

# Metric Embeddings of Laakso Graphs Into Banach Spaces

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Let  $X$  be a Banach space which is not super-reflexive, i.e., which does not admit an equivalent uniformly convex norm. Then, for each  $n \geq 1$  and  $\varepsilon > 0$ , we exhibit metric embeddings of the Laakso graph  $\mathcal{L}_n$  into  $X$  with distortion less than  $2 + \varepsilon$  and into  $L_1[0, 1]$  with distortion  $4/3$ . These results improve previous estimates although we do not know whether they are optimal. However, we show that the distortion of an embedding of  $\mathcal{L}_2$  (respectively, the diamond graph  $D_2$ ) into  $L_1[0, 1]$  is at least  $9/8$  (respectively,  $5/4$ ).

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