimensional $\mathbb{Z}_2 \times G$-graded simple algebras over $F$. It turns out that answer is only “half” correct. Joint work with Yakov Karasik.
On classification
of four-dimensional division algebras
over finite fields \( \mathbb{F}_q \)

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Keywords: Division algebra, automorphism group.
2010 Mathematics Subject Classification: 17A75, 17A45.

Let \( A \) be a four-dimensional division-algebra over a finite field \( \mathbb{F}_q \), where \( q \) is an odd prime, admitting an elementary abelian four-group of automorphisms \( E \leq \text{Aut}(A) \). By [1, Proposition 1], \( E \) acts freely on \( A \). We assume that \( E \) acts freely of rank 1 on \( A \), i.e., \( A \cong \mathbb{F}_q[E] \). The aim of this talk is to classify this class of division algebras using tools from algebraic geometry. It is remarkable to mention that division algebras of dimension 4 over \( \mathbb{F}_q \), \( q \) odd prime, admitting Klein’s four group of automorphisms and having \( \mathbb{F}_{q^2} \) in the left nucleus are classified in [2].

References

Asymptotics for graded Capelli polynomials

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Keywords: Superalgebras, polynomial identities, codimensions, growth.
2010 Mathematics Subject Classification: 16R10, 16P90, 16W55.

The finite dimensional simple superalgebras play an important role in the theory of PI-algebras in characteristic zero. In this talk we present a characterization of the $T_2$-ideal of graded identities of any such algebra by considering the growth of the corresponding supervariety [1]. We consider the $T_2$-ideal $\Gamma_{M+1,L+1}$ generated by the graded Capelli polynomials $\text{Cap}_{M+1}[Y,X]$ and $\text{Cap}_{L+1}[Z,X]$ alternating on $M+1$ even variables and $L+1$ odd variables, respectively. We prove that the graded codimensions of a simple finite dimensional superalgebra are asymptotically equal to the graded codimensions of the $T_2$-ideal $\Gamma_{M+1,L+1}$, for some fixed natural numbers $M$ and $L$. In particular

$$c_{n}^{\text{sup}}(\Gamma_{k^2+l^2+1,2kl+1}) \simeq c_{n}^{\text{sup}}(M_{k,l}(F))$$

and

$$c_{n}^{\text{sup}}(\Gamma_{s^2+1,s^2+1}) \simeq c_{n}^{\text{sup}}(M_{s}(F \oplus tF)).$$

These results extend to finite dimensional superalgebras a theorem of Giambruno and Zaicev [2] giving in the ordinary case the asymptotic equality

$$c_{n}^{\text{sup}}(\Gamma_{k^2+1,1}) \simeq c_{n}^{\text{sup}}(M_{k}(F))$$

between the codimensions of the Capelli polynomials and the codimensions of the matrix algebra $M_{k}(F)$.
References


Minimum distance and covering radius of some Melas-like cyclic codes

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\textbf{Keywords:} Coding theory, cyclic codes, Melas codes, minimal distance, covering radius, equations over finite fields.

\textbf{2010 Mathematics Subject Classification:} 94B15, 94B65, 11T71.

Let $\mathbb{F}_P$ be the finite field of $P = p^k$ ($p$ is a prime) elements. Let $\mathbb{F}_q (q = P^m)$ be an extension of degree $m$ of $\mathbb{F}_P$ and let $\alpha$ be a generator of the multiplicative group $\mathbb{F}_q^\ast$. The codes of Melas [2] are a class of cyclic codes generated by the product $M_{\alpha}M_{\alpha^{-1}}$, where $M_{\alpha}$ and $M_{\alpha^{-1}}$ are the minimal polynomials over $\mathbb{F}_P$ of $\alpha$ and $\alpha^{-1}$, respectively. In [3] the second author together with E. Velikova considered these codes and found an upper bound for their covering radius.

In this talk we consider a family of similar codes generated by the product $M_{\alpha}M_{\alpha^i}$ of minimal polynomials over $\mathbb{F}_P$ for $i = 2, 3$. This class of codes for $i = 2$ and the Melas codes are more thoroughly considered in the Master Thesis of the first author [1] written under the supervision of the second. The minimal distance and the covering radius of these codes are exactly specified there. For $i = 3$ the minimal distance is determined for all codes. Exact values and bounds for the covering radius are found in some cases. An essential moment of the proofs of our results is to check for existence of solutions of suitable systems of algebraic equations such that a part of the unknowns belongs to $\mathbb{F}_P$ and the rest belongs to $\mathbb{F}_q$. 

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References


Self-dual codes and their automorphisms

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Keywords: Coding theory, self-dual codes, automorphism group, construction.

2010 Mathematics Subject Classification: 11T71, 94B05, 08A35, 20B25.

The purpose of this talk is to present the connection between self-dual codes over a finite field with \( q \) elements and their automorphism groups [3]. Methods to construct and classify binary self-dual codes under the assumption that they have an automorphism of a given prime order are described [1, 2, 4]. These methods are extended in four directions: automorphisms of odd composite order, automorphisms of order 2, binary linear codes (not necessarily self-dual) invariant under automorphisms of odd order, and self-dual codes over larger fields with nontrivial automorphisms.

A linear \([n, k]\)-code \( C \) is a \( k \)-dimensional subspace of the vector space \( \mathbb{F}_q^n \), where \( \mathbb{F}_q \) is the finite field of \( q \) elements. Let \( (u, v) : \mathbb{F}_q^n \times \mathbb{F}_q^n \rightarrow \mathbb{F}_q \) be an inner product in the vector space \( \mathbb{F}_q^n \). If \( C \) is an \([n, k]\)-linear code, then its orthogonal complement \( C^\perp = \{ u \in \mathbb{F}_q^n : (u, v) = 0 \text{ for all } v \in C \} \) is a linear \([n, n-k]\)-code. If \( C \subseteq C^\perp \), \( C \) is termed self-orthogonal and if \( C = C^\perp \), \( C \) is self-dual. Self-dual codes are an important class of codes for practical reasons, since many of the best codes known are of this type, and for theoretical reasons, because of their connections with groups, lattices and designs.

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We say that two codes $C_1$ and $C_2$ of the same length over $\mathbb{F}_q$ are equivalent provided there is a monomial matrix $M$ and an automorphism $\gamma$ of the field such that $C_2 = C_1 M \gamma$. If $M$ is a monomial matrix with entries only from \{0, -1, 1\}, then $C$ is self-dual if and only if $CM$ is self-dual. The set of coordinate permutations that map the code $C$ to itself forms a group, called the permutation automorphism group of $C$ and denoted by $\text{PAut}(C)$, $\text{PAut}(C) < S_n$. Two more groups can be considered – the monomial automorphism group $\text{MAut}(C)$, and the group $\Gamma \text{Aut}(C)$ consisting of the maps of the form $M\gamma$, that map $C$ to itself. If $q$ is a prime then $\text{MAut}(C) = \Gamma \text{Aut}(C)$. If $q = 2$ then $\text{PAut}(C) = \text{MAut}(C) = \Gamma \text{Aut}(C)$. We focus on the binary self-dual codes and consider their automorphism groups as subgroups of the symmetric group of a corresponding degree. Many interesting finite groups appear as the group of some self-dual code. For example, the automorphism group of the extended Golay code, which is a $[24,12,8]$-self-dual doubly-even code, is the 5-transitive Mathieu group $M_{24}$.

If $C$ is a binary self-dual code having an automorphism $\sigma$ of odd prime order $p$, then $C = F_{\sigma}(C) \oplus E_{\sigma}(C)$ where $E_{\sigma}(C) = \{v \in C : v \text{ has even weight on each cycle of } \sigma\}$ and $F_{\sigma}(C) = \{v \in C : \sigma(v) = v\}$ are subcodes of $C$. There is a bijection from $F_{\sigma}(C)$ to a binary self-dual code of length $c + f$ where $c$ is the number of independent cycles of $\sigma$ of length $p$, and $f$ is the number of the fixed points. The other subcode can be mapped into a Hermitian self-dual code of length $c$ over the field $\mathbb{F}_{2^{p-1}}$ if 2 is a primitive root modulo $p$. We can use this structure to construct binary self-dual codes on the base of these smaller images of their subcodes. If we take the permutation $\sigma$ to be an automorphism of odd composite order, the structure of the subcode $E_{\sigma}(C)$ is much more complicated. Considering automorphisms of even order, we lose the direct sum. That is why all these cases have to be considered separately.

References
Free subgroups in group rings

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Let $V(KG)$ be the normalized group of units of the group ring $KG$ of a non-Dedekind group $G$ with nontrivial torsion part $t(G)$ over the integral domain $K$ with $\text{char}(K) = 0$.

B. Hartley and P. F. Pickel (see [2]) proved that if $G$ is a finite non-Dedekind group, then $V(ZG)$ contains a free group of rank 2. A. Salwa (see [4]) showed that two noncommuting unipotent elements $\{1+x, 1+x^*\}$ of $ZG$ always generate a free group of rank 2, where $x$ is a nilpotent element and $*$ is the classical involution of $ZG$.

In [1] we introduce a new family of torsion and non-torsion units in $V(KG)$. Using these units we prove that the group ring $K G$ of a non-Dedekind group $G$ which has at least one non-normal finite cyclic subgroup of order $n$ always contains the free product $C_n \star C_n$ as a subgroup. Moreover, this subgroup $C_n \star C_n$ can be normally generated by a single element. Note that several problems in group theory and the theory of small dimensional topology (the Relation Gap problem, Wall’s $D2$ Conjecture, the Kervaire Conjecture, Wiegold’s Problem, Short’s Conjecture and the Scott-Wiegold Conjecture (Questions 5.52, 5.53 and 17.94 in [3])) can be reduced to the question whether a given group can be normally generate by a single element.

For some classes of groups $G$ we provide an alternative proof of the main result of [2] constructing a free subgroup of rank 2 normally generated by a single element.

Finally, note that unlike other proofs in this subject our proof does not use the well-known result of I. N. Sanov [5].
References


Levenshtein-type bound on maximal codes

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Keywords: Spherical codes, potential energy, bounds for codes.

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Let $S^{n-1}$ be the unit sphere in $\mathbb{R}^n$. We refer to a finite set $C \subset S^{n-1}$ as a spherical code and, for a given (extended real-valued) function $h(t) : [-1,1) \rightarrow [0, +\infty)$, we consider the $h$-energy (or the potential energy) of $C$ defined by

$$E(n,C; h) := \sum_{x,y \in C \atop x \neq y} h(\langle x, y \rangle),$$

where $\langle x, y \rangle$ denotes the inner product of $x$ and $y$. The potential function $h$ is called absolutely monotone on $[-1,1)$ if its $k$-th derivative satisfies $h^{(k)}(t) \geq 0$ for all $k \geq 0$ and $t \in [-1,1)$.

Suppose $-1 < \ell < s < 1$ and denote by $C_{\ell,s}$ the class of spherical codes on $S^{n-1}$ that have all their inner products of distinct points in the interval
We introduce a method to find a universal lower bound on the potential energy of the codes from $C_{l,s}$ for an absolute monotone potential function. In the process we also answer a question posed by Levenshtein [4, 5] about finding a Levenshtein-type bound for spherical codes whose inner products between distinct points lie in an interval $[\ell, s]$.

Our results are continuation of previous work on universal bounds for spherical codes and designs [2, 3] (see also [1]).

References


Absolute Brauer $p$-dimensions,
Henselian fields and the inverse problem
for index-period pairs

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**Keywords:** Index-exponent pair, Brauer pair, finitely-generated extension, Brauer/absolute Brauer $p$-dimension, valued field.

**2010 Mathematics Subject Classification:** 16K20, 16K50, 12F20, 12J10, 16K40.

Let $E$ be a field, $E_{\text{sep}}$ a separable closure of $E$, $s(E)$ the class of associative finite-dimensional central simple $E$-algebras, $d(E)$ the subclass of division algebras $D \in s(E)$, and for each $A \in s(E)$, let $\text{ind}(A)$ be the Schur index of $A$, $[A]$ the equivalence class of $A$ in the Brauer group $\text{Br}(E)$, and $\text{exp}(A)$ the exponent of $A$, i.e., the order of $[A]$ in $\text{Br}(E)$. It is known that $\text{Br}(E)$ is an abelian torsion group, so it decomposes into the direct sum of its $p$-components $\text{Br}(E)_p$, taken over the set $\mathbb{P}$ of prime numbers. More precisely, by Brauer’s theorem, $(\text{ind}(A), \text{exp}(A))$ is a Brauer pair, i.e., $\text{exp}(A)$ divides $\text{ind}(A)$ and shares with $\text{ind}(A)$ one and the same set of prime divisors. Since the Schur index function $\text{Ind}: \text{Br}(E) \to \mathbb{N}$ is multiplicative, Brauer’s theorem implies $A$ has a primary tensor product decomposition. It reduces the study of index-exponent pairs $\text{ind}(A), \text{exp}(A), A \in s(E)$, to the study of index-exponent $p$-primary pairs over $E$, for each $p \in \mathbb{P}$, which attracts interest in the Brauer $p$-dimensions $\text{Brd}_p(E): p \in \mathbb{P}$, and in their supremum $\text{Brd}(E)$, the Brauer dimension of $E$. We say that $\text{Brd}_p(E) = n < \infty$, for a given $p \in \mathbb{P}$, if $n$ is the least integer $\geq 0$, for which $\text{ind}(A_p) \mid \text{exp}(A_p)$ whenever $A_p \in s(E)$ and $[A_p] \in \text{Br}(E)_p$; if no such $n$ exists, we put $\text{Brd}_p(E) = \infty$. 

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The absolute Brauer $p$-dimension $\text{abrd}_p(E)$ is defined as the supremum of $\text{Brd}_p(R)$, where $R$ runs across the set of finite extensions of $E$ in $E_{\text{sep}}$, and the absolute Brauer dimension of $E$ is the supremum $\text{abrd}(E)$ of $\text{abrd}_p(E)$, $p \in \mathbb{P}$. It is known that $\text{Brd}_p(E) = \text{abrd}_p(E)$, for every $p \in \mathbb{P}$ if $E$ is a global or local field (class field theory), or the function field of an algebraic surface over an algebraically closed field $[3, 6]$. By [7] we have $\text{abrd}_p(F_m) < p^{m-1}$, $p \in \mathbb{P}$, in case $F_m$ is a field of $C_m$-type, for some $m \in \mathbb{N}$. Hence, by the Lang-Tsen theorem [4] this holds, if $F$ is the function field of an $m$-dimensional algebraic variety over an algebraically closed field.

This talk considers the set of sequences $\text{abrd}_p(E), \text{Brd}_p(E), p \in \mathbb{P}$, taken over the class of fields of zero characteristic and the class of fields containing finitely many roots of unity. Our main result shows that if $N_\infty = \mathbb{N} \cup \{0, \infty\}$ and $a_p, b_p \in N_\infty$, $p \in \mathbb{P}$, is a sequence, such that $a_p \geq b_p$, for each $p \in \mathbb{P}$, and $a_2 \leq 2b_2 < \infty$, then there is a field $\Phi$ with $(\text{abrd}_p(\Phi), \text{Brd}_p(\Phi)) = (a_p, b_p), p \in \mathbb{P}$. It proves that $\Phi$ can be chosen so that $\text{char}(\Phi) = q > 0$ and $\Phi$ contains only $q-1$ roots of unity, provided that $a_\pi \leq 2b_\pi < \infty$, $\pi \mid (q - 1)$, and $a_q \leq b_q + 1 < \infty$. The method of proving the main result enables one to deduce from Kollár’s theorem [5] and the Lang-Nagata-Tsen theorem [8] that if $a_p \leq d$, $p \in \mathbb{P}$, for some $d \in \mathbb{N}$, then $\Phi$ can be chosen to be of $C_{d+1}$-type and zero characteristic.

The second main result of this talk (to appear in [2]) states that if $p \in \mathbb{P}$ and $F/E$ is a transcendental finitely-generated field extension (an FG-extension) of transcendency degree $t$, then: $(p^s, p^m)$, $s, m \in \mathbb{N}$, $s \geq m$, are index-exponent pairs over $F$, provided $\text{abrd}_p(E) = \infty$; $\text{Brd}_p(F) \geq \text{abrd}_p(E) + t - 1$ in case $\text{abrd}_p(E) < \infty$ and $F/E$ purely transcendental. When $p = \text{char}(E)$, it yields $\text{Brd}_p(F) = \infty$ iff the degree $[E : E^p]$ is infinite; $\nu + t - 1 \leq \text{Brd}_p(F) \leq \nu + t$, if $[E : E^p] = p^\nu < \infty$.

The results show that $\text{Brd}(F) = \infty$ whenever $F/E$ is a transcendental FG-extension and $\text{abrd}(E) = \infty$. Therefore, $\text{abrd}(E) < \infty$, provided that $E$ is a field whose proper FG-extensions $F$ possess a (field) dimension $\text{dim}(F)$, such that $\text{Brd}(F) < \text{dim}(F) < \infty$ and $\text{dim}(F(t)) = \text{dim}(F) + 1$, where $F(t)/F$ is transcendental. Our main results also show that for each pair $(q, k) \in (\mathbb{P} \cup \{0\}) \times \mathbb{N} \cup (0, 0)$, there is a field $\Phi_{q,k}$ with $\text{char}(\Phi_{q,k}) = q$, $\text{Brd}(\Phi_{q,k}) = k$, and $\text{Brd}_p(\Phi_{q,k}) = \infty$, for every $p \in \mathbb{P} \setminus \{2\}, p \nmid (q - 1)$. When $q \neq 0$, $\Phi_{q,k}$ can be chosen so that $[\Phi_{q,k} : \Phi_{q,k}^q] = \infty$. Hence, a Brauer pair $(n, m)$ is an index-exponent pair over each transcendental FG-extension $F_{q,k}$ of $\Phi_{q,k}$ in the following cases: $q = 0$ and $2 \nmid mn$; $q > 0$ and g.c.d.$\{mn, q - 1\} = 1$. This solves a problem posed in of [1], Sect. 4.

The proofs of the presented results rely on valuation theory. For the first one, we find explicit formulae for $\text{Brd}_p(K)$ and $\text{abrd}_p(K)$, $p \in \mathbb{P} \setminus \{q\}$, assuming
that \( K \) is a field which has a Henselian valuation \( v \) with a residue field \( \hat{K} \) of characteristic \( q \geq 0 \), whose absolute Galois group \( \hat{G}_K \) is a projective profinite group. Such a formula is also obtained for \( \text{Brd}_q(K) \) in case \((K, v)\) is a maximally complete field, \( \text{char}(K) = q \) and \( \hat{K} \) is perfect. This is used for proving the existence of a Henselian field \((\Phi, \varphi)\) with \( \hat{\Phi} \) perfect, \( \hat{G}_\Phi \) projective profinite and \((\text{abrd}_p(\Phi), \text{Brd}_p(\Phi)) = (a_p, b_p), p \in \mathbb{P} \).

References


Degree bounds for generators
of invariant rings

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Keywords: Polynomial invariants, finite groups, degree bounds.

2010 Mathematics Subject Classification: 13A50, 11B30.

Let $G \subset \text{GL}(V)$ be a finite group where $V$ is a finite dimensional vector space over a field $\mathbb{F}$. The action of $G$ on $V$ extends naturally to an action on the coordinate ring $\mathbb{F}[V]$. The invariant ring $\mathbb{F}[V]^G$ is the subring of $\mathbb{F}[V]$ consisting of those polynomials which are fixed under this action of $G$. We know from a classic result of Hilbert that this invariant ring is finitely generated. However the task of actually determining a minimal set of generators is computationally so difficult that practically it is unfeasible in most of the cases. This gives the relevance of the attempt of finding better a priori bounds on the possible degrees of the generators. Already E. Noether showed that $\mathbb{F}[V]^G$ is generated by its elements of degree at most $|G|$. But later it turned out that this bound is sharp only for cyclic groups and it can be substantially smaller in general. In our talk we will present some new methods developed for estimating these degree bounds and we will show their applications through some concrete examples. The results presented here are joint with Mátyás Domokos.

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The center of the generic $G$-crossed product

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Keywords: Crossed-product, generic matrices, rational extensions.

2010 Mathematics Subject Classification: 14E08, 16S35, 16S10.

It is well known that finite dimensional central simple algebras are “almost” matrix algebras, in the sense that they become isomorphic to matrix algebras after a suitable extension of scalars. This leads to the construction of the generic division algebra which by definition is a suitable central localization of the $F$-algebra generated by two $n \times n$ generic matrices $X = (x_{i,j})$ and $Y = (y_{i,j})$. Its main attractiveness rises from the fact that central simple algebras of rank $n$ over field extensions of $F$ inherit many of the properties of the generic division algebra. For example, they inherit the property of being Brauer equivalent to a product of cyclic algebras.

The fraction field of the center of the generic division algebra has a central role in this study, where the main question asked is whether it is a purely transcendental extension of $F$, and if not, how close it is to being as such.

Since every central simple algebra is Brauer equivalent to a crossed product, it is only natural to consider constructing also a generic $G$-crossed product. In this talk, we will describe such a construction using generic graded matrices. We will give a description of the center of this generic crossed product using flows in graphs, and will present some results regarding how close the center is to being a purely transcendental extension of $F$. 

Minimal superalgebras
generating minimal supervarieties
with respect to the graded PI-exponent

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Keywords: Minimal superalgebras, PI-exponent, graded polynomial identities.

2010 Mathematics Subject Classification: 16R10, 16R50.

In this talk we shall discuss some results concerning the $\mathbb{Z}_2$-graded polynomial identities of associative superalgebras. More precisely, we consider the minimal superalgebras introduced by Giambruno and Zaicev in their characterization of varieties of associative PI-algebras over a field of characteristic zero which are minimal of fixed exponent, [2]. These superalgebras are also involved in the description of generators of minimal supervarieties of finite basic rank. In fact it is know that any minimal (with respect to the value of its graded exponent) supervariety of finite basic rank is generated by a suitable minimal superalgebra, [1].

In this talk we provide an example of a minimal superalgebra not generating a minimal supervariety. Moreover, we discuss in its generality the case when the semisimple component of the minimal superalgebra has exactly three simple-graded components.

The last part is a joint work with Ernesto Spinelli and Viviane Ribeiro Tomaz da Silva.

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References


On certain modules of covariants in exterior algebras

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Keywords: Invariant theory, symmetric spaces, exterior algebras, polynomial trace identities.

2010 Mathematics Subject Classification: 17B20.

We study the structure of the space of covariants $B := \bigwedge (\mathfrak{g}/\mathfrak{k})^* \otimes \mathfrak{g}^\mathfrak{k}$, for a certain class of infinitesimal symmetric spaces $(\mathfrak{g}, \mathfrak{k})$ such that the space of invariants $A := \bigwedge (\mathfrak{g}/\mathfrak{k})^*$ is an exterior algebra $\bigwedge (x_1, \ldots, x_r)$, with $r = rk(\mathfrak{g}) - rk(\mathfrak{k})$.

We prove that they are free modules over the subalgebra $A_{r-1} = \bigwedge (x_1, \ldots, x_{r-1})$ of rank $4r$. In addition we will give an explicit basis of $B$.

As particular cases we will recover some classical results. In fact we will describe the structure of $(\bigwedge (M_{m}^{\pm})^* \otimes M_{m})^G$, the space of the $G$–equivariant matrix valued alternating multilinear maps on the space of (skew-symmetric or symmetric with respect to a specific involution) matrices, where $G$ is the symplectic group or the odd orthogonal group. We deduce corresponding results for the spaces $B^{\pm} := (\bigwedge (M_{m}^{\bullet})^* \otimes M_{m}^{\pm})^G$. Furthermore we prove new polynomial trace identities.

The even orthogonal case is strictly different from the other classical groups, in fact we will see that similar results are valid for the spaces $B^{+}$, namely they are free modules on a certain subalgebra of $A$. On the other hand the cases of type $B^{-}$ don’t respect this rule.
We discuss the statement only for the symmetric case
\[
\left( \bigwedge (M_{2n}^+)^* \otimes M_{2n}^- \right)^G,
\]
because the skew-symmetric case is already known by [1].

References


Games on partitions

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Keywords: Bulgarian solitaire, partitions, oriented graphs, discrete dynamical systems, card games.

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The main object of the talk is the Bulgarian solitaire. This is a mathematical card game played by one person. A pack of n cards is divided into several decks (or “piles”). Each move consists of the removing of one card from each deck and collecting the removed cards to form a new deck. The game ends when the same position occurs twice. It has turned out that when \( n = k(k + 1)/2 \) is a triangular number, the game reaches the same stable configuration with size of the piles 1, 2, . . . , k. The problem was brought to Bulgaria from Russia in 1980 and then was spread to the world. The first solutions appeared in 1981 in Bulgarian and Russian [1, 5]. The name was given by Henrik Eriksson [3] and then popularized by Martin Gardner [4].

In the language of partitions, one starts with a partition \( \lambda = (\lambda_1, \ldots, \lambda_c) \) with \( \lambda_c > 0 \) and obtains the partition \( B(\lambda) = (c, \lambda_1 - 1, \ldots, \lambda_c - 1) \). (If \( \lambda_i - 1 > c \geq \lambda_{i+1} - 1 \), then we assume that \( B(\lambda) = (\lambda_1 - 1, \ldots, \lambda_i - 1, c, \lambda_{i+1} - 1, \ldots, \lambda_c - 1) \).

The Bulgarian solitaire has several “younger brothers”, e.g., the Austrian, Carolina, and Montreal solitaires, the Red-green, Three-dimensional, Dual, and Multiplayer Bulgarian solitaires, Stochastic Bulgarian solitaires. The purpose

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of the talk is to survey the (quite amusing) story of these games and the mathematical problems related with them. In particular, we discuss the relations with combinatorics, graph theory, discrete dynamical systems, cellular automata, linear algebra, statistics, economical models. The topic has the advantage that most of the problems can be stated in an elementary way which allows to use it to attract young people to mathematical research. The talk is based on the resent paper [2] with additional information collected after its publishing.

References


Some classes of automorphisms
of relatively free nilpotent Lie algebras

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Keywords: Free metabelian Lie algebras, inner, outer, normal automorphisms, Baker-Campbell-Hausdorff formula, generic Lie matrices.

2010 Mathematics Subject Classification: 17B01, 17B30, 17B40, 16R30.

Let \( L_{m,c} \) be the free \( m \)-generated metabelian nilpotent of class \( c \) Lie algebra over a field of characteristic 0. An automorphism \( \varphi \) of \( L_{m,c} \) is called normal if \( \varphi(I) = I \) for every ideal \( I \) of the algebra \( L_{m,c} \). Such automorphisms form a normal subgroup \( N(L_{m,c}) \) of \( \text{Aut}(L_{m,c}) \) containing the group of inner automorphisms. We describe the groups of inner and outer automorphisms of \( L_{m,c} \) [2]. To obtain this result we first describe the groups of inner and continuous outer automorphisms of the completion \( \hat{F}_m \) with respect to the formal power series topology of the free metabelian Lie algebra \( F_m \) of rank \( m \). We also describe the group of normal automorphisms of \( L_{m,c} \) and the quotient group of \( \text{Aut}(L_{m,c}) \) modulo \( N(L_{m,c}) \) [4].

In the second half of the talk, we consider the relatively free algebra \( L_2 = L_2(\text{vars}l_2(K)) \) of rank 2 in the variety of Lie algebras generated by the algebra \( sl_2(K) \) over a field \( K \) of characteristic 0. The algebra \( L_2 \) is generated by two generic traceless \( 2 \times 2 \) matrices. Translating an old result of Baker [1] from 1901 we present a multiplication rule for the inner automorphisms of the completion \( \hat{L}_2 \) of \( L_2 \) and give a complete description of the group of inner automorphisms of \( \hat{L}_2 \) [3]. We also describe the group of outer automorphisms of \( \hat{L}_2 \) [5]. As a

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consequence we obtain similar results for the automorphisms of the relatively free algebra $L_2/L_2^{c+1} = L_2(\text{var}(sl_2(K)) \cap \mathfrak{N}_c)$ in the subvariety of $\text{var}(sl_2(K))$ consisting of all nilpotent algebras of class at most $c$ in $\text{var}(sl_2(K))$.

Some of the results were obtained jointly with Vesselin Drensky during the visit of the author at the Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences.

References
Tensor product theorems for different types of fields

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Keywords: Polynomial identities, variety of algebras, T-prime ideal.

2010 Mathematics Subject Classification: 16R10, 16R40.

Let $F$ be a field (maybe finite). We consider only associative algebras over $F$. For an algebra $A$ we denote by $T(A)$ the ideal of polynomial identities satisfied by $A$ and by $M_n(A)$ the algebra of all $n \times n$ matrices over $A$.

Let $E$ be the infinite-dimensional Grassmann algebra without identity element:

$$E = \langle e_1, e_2, \ldots | e_i^2 = 0, e_i e_j = -e_j e_i \rangle,$$

and let $E^1$ be the Grassman algebra with identity element 1. It is easy to see that $E = E_0 \oplus E_1$, where $E_0$ is the span of all words of even length and $E_1$ is the span of all words of odd length. For $E^1$ we have $E^1 = E^1_0 + E_1$, where $E^1_0 = E_0 + F \cdot 1$.

Denote by $M_{a,b}(E)$ the subalgebra of $M_{a+b}(E)$ which consists of matrices

$$\begin{pmatrix} A & C \\ D & B \end{pmatrix},$$

where $A \in M_a(E_0)$, $B \in M_b(E_0)$, and $C$, $D$ are matrices of size $a \times b$ and $b \times a$, respectively, whose entries belong to $E_1$. We obtain $M_{a,b}(E^1)$ if we assume that $A \in M_a(E^1_0)$ and $B \in M_b(E^1_0)$.

In the case of a field of zero characteristic A. Kemer ([2]) proved the Tensor Product Theorem (TPT):

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\[
T(M_{a,b}(E^1) \otimes M_{c,d}(E^1)) = T(M_{ac+bd,ad+bc}(E^1)), \\
T(M_{a,b}(E^1) \otimes E^1) = T(M_{a+b}(E^1)), \\
T(E^1 \otimes E^1) = T(M_{1,1}(E^1)).
\]

If \( F \) is of positive characteristic, then \( T(E^1 \otimes E^1) \neq T(M_{1,1}(E^1)) \) (see [1]). But the TPT is true for the Grassmann algebra without 1.

**Theorem.** If \( \text{char} F > 2 \), then

1) \( T(M_{a,b}(E) \otimes M_{c,d}(E)) = T(M_{ac+bd,ad+bc}(E)) \),

2) \( T(M_{a,b}(E) \otimes E) = T(M_{a+b}(E)) \),

3) \( T(E \otimes E) = T(M_{1,1}(E)) \).

**Proposition.** If \( \text{char} F > 2 \), then

1) \( T(M_{a,b}(E^1) \otimes M_{c,d}(E^1)) \neq T(M_{ac+bd,ad+bc}(E^1)) \),

2) \( T(M_{a,b}(E^1) \otimes E^1) \neq T(M_{a+b}(E^1)) \).

**References**


Semigroup graded algebras
and graded polynomial identities

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Keywords: Polynomial identity, semigroup grading, Amitsur’s conjecture, graded-simple associative algebra, non-integer PI-exponent.

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Study of polynomial identities is an important aspect of study of algebras themselves. It turns out that the asymptotic behaviour of the numeric characteristics of polynomial identities of an algebra (called codimensions) is tightly related to the structure of the algebra itself.

In 2013–2014 the author proved that for every finite dimensional associative algebra graded by a cancellative semigroup (e.g. by a group) there exists an integer exponent of graded codimension growth. However, if the semigroup is not cancellative, the exponent can be non-integer.

In the talk we will describe all finite dimensional associative graded-simple algebras graded by a finite semigroup with trivial maximal subgroups. In addition, we will provide a series of such algebras having arbitrarily large non-integer graded PI-exponents. (Joint with E. Jespers and G. Janssens.)
Structure and chains in non-commutative spectra

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Keywords: Prime spectrum, Krull dimension, growth.

2010 Mathematics Subject Classification: 16P99.

We consider Spec$(R)$ for $R$ non-commutative and investigate its chains. We are interested in infinite chains which have unions that exceed the spectrum, and prove several constraints on this phenomenon for certain classes of rings (e.g., rings satisfying a polynomial identity). Under suitable conditions, we are able to prove that every chain can be “cut” in a way generalizing classical results of Kaplansky from the commutative case.

We are also interested in the very opposite, where an (efficient) bound can be put on the classical Krull dimension – and prove it is possible under suitable growth conditions. For example, this is the case in the class of finitely generated graded domains with cubic growth, which naturally arises in non-commutative geometric objects.

If time permits, we will turn to discuss the connection between the distribution of the co-dimensions of the maximal ideals of an algebra and the growth of the algebra.

References


PI-Exponent and structure of Lie algebras with a generalized action

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Keywords: PI-Lie algebras, semigroup gradings, PI-exponent, graded Lie algebra structure theorems.

2010 Mathematics Subject Classification: 17B05, 16R10, 17B70.

One way to study, not necessarily associative, PI-algebras $A$ is through the so called sequence of codimensions $c_n(A)$. The asymptotics of the sequence $c_n(A)$, i.e. $\lim_{n \to \infty} \sqrt[n]{c_n(A)}$, has proven to be an integer and contains much structural information about $A$. It is called the PI-exponent of $A$. One can add more refined information into the polynomials, such as a grading by a group, Hopf algebra and group actions and study the corresponding polynomials and sequences. For (finite dimensional) Lie and associative algebras it is well known that in the above cases the graded/Hopf PI-exponent is also an integer and delivers information about the graded/Hopf structure of $A$.

In this talk I will tell about recent results concerning the case that a general associative algebra is acting on a Lie algebra $L$ (e.g $L$ is semigroup-graded). It turns out that in general the PI-exponent has no longer to be an integer and a positive answer for some classes of Lie algebras depends heavily on the behaviour of the typical structural theorems of Lie algebras under the generalized action.

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PI-representability and exponent in the framework of Hopf algebras

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Keywords: PI-algebras, Hopf algebras, PI-exponent.

2010 Mathematics Subject Classification: 16R10, 16T05.

Kemer’s representability theorem is one of the less understood gems of PI-theory, whereas the PI-exponent is, by now, a well known and incredibly handy tool in the arsenal of PI-theory. It is remarkable however that these two topics have a lot in common and one can greatly benefit the other. One such occasion occurs when one generalizes the representability theorem and Amitsur’s PI-exponent conjecture to the framework of $H$-module $F$-algebras satisfying an ordinary polynomial identity. Here $F$ is a characteristic zero field and $H$ is a semisimple finite dimensional Hopf algebra over $F$. In particular, this includes (finite) group graded and group acted algebras.

In this talk I will tell about recent and (less recent) results concerning the above with emphasis on the intersection of these two theories.
Toroidal compactifications
of discrete quotients of period domains

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Keywords: Period map, local homogeneous spaces of orthogonal and symplectic groups, toric variety, toroidal compactification.

2010 Mathematics Subject Classification: 14D07, 32J05, 14M25, 51N30, 14C30.

Let $S = G_\mathbb{R}/K$ be a Hermitian symmetric space of non-compact type and $\Gamma$ be a lattice of $G_\mathbb{R}$. Then $S/\Gamma$ admits a toroidal compactification $(S/\Gamma)_\Sigma$, depending on a $\Gamma$-admissible family $\Sigma = \{\Sigma(P)\}_P$ of fans $\Sigma(P)$ in the centers $U_P$ of the unipotent radicals of the maximal $\Gamma$-rational parabolic subgroups $P \in \text{MPar}_\Gamma$ of $G_\mathbb{R}$. In the case of a neat lattice $\Gamma$, $(S/\Gamma)_\Sigma$ are smooth projective varieties.

The period domains $D$ are classifying spaces for the Hodge structures $H^n = \bigoplus_{j=0}^n H^{n-j,j}$ on the primitive cohomologies $H^n = H^n_{\text{prim}}(X, \mathbb{C})$ of projective algebraic manifolds $X$ of $\dim_\mathbb{C} X = n$ with a fixed polarization $Q$ and Hodge numbers $h^{n-j,j} = \dim_\mathbb{C} H^{n-j,j}$. They are acted transitively by non-compact simple Lie groups $G_\mathbb{R}$, preserving $Q$ and the induced Hermitian inner product on $H^n$.

Let $f : \mathcal{X} \to S$ be a family of smooth projective varieties $X_s = f^{-1}(s)$, $s \in S$, whose base $S$ admits a smooth projective compactification $\overline{S}$ by a divisor $\overline{S} \setminus S$ with normal crossings. Assume that $\mathcal{X} \subset \mathbb{P}^n(\mathbb{C})$ is a quasi-projective variety and fix the polarization $Q$ on $\mathcal{X}$, given by the Kähler class of the Fubini-Study metric. The correspondence, associating to $X_s$ the Hodge structure on the primitive cohomologies $H^n := H^n_{\text{prim}}(X_s, \mathbb{C})$, polarized by $Q|_{X_s}$ is a holomorphic
map \( \Phi : S \to D/\Gamma \) for an arithmetic lattice \( \Gamma \leq G_{\mathbb{Z}} \). Towards a holomorphic extension of \( \Phi \) to \( \overline{S} \), one needs to adjoin to \( D/\Gamma \) the \( \Gamma \)-orbits of some boundary points of \( D \) in its compact dual \( \hat{D} \). The talk proposes a construction of a complex analytic compactification \((D/\Gamma)_{\Sigma}\) of \( D/\Gamma \), such that any period map \( \Phi : S \to D/\Gamma \) has a (unique) holomorphic extension to \( \Phi : \overline{S} \to (D/\Gamma)_{\Sigma} \). In the case of a Hermitian symmetric space \( D \), \((D/\Gamma)_{\Sigma}\) coincides with Ash-Mumford-Rapoport-Tai’s toroidal compactification of \( D/\Gamma \) (cf. [1]). That is why \((D/\Gamma)_{\Sigma}\) are called toroidal compactifications of \( D/\Gamma \), associated with \( \Sigma \).

Kato and Usui’s monograph [3] provides compactifications \( D_{\Sigma}/\Gamma \) of \( D/\Gamma \), which are log-manifolds or complex analytic spaces with “slits”. An arbitrary period map \( \Phi : S \to D/\Gamma \) is shown to extend to \( \Phi : \overline{S} \to D_{\Sigma}/\Gamma \). Hayama’s article [2] reveals that \( D_{\Sigma}/\Gamma \simeq (D/\Gamma)_{\Sigma} \times (D/\Gamma)_{\Sigma} \) for our toroidal compactifications \((D/\Gamma)_{\Sigma}\) and their complex conjugates \((D/\Gamma)_{\Sigma}^{\overline{\cdot}}\).

The toroidal compactifications \((D/\Gamma)_{\Sigma}\) for period domains \( D \) of odd weight \( n = 2m + 1 \) are pulled back from the corresponding Ash-Mumford-Rapoport-Tai’s toroidal compactifications \((G/\Gamma)_{\Sigma}\) for \( G = \text{Sp}(N, \mathbb{R})/U(N) \), \( N = h^{2m+1,0} + h^{2m-1,2} + \cdots + h^{1,2m} \). The construction of the toroidal compactifications \((D/\Gamma)_{\Sigma}\) for period domains \( D \) of even weight is reduced to appropriate ones \( \mathcal{D} = G_{\mathbb{R}}/V_0 \) of weight 2. To this end, the parabolic subgroups \( P \) of \( G_{\mathbb{R}} \) are described by the means of isotropic subspaces \( \mathcal{H}_P \) of a reference Hodge structure \( H^2 \).

Let \( P \) be a parabolic subgroup of \( G_{\mathbb{R}} \) with unipotent radical \( N_P \). If \( P \) is \( \Gamma \)-rational then \( N_P \cap \Gamma \) is a lattice of \( N_P \), \( \Upsilon_P := U_P \cap \Gamma \) is a lattice of the center \( U_P \) of \( N_P \) and \( \mathbb{T}(P) := (U_P \otimes_{\mathbb{R}} \mathbb{C})/\Upsilon_P \simeq (\mathbb{C}^*)^m \) is a complex torus. There is a reductive complement \( G_P = G_{P,v} \times G_{P,h} \) of \( N_P \) to \( P \), which splits in a product of a vertical reductive part \( G_{P,v} \) and a horizontal semisimple part \( G_{P,h} \). The quotient \( F(P) := G_{P,h}/G_{P,h} \cap V_0 \) of \( G_{P,h} \) is a period domain of weight 2, called the analytic boundary component of \( \mathcal{D} \), associated with \( P \). We show that \( \mathcal{G}_{P,v} := G_{P,v}/G_{P,v} \cap V \) can be embedded in \( U_P \). Then there is a diffeomorphic Siegel domain presentation

\[
\mathcal{D} \simeq (U_P + i\mathcal{G}_{P,v}) \times (N_P/V_P) \times \mathcal{G}_{P,h} \subset (U_P \otimes_{\mathbb{R}} \mathbb{C}) \times (N_P/U_P) \times \mathcal{G}_{P,h}.
\]

Let \( \Sigma(P) \) be a \( \Gamma \)-admissible fan in \( U_P \simeq (\mathbb{R}^m, +) \) and \( X_{\Sigma(P)} \supseteq \mathcal{T}(P) \) be the toric variety, associated with \( \Sigma(P) \). The partial toroidal compactification is \((\mathcal{D}/\Upsilon_P)_{\Sigma(P)} = Y_{\Sigma(P)} \times (N_P/V_P) \times \mathcal{G}_{P,h} \) for the interior \( Y_{\Sigma(P)} \) of the closure of \((U_P + i\mathcal{G}_{P,v})/\Upsilon_P \) in \( X_{\Sigma(P)} \). The toroidal compactification

\[
(\mathcal{D}/\Gamma)_{\Sigma} = \coprod_{P \in \text{MParr}} (\mathcal{D}/\Upsilon_P)_{\Sigma(P)}/ \sim \Gamma
\]
is glued from \((\mathfrak{D}/\Upsilon_P)_{\Sigma(P)}\), according to a \(\Gamma\)-equivalence relation.

**Theorem 1** Let \(\Gamma \leq G_\mathbb{Z}\) be an arithmetic lattice and \(\Phi : S \to D/\Gamma\) be a period map of a quasi-projective variety \(S\), which admits a smooth compactification \(\overline{S}\) by a divisor \(\overline{S}\ \setminus S\) with normal crossings. Denote \(d = \dim_{\mathbb{C}} S\) and consider a neighborhood \(W \simeq (\Delta^*)^t \times \Delta^{d-t}\) of a boundary point \(s_\infty \in \overline{S}\ \setminus S\) with closure \(\overline{W} \simeq \Delta^d\) in \(\overline{S}\). Then there exists a maximal \(\Gamma\)-rational parabolic subgroup \(P\) of \(G_\mathbb{R}\), such that:

(i) the fundamental group \(\pi_1(W) \simeq \pi_1(\Delta^*)^t \simeq (\mathbb{Z}^t, +)\) is mapped in \(\Upsilon_P := U_P \cap \Gamma\) for the center \(U_P\) of the unipotent radical of \(P\);

(ii) \(\Phi(\Delta^*)^t\) is tangent to the complex torus \(T(P) = (U_P \otimes_{\mathbb{R}} \mathbb{C})/\Upsilon_P\);

(iii) \(\Phi : W \to D/\Gamma\) has a lifting \(\Phi_P : \overline{W} \simeq \Delta^d \to (D/\Upsilon_P)_{\Sigma(P)}\) in a partial toroidal compactification \((D/\Upsilon_P)_{\Sigma(P)}\), associated with \(P\).

After checking the compatibility of the gluings of \((D/\Upsilon_P)_{\Sigma(P)}\), \(P \in \text{MPar}_\Gamma\) in \((D/\Gamma)_{\Sigma}\) with the gluings of \(S\) and of the neighborhoods \(W\) of \(s_\infty \in \overline{S}\ \setminus S\) in \(\overline{S}\), one obtains the following

**Corollary 2** Let \(\Phi : S \to D/\Gamma\) be a period map in a quotient of a period domain \(D = G_\mathbb{R}/V\) by an arithmetic lattice \(\Gamma \leq G_\mathbb{Z}\). Suppose that the source \(S\) admits a smooth projective compactification \(\overline{S}\) by a normal crossing divisor \(\overline{S}\ \setminus S\). Then there is a holomorphic extension \(\Phi : \overline{S} \to (D/\Gamma)_{\Sigma}\) to a toroidal compactification \((D/\Gamma)_{\Sigma}\).

**References**


Hopf algebras and representation varieties

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Keywords: Hopf algebras, representation varieties, automorphism groups of free groups.

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We show that if $H$ is a cocommutative Hopf algebra, then there is a natural action of $\text{Aut}(F_n)$ on $H^n$ which induces an $\text{Out}(F_n)$-action on a quotient. In the case when $H = T(V_{2g})$ is the tensor algebra, we show that there is a surjection from cokernel of the Johnson homomorphism for the mapping class group of genus $g$ to the top coholomogy groups of $\text{Out}(F_n)$ with coefficients in this representation.

The same construction can be used to construct a representation of $\text{Aut}(\Gamma)$ for any finitely generated group $\Gamma$. In the case of $H = U(\mathfrak{g})$ the resulting representations are related to the representation variety of $\Gamma$ into $G$, where $G$ is a Lie group with Lie algebra $\mathfrak{g}$.

(This is a partially joint work with J. Connant.)

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Volume gradients and homology in towers of residually-free groups

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Keywords: Volume gradient, $\ell^2$-Betti numbers, residually-free groups.
2010 Mathematics Subject Classification: 20E26, 20J05.

I will present the results from the joint work with Martin Bridson (University of Oxford) from the preprint [1]. One of the main results is the calculation of the $\ell^2$-Betti numbers in dimension at most $m - 1$ of all residually free groups of type FP$_m$, using Lück’s approximation theorem. If the time permits I will discuss some new corollaries of the techniques introduced in the above preprint.

References

Wishart matrices of arbitrary dimension
over the reals

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Keywords: Wishart ensemble, random matrix, Jack polynomial, hypergeometric function of matrix argument.

2010 Mathematics Subject Classification: 15B52.

In multivariate statistics, the Wishart matrices serve as an idealized model for multivariate data \((m\) observations of \(n\) objects). Questions about classification, correlations, etc., are typically cast as tests on the eigenvalues of the observed covariance matrices against the Wishart ones. Applications are wide ranging: from finance to genomics, to target classification, etc.

To form a Wishart matrix \(W\) (in the simplest case of identity covariance), one starts with a normal random matrix \(A\) and forms the product \(W = A^T A\).

When the entries of \(A\) are real normals (i.e., \(a_{kl} \sim N(0, 1)\) and independent), then \(W = A^T A\) is a real Wishart matrix. When \(a_{kl} \sim b_{kl} + ic_{kl}\), where \(b_{kl}, c_{kl} \sim N(0, 1)\) are independent, one gets a complex Wishart matrix. The quaternion case is analogous.

These are the real, complex, or quaternion Wishart matrices of dimension \(\beta = 1, 2,\) and \(4\), respectively over the reals.

In this talk we present a method to generate a Wishart matrix of arbitrary \(\beta > 0\) dimension over the reals, using the method of “ghosts” and “shadows” introduced by Alan Edelman. The “ghosts” are abstract variables, which act (in every way we care about) as \(\beta\)-dimensional variables over the reals. The “shadows” are their (real) norms, which allow us to perform calculations with these Wishart matrices in practice.
We also pose questions about the algebraic properties of these “ghost” variables which, despite the recent progress, are not yet very well understood.

This is joint work with Alex Dubbs, Alan Edelman, and Praveen Venkataramana.

References


Gradings on Lie and Jordan algebras
and their graded identities

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Keywords: Gradings, elementary gradings, graded identities, upper triangular matrices.

2010 Mathematics Subject Classification: 16R50, 16W50, 17B01, 17B70, 17C05, 17C50.

In this talk we shall discuss results concerning group gradings on Lie and Jordan algebras and the corresponding graded identities. We shall focus mainly on the algebra $UT_n$ of upper triangular matrices over a field. We shall describe the elementary grading for the Lie algebra $UT_n^-$. We shall produce a basis of the graded identities when the grading is the canonical one with the cyclic group $C_n$ of order $n$. It turns out that the description is rather more complicated than in the associative case. (The latter was given by Valenti and Zaicev, see [1]). We shall discuss also general gradings on $UT_n^-$. There are gradings on it that are not elementary nor are isomorphic to elementary ones.

We shall also discuss briefly gradings on the Jordan algebra $UT_n^+$. The elementary ones are very close to the associative ones.

Whenever possible we shall make parallels with the associative case.

Parts of the results are joint with Yukihide, and/or with Martino.

References


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Application of rational generating functions to algebras with polynomial identities

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Keywords: Diophantine equations and inequalities, generating functions, algebras with polynomial identity, symmetric and Schur functions, multiplicity, Hilbert series.

2010 Mathematics Subject Classification: 05A15, 05E05, 05E10, 11D04, 11D45, 11D72, 116R10, 16R30, 20C30.

In 1903 Elliott [5] discovered a method for computing the generating function of the nonnegative solutions of a system of homogeneous linear diophantine equations and inequalities. This method was further developed by MacMahon [10]. The idea is, for a given formal power series

\[ f(t_1, \ldots, t_d) = \sum_{n_1 \geq 0} a_{n_1, \ldots, n_d} t_1^{n_1} \cdots t_d^{n_d} \in \mathbb{C}[[t_1, \ldots, t_d]], \]

to compute the part of \( f(t_1, \ldots, t_d) \) obtained as a result of the summation on those \((n_1, \ldots, n_d)\) which satisfy the diophantine system. The method works successfully for any formal power series \( f(t_1, \ldots, t_d) \) expressed as a rational function with denominator which is a product of binomials \( 1 - t_1^{k_1} \cdots t_d^{k_d} \). In particular, it can be applied to a rational symmetric formal power series

\[ f(t_1, \ldots, t_d) = \sum_{\lambda} m_{\lambda}s_{\lambda}(t_1, \ldots, t_d) \in \mathbb{C}(t_1, \ldots, t_d), \]

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where $s_{\lambda}(t_1, \ldots, t_d)$ is the Schur function indexed by the partition $\lambda = (\lambda_1, \ldots, \lambda_d)$, and the result is the multiplicity series

$$M(f; t_1, \ldots, t_d) = \sum_{\lambda} m_{\lambda} t_1^{\lambda_1} \cdots t_d^{\lambda_d}.$$ 

Applications of the method of Elliott to algebras with polynomial identities and invariant theory are surveyed in [1].

Let $T(R_{p,q}(K))$ be the T-ideal of the polynomial identities of the algebra of upper block triangular $(p+2q) \times (p+2q)$ matrices over a field $K$ of characteristic zero with diagonal consisting of $p$ copies of $1 \times 1$ and $q$ copies of $2 \times 2$ matrices. We give an algorithm which calculates the generating function of the cocharacter sequence

$$\chi_n(R_{p,q}(K)) = \sum_{\lambda \vdash n} m_{\lambda}(R_{p,q}(K)) \chi_{\lambda}$$

of the T-ideal $T(R_{p,q}(K))$. We have found the explicit form of the multiplicities $m_{\lambda}(R_{p,q}(K))$ and their asymptotic behaviour for small values of $p$ and $q$. The proofs use techniques from [1] and [3], the explicit form of the multiplicities $m_{\lambda}(M_2(K))$ found in [2, 6] and the formula of Formanek [7] (see [8] for the proof) for the Hilbert series of the product $T(R) = T(R_1)T(R_2)$ of two T-ideals expressed in terms of the Hilbert series of the factors $T(R_1)$ and $T(R_2)$.

The presentation is based on the Master Thesis of the author [9] and his joint results with Vesselin Drensky [4].

References


Algebras with polynomial codimension growth

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Keywords: Polynomial identities, codimensions, growth.
2010 Mathematics Subject Classification: 16R10, 16P90.

Let $A$ be an associative algebra over a field $F$ of characteristic zero and let $c_n(A)$, $n = 1, 2, \ldots$, be its sequence of codimensions.

It is well known that if $A$ satisfies some non-trivial polynomial identity then the sequence of codimensions of $A$ is exponentially bounded. Moreover either $c_n(A)$, $n = 1, 2, \ldots$, is polynomially bounded or $c_n(A)$ grows exponentially.

The purpose of this note is to present some results about algebras whose codimensions are polynomially bounded.

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The polynomial method in finite geometry

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Keywords: Finite fields, projective geometries, linearized polynomials, arcs, blocking sets, maximal arcs, divisible arcs.

2010 Mathematics Subject Classification: 51E15, 51E21, 51E22, 94B05, 94B27, 94B65.

We survey the use of polynomials in problems from finite geometry and the related fields of combinatorics and coding theory. The general idea is to consider polynomials whose zeros correspond to subspaces of Desarguesian affine or projective spaces. In this talk, we focus on the following problems.

1. Lower bounds on the size of affine blocking sets. J. Doyen conjectured at a lecture in Oberwolfach in 1976 that an affine blocking set in $\text{AG}(2, q)$ has at least $2q - 1$ points. This was proved by R. Jamison [9] and independently by A. Brouwer and A. Schrijver [4] by what is considered to be the first use of the polynomial method. A. Bruen extended their result to higher dimensions, proving that a $t$-fold blocking set in $\text{AG}(n, q)$ has at least $(n + t - 1)(q - 1) + 1$ points [5]. Further improvements on Bruen’s bound were proved by Ball in [1]. Examples of affine blocking sets attaining the lower bounds by Bruen and Ball were given in [11, 15].

2. The non-existence of maximal arcs in projective planes of odd order. A $(k, n)$-arc in $\text{PG}(2, q)$ is a set of $k$ points in the Desarguesian projective plane of order $q$ with at most $n$ points on a line. An obvious upper bound on the size of a $(k, n)$-arc in $\text{PG}(2, q)$ is $k \leq (n - 1)(q + 1) + 1$. An arc for

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which equality occurs is called a maximal arc. R. Denniston [7] constructed maximal arcs in PG(2, q), q even, for all n dividing q. J. Thas [12, 14] gave further constructions for maximal arcs in planes of even order. Later on, N. Hamilton and C. Quinn [8] constructed maximal arcs from m-systems in polar spaces. All these examples were in planes of even order.

The nonexistence of maximal arcs in PG(2, q) for odd q was formulated as a conjecture in the 1960’s. A. Cossu [6] proved the initial case \((n, q) = (3, 9)\) and J. Thas proved the nonexistence for \((n, q) = (3, 3^h)\) [13]. The conjecture was proved in its full generality using the polynomial method in [2, 3].

3. Divisibility of codes and arcs. One of the remarkable results about linear codes in the 1990’s was the divisibility theorem by H. N. Ward [16]. The polynomial method gives an alternative proof for Ward’s theorem and allows a generalization for some non-Griesmer codes [10].

References


Unramified cohomology and Noether’s problem

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**Keywords:** Bogomolov multiplier, unramified cohomology, Noether’s problem, rationality problem.

**2010 Mathematics Subject Classification:** 14E08, 14M20, 13A50, 12F12.

Let $K$ be a field, $G$ a finite group and $V$ a faithful representation of $G$ over $K$. Then there is a natural action of $G$ upon the field of rational functions $K(V)$. The **rationality problem** (also known as **Noether’s problem** when $G$ acts on $V$ by permutations) then asks whether the field of $G$-invariant functions $K(V)^G$ is rational (i.e., purely transcendental) over $K$. A question related to the above mentioned is whether $K(V)^G$ is stably rational, that is, whether there exist independent variables $x_1, \ldots, x_r$ such that $K(V)^G(x_1, \ldots, x_r)$ becomes a purely transcendental extension of $K$. This problem has close connection with Lüroth’s problem [4] and the inverse Galois problem [3, 5].

Saltman [3] found examples of groups $G$ of order $p^9$ such that $\mathbb{C}(V)^G$ is not stably rational over $\mathbb{C}$. His main method was application of the unramified cohomology group $H^2_{nr}(\mathbb{C}(V)^G, \mathbb{Q}/\mathbb{Z})$ as an obstruction. Bogomolov [1] proved that $H^2_{nr}(\mathbb{C}(V)^G, \mathbb{Q}/\mathbb{Z})$ is canonically isomorphic to

$$B_0(G) = \bigcap_A \ker\{\text{res}^A_G : H^2(G, \mathbb{Q}/\mathbb{Z}) \to H^2(A, \mathbb{Q}/\mathbb{Z})\}$$

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where $A$ runs over all the bicyclic subgroups of $G$ (a group $A$ is called bicyclic if $A$ is either a cyclic group or a direct product of two cyclic groups). The group $B_0(G)$ is a subgroup of the Schur multiplier $H^2(G, \mathbb{Q}/\mathbb{Z})$, and Kunyavskiǐ [2] called it the Bogomolov multiplier of $G$. Thus the vanishing of the Bogomolov multiplier is an obstruction to Noether’s problem.

The aim of this talk is to calculate the Bogomolov multiplier for various $p$-groups, and in particular to show that there is a nilpotency class two group of order $p^7$ with non-vanishing Bogomolov multiplier. For some of the groups with vanishing Bogomolov multipliers we are able to give a positive answer to Noether’s problem.

References


Free center-by-metabelian and nilpotent groups

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**Keywords:** Automorphisms, free center-by-metabelian and nilpotent groups, associated Lie algebras.

**2010 Mathematics Subject Classification:** 20F28, 20F40, 17B40.

For positive integers $n$ and $c$, with $n \geq 2$, let $G_{n,c}$ be a free center-by-metabelian and nilpotent group of rank $n$ and class $c$. We prove that the subgroup of $\text{Aut}(G_{2,c})$ generated by the tame automorphisms and three more IA-automorphisms of $G_{2,c}$ has finite index in $\text{Aut}(G_{2,c})$. For $n \geq 3$, the subgroup of $\text{Aut}(G_{n,c})$ generated by the tame automorphisms and two more IA-automorphisms of $G_{n,c}$ has finite index in $\text{Aut}(G_{n,c})$. 
Primitive ideals
in $U(\mathfrak{sl}(\infty)), U(\mathfrak{o}(\infty)), U(\mathfrak{sp}(\infty))$

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Keywords: Primitive ideals, enveloping algebras, finitary Lie algebras.
2010 Mathematics Subject Classification: 17B35, 17B65.

The enveloping algebras of the three simple infinite-dimensional complex Lie algebras $\mathfrak{sl}(\infty), \mathfrak{o}(\infty), \mathfrak{sp}(\infty)$ are interesting associative algebras whose ideal structure is quite different from the ideal structure of $U(\mathfrak{g})$ for a finite-dimensional simple Lie algebra $\mathfrak{g}$. Alexey Petukhov and I are working on providing a complete description of primitive (and possibly of all) ideals in $U(\mathfrak{sl}(\infty)), U(\mathfrak{o}(\infty)), U(\mathfrak{sp}(\infty))$. This work makes use of the pioneering work of A. Zhilinskii from the 1990’s. In this talk I will report on the class of ideals we understand, and in particular will provide a description of all annihilators of simple highest modules over $U(\mathfrak{sl}(\infty)), U(\mathfrak{o}(\infty)), U(\mathfrak{sp}(\infty))$. I will mention also the problem of noetherianity (with respect to two-sided ideals) of $U(\mathfrak{sl}(\infty)), U(\mathfrak{o}(\infty)), U(\mathfrak{sp}(\infty))$. 
Pre-plastic algebra

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Keywords: Pre-plastic algebra, symmetric functions, combinatorial aspects of representation theory.

2010 Mathematics Subject Classification: 05E05, 05E10.

The Poirier-Reutenauer algebra is a Hopf structure on the set of the standard Young tableaux [1]. In a previous work [2] it was shown that the Schur-Weyl duality interpolates between the Poirier-Reutenauer algebra and the algebra of the plactic monoid (providing an associative structure on the semistandard Young tableaux). We introduce a Hopf algebra whose quotient is the Poirier-Reutenauer algebra. This algebra is connected to the quantum plactic algebra introduced by Jean-Yves Thibon and Daniel Krob [3] with relation to noncommutative Schur functions.

References

On the PI-properties of some matrix algebras with involution

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Keywords: PI-algebras, matrix algebras over Grassmann algebras, algebras with involution $\phi$, $\phi$-variables, $\phi$-identities.

2010 Mathematics Subject Classification: 16R10, 15A75, 16R50.

In the theory of PI-algebras an important role plays the study of the PI-properties of an algebra $R$ having an involution $\phi$, i.e., a second order antiautomorphism with $\phi(ab) = \phi(b)\phi(a)$ for $a, b \in R$. A special case is the involution $\flat$ defined on the $2 \times 2$ matrix algebra $M_2(R)$ as

$$
\begin{pmatrix}
a & b \\
c & d
\end{pmatrix}^{\flat} = 
\begin{pmatrix}
d^* & b^* \\
c^* & a^*
\end{pmatrix},
$$

where $\ast$ is an involution on $R$.

The talk consists of two parts. In the first one we present investigations which continue those started in [1]. We define the index of nilpotency of the $\flat$-variables in the $2 \times 2$ matrix algebra $M_2(E'_4)$ with involution $\flat$ and entries from the non-unitary Grassmann algebra with four generators $E'_4$.

In the second part we consider a special $4 \times 4$ matrix algebra $AM_4(K)$ over a field of characteristic zero and find an identity of low degree for it. If additionally the algebra is with symplectic involution $\ast$, then we give a $\ast$-identity of minimal degree.

When the $4 \times 4$ matrix algebra has entries from a finite dimensional Grassmann algebra we show that the algebra $AM_4(E'_4)$ is nil and find its nil index.

We introduce an involution, denoted as $(\flat)$, in the matrix algebra $AM_4(E)$. For its graded subalgebra over $E'_3$ some $(\flat)$-identities are given.

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References

In this talk, I will introduce a notion which first appeared in Sageev’s work, later called *bounded packing* by Hruska and Wise. It is a group theoretic notion which has an intimate connection with the geometry of Sageev’s cube complex. In short, bounded packing of a codimension-1 subgroup is a sufficient condition for Sageev’s cubing coming from this subgroup to be finite dimensional. More interestingly, I will tell the story of my solution to the bounded packing problem in the discrete lattice quasi-isometric to $Sol$, which I successfully generalized to a solution to the bounded packing problem in $\mathbb{Z}^n \rtimes \mathbb{Z}$ in certain cases. I will also show how one can prove bounded packing of any subgroup of Hirsch length 1 in any polycyclic group, if one could keep track of the orbits of certain automorphisms of $\mathbb{R}^n$. Time permitting, I will introduce a function which I have called *coset growth*, which I show how to obtain an upper bound on in $\mathbb{Z}^2 \rtimes \mathbb{Z}$. 
The dimension problem for groups and Lie rings

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Keywords: Group rings, Lie rings, dimension problem, central series, universal enveloping algebra.
2010 Mathematics Subject Classification: 05E15, 17B01, 17B99, 20C05, 20C07.

The so-called “dimension problem” for groups can be stated as follows: Take a group $G$ and its integral group ring $\mathbb{Z}G$. Let $\varepsilon : \mathbb{Z}G \to \mathbb{Z}$ be the augmentation map, i.e., the linear extension of the map $g \mapsto 1$ to $\mathbb{Z}G$ and set $\Delta(G) = \ker(\varepsilon)$. Then the group $D_n(G) := (1 + \Delta(G)^n) \cap G$ is a normal subgroup of $G$. One easily sees that $G_n$, the $n$-th term of the lower central series of $G$, is always contained in $D_n(G)$. One can ask whether those groups are always the same, and it turns out that for $n \leq 3$ they are, and for a free group they are equal for any $n$. But in 1972 E. Rips found an example of a group $G$ with $D_4(G) \neq G_4$, and furthermore N. Gupta proved in 1991 that for any $n \geq 4$ there is a group $G$ with $D_n(G) \neq G_n$. However, a bound on the exponent of $D_n(G)/G_n$ depending on $n$ was found by J. A. Sjogren in 1979.

The dimension problem can also be formulated for Lie rings: Let $L$ be a Lie ring and $UL$ its universal enveloping algebra, then the augmentation map $\varepsilon : UL \to \mathbb{Z}$ is the extension of the map $L \to 0$, and again $\Delta(L) = \ker(\varepsilon)$ and $D_n(L) = \Delta(L)^n \cap L$. It is again easily seen that the $n$-th term of the lower central series of $L$ is contained in $D_n(L)$, and the reverse inclusion does not hold in general. However the methods of a second proof of Sjogren’s theorem due to G. Cliff and B. Hartley can be used to proof a similar result in the world of Lie rings.
Test elements in (pro-$p$) groups

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Keywords: Free groups, pro-$p$ groups, free pro-$p$ groups, automorphisms of groups.

2010 Mathematics Subject Classification: 20E18, 20E05, 20E36.

An element $g$ of a group $G$ is called a test element if for any endomorphism $\varphi$ of $G$, $\varphi(g) = g$ implies that $\varphi$ is an automorphism. The idea is to distinguish automorphisms among arbitrary endomorphisms by means of their action on a single element. The first example of a test element was given by Nielsen in 1918, when he proved that every endomorphism of a free group of rank 2 that fixes the commutator $[x_1, x_2]$ of a pair of generators must be an automorphism. Further examples of test elements in free groups of finite rank were obtained by Zieschang, Rosenberger, Rips, Shpilrain, etc. An important characterization of test elements in free groups of finite rank was obtained by Turner, who proved that an element of a free group $F$ of finite rank is a test element if and only if it is not contained in a proper retract of $F$.

A pro-$p$ group is the inverse limit of an inverse system of finite $p$-groups. In this talk I will discuss test elements in pro-$p$ groups. The emphasis will be on free pro-$p$ groups and Demushkin groups (which are precisely the Poincaré pro-$p$ groups of dimension 2). Furthermore, I will give applications of these results in free discrete groups and surface groups. This is a joint work with Slobodan Tanushevski.
Applications of matrix invariants in free function theory and their non-commutative resolutions

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**Keywords:** Matrix invariants, free function theory, non-commutative resolutions of singularities.

**2010 Mathematics Subject Classification:** 16R30, 13A50, 46L52, 47A56.

The talk will be divided in two parts. The first part is joint work with Igor Klep, and the second with Michel Van den Bergh.

Free function theory studies functions in several non-commuting variables evaluated on matrices of arbitrary size. We present an alternative, algebraic approach to free function theory through matrix invariants.

We further study non-commutative resolutions, as introduced by Michel Van den Bergh, of quotient singularities for reductive groups. We show in particular that matrix invariants admit (twisted) non-commutative crepant resolutions.
A new class of matrix algebras

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Keywords: Ring, Lie nilpotent ring, endomorphism, fixed ring, skew polynomial ring, symmetric adjoint, right determinants and characteristic polynomials, right (left) integrality.

2010 Mathematics Subject Classification: 15A15, 15A33, 16R40, 16S36, 16W20, 16W50, 16W55.

Using an endomorphism \( \delta : R \longrightarrow R \) of a ring \( R \) and an invertible matrix \( W \in \text{GL}_n(R) \), we consider the subring

\[
M_n(R, \delta, W, W^{-1}) = \{ A = [a_{i,j}] \in M_n(R) \mid [\delta(a_{i,j})] = WAW^{-1} \}
\]

of the full \( n \times n \) matrix ring \( M_n(R) \).

The classical supermatrix algebra \( M_{n,d}(E) \) over the Grassmann algebra \( E \) plays an important role in Kemer’s classification of \( T \)-prime \( T \)-ideals and appears in the form \( M_n(E, \varepsilon, D_d, D_d^{-1}) \), where \( \varepsilon \) is the automorphism of \( E \) arising from the natural \( \mathbb{Z}_2 \)-grading \( E = E_0 \oplus E_1 \) and \( D_d \) is a certain diagonal matrix. Notice that \( E \) is Lie nilpotent of index 2.

The ring \( M_n^g(R) \) of the so called graded \( n \times n \) matrices over a \( \mathbb{Z}_n \)-graded base ring \( R = R_0 \oplus R_1 \oplus \cdots \oplus R_{n-1} \) appears as \( M_n(R, \hat{\varepsilon}, P, P^{-1}) \), where \( \hat{\varepsilon} \) is an endomorphism of \( R \) naturally defined by a primitive \( n \)-th root of unity \( e \) and \( P = \sum_{i=1}^{n} e^{i-1} E_{i,i} \) is a diagonal matrix.

If \( \delta^n = \text{id}_R \), then for a certain cyclic permutation matrix \( H \) and for a special invertible diagonal matrix \( G \), we exhibit the natural embeddings

\[
\delta : R \longrightarrow M_n(R, \delta, H, H^{-1}) , \quad \delta^w : R[w, \delta] \longrightarrow M_n(R[z], \delta_z, H, H^{-1})
\]

and \( \tilde{\delta} : R \longrightarrow M_n(R, \delta, G, G^{-1}) \), where \( R[w, \delta] \) is a skew polynomial ring. If \( W \) and \( W^{-1} \) are over the centre \( Z(R) \), then we prove that \( M_n(R, \delta, W, W^{-1}) \)
is closed with respect to taking the symmetric adjoint. If $R$ is Lie nilpotent
of index $k$ and $A \in M_n(R, \delta, W, W^{-1})$, then the symmetric adjoint and
the corresponding right determinants and right characteristic polynomials provide
a Cayley-Hamilton identity (of degree $n^k$) for $A$ with right coefficients in the
fixed ring $\text{Fix}(\delta)$.

Using the embeddings $\overline{\delta}$ and $\delta^w$, for a Lie nilpotent $R$ with $\delta^n = \text{id}_R$, we prove that $R$ is right integral over $\text{Fix}(\delta)$ and $R[w, \delta]$ is right integral over $\text{Fix}(\delta)[w^n]$. 
(2, 3)-Generation of some finite groups

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Keywords: (2,3)-Generated group, Hurwitz group.

2010 Mathematics Subject Classification: 20F05, 20D06.

A group $G$ is (2, 3)-generated if and only if it is a homomorphic image of the modular group $PSL_2(\mathbb{Z})$. The most powerful result in this area is the theorem of Liebeck-Shalev and Lübeck-Malle (see [1]), which states that all finite simple groups, except the symplectic groups $PSp_4(2^m)$, $PSp_4(3^m)$, the Suzuki groups $Sz(2^m)$ ($m$ odd), and finitely many other groups, are (2, 3)-generated.

A finite group $G$ is called Hurwitz or (2, 3, 7)-generated, if it is generated by the elements of order 2 and 3, respectively, and their product has order 7. These kind of groups have a close relations with Riemann surfaces, namely: the automorphism group of a compact Riemann surface with genus $g > 1$ always has order at most $84(g - 1)$ and this upper bound is attained precisely when the group is (2, 3, 7)-generated.

The talk is split in two parts: the first one is dedicated to the proof of the (2, 3)-generation of some classical finite groups – namely $PSL_6(q)$ and $PSL_7(q)$. The second part of the talk is a classification of all pairs of elements of order 2 and 3 in the Ree group $^2G_2(q)$, such that the group is (2, 3)-generated. Finally we classify all those pairs which have a Hurwitz property.

References

A new class of generalized Thompson’s groups

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Keywords: Thompson’s groups, generalized Thompson’s groups, Thompson’s group $F$.

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In the mid-1960s three infinite finitely presented groups, commonly denoted by $F, T$ and $V$, were introduced by Richard J. Thompson. These groups have several unusual properties, on several occasions being the first known groups with a certain property. For example, Thompson proved that $T$ and $V$ are simple, which made them the first known examples of infinite finitely presented simple groups. Since then numerous generalizations of Thompson’s groups have appeared in the literature. Thompson’s groups as well as their generalizations are important source of illuminating examples in the study of infinite groups.

In this talk, I will introduce a new class of generalized Thompson’s groups. For a given group $G$ and a homomorphism $\phi : G \to G \times G$, I will describe groups $F_\phi(G)$, $T_\phi(G)$ and $V_\phi(G)$ that blend Thompson’s groups $F$, $T$ and $V$ with $G$, respectively. Several properties of the groups $F_\Delta(G)$ where $\Delta : G \to G \times G$ is the diagonal homomorphism will be discussed. In particular, I will explain how the finiteness properties (resp. conjugacy problem) of $F_\Delta(G)$ are related to the finiteness properties (resp. conjugacy problem) of $G$. In addition, I will describe the lattice of normal subgroups of $F_\Delta(G)$. 
A Bombieri-Vinogradov type exponential sum result

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Keywords: Exponential sums, primes in arithmetic progressions, almost primes.

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We improve Matomäki’s [7] Bombieri-Vinogradov type result for linear exponential sums over primes. Then we apply it to show that, for $B > 1$ and some constants $\lambda_j, j = 1, 2, 3, \eta$, subject to the following restrictions:

$$\lambda_i \in \mathbb{R}, \lambda_i \neq 0, i = 1, 2, 3;$$
$$\lambda_1, \lambda_2, \lambda_3 \text{ not all of the same sign;}$$
$$\lambda_1/\lambda_2 \in \mathbb{R} \setminus \mathbb{Q};$$
$$\eta \in \mathbb{R},$$

there are infinitely many prime triples $p_1, p_2, p_3$ satisfying the inequality

$$|\lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 p_3 + \eta| < [\log(\max p_j)]^{-B}$$

and such that

$$p_1 + 2 = P_5', \quad p_2 + 2 = P_5'', \quad p_3 + 2 = P_5'''.$$

This result improve a previous result of Dimitrov and Todorova (see [9]).

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