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Faculty of Mathematics and Informatics at the Sofia University "St. Kliment Ohridski"
Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences

16-th International Workshop on

Well-Posedness of Optimization Problems and Related Topics

3 – 7 July 2023, Borovets, Bulgaria

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About

This Meeting is Dedicated to the 80th Anniversary of Academician Petar Stoyanov Kenderov and to the 70th Anniversary of Professor Vladimir Mihailov Veliov.

The workshop is a continuation of a series of meetings with the same title that was initiated in 1987 in Milan, Italy, by a small group of Bulgarian and Italian mathematicians working in the field of Well-Posedness in Optimization and Related Topics. The subsequent editions of the workshop were held in Sofia (1989), Santa Margherita Ligure (1991), Sozopol (1993), Marseille (1995), Sozopol (1997), Gargnano (1999), Warsaw (2001), Marseille (2003), Borovets (2005), Alicante (2007), Levico Termi (2009), Borovets (2011), Borovets (2018) and Borovets (2021), where the scope and the attendance were substantially enlarged.

The aim of the series of workshops is to bring together people that are interested in stability of optimization problems in a broad sense. In particular, the organizers want to attract young researchers to that area. Throughout the years the workshops have become broader in scope covering variety of topics in Variational Analysis, Topological and Functional Analytic Methods in Optimization, Optimal Control, and related applications. At the meeting we will celebrate the anniversaries of Petar S. Kenderov and Vladimir M. Veliov.

Topics

- well-posedness, optimality and stability of problems in optimization;
- calculus of variations, variational analysis;
- optimal control and stabilization;
- variational principles in optimization;
- numerical optimization methods and their stable behavior under perturbations.

Program committee

Robert Deville (Bordeaux, France)
Marian Fabian (Prague, Czechia)
Helene Frankowska (Paris, France)
Alexander Ioffe (Haifa, Izrael)

Organizers

Mikhail I. Krastanov (Sofia, Bulgaria)
Nadezhda K. Ribarska (Sofia, Bulgaria)
Nadia P. Zlateva (Sofia, Bulgaria)

Scientific Program

July 3, Monday

Chairman:	**Petar Kenderov**
9:00–9:30	**Julian Revalski**, The use of Banach-Mazur game In variational analysis: some recent results
9:30–10:00	**Aris Daniilidis**, Determination of functions by metric slopes
10:00–10:30	**Roberto Luccetti**, Characterizing a class of social ranking functions
10:30–11:00	**Coffee Break**
Chairman:	**Stanimir Troyanski**
11:00–11:30	**Robert Deville**, Recurrent points in linear dynamics
11:30–12:00	**Milen Ivanov**, Hadamard inverse function theorem proved by variational analysis
12:30–14:00	**Lunch**
Chairman:	**Julian Revalski**
14:00–14:30	**Robert Baier**, (Strengthened) one-sided Lipschitz set-valued maps: I. Examples and Applications
14:30–15:00	**Elza Farkhi**, Continuousand discrete Filippov-type stability for one-sided Lipschitzian inclusions
15:00–15:30	**Coffee Break**
Chairman:	**Luca Lussardi**
15:30–16:00	**Tzanko Donchev**, Conformable evolution inclusions with causal operators
16:00–16:30	**Ognyan Kounchev**, The solution of the most ill- of all ill-posed inverse problems - Dimiter Zidarov?s legacy in potetial theory, gravimetry and magnetometry in Bulgaria
19:00–20:00	**Dinner**
20:00	**Wellcome party**

July 4, Tuesday

Chairman:	**Vladimir Veliov**
9:30–10:00	**Helene Frankowska**, Hamilton-Jacobi-Bellman equation on the Wasserstein Space $\mathcal{P}_2(\mathbb{R}^d)$
10:00–10:30	**Antonio Marigonda**, Dynamical Systems in the Wasserstein Space and Their L^2 Representation
10:30–11:00	**Giulia Cavagnari**, Lagrangian approach to totally dissipative evolutions in Wasserstein spaces
11:00–11:30	**Coffee Break**
Chairman:	**Roberto Lucchetti**
11:30–12:00	**Alexander Ioffe**, On the strong minimum in the classical problem of calculus of variations
12:00–12:30	**Luca Lussardi**, Some uniqueness results for minimizers of classical integral functionals
12:30–14:00	**Lunch**
Chairman:	**Pierpaolo Soravia**
14:00–14:30	**Laura Poggiolini**, An Hamiltonian approach to state constrained optimal control problems
14:30–15:00	**Margarita Nikolova**, On the small-time local controllability
15:00–15:30	**Coffee Break**
Chairman:	**Denka Kutsarova**
15:30–16:00	**Todor Manev**, On locally uniformly rotund renorming of the space of continuous functions on a compact admitting a fully closed projection
16:00–16:30	**Sebastian Lajara**, Invertibility of weakly Gateaux differentiable maps between Banach spaces
19:00–20:30	**Dinner**

July 5, Wednesday

Excursion

July 6, Thursday

Chairman:	**Robert Deville**
9:30–10:00	**Denka Kutzarova**, From Property (β) of Rolewicz to Metric Geometry
10:00–10:30	**Mikhail Ostrovskii**, Weak? closures and derived sets for convex sets in dual Banach spaces
10:30–11:00	**Stephen J. Dilworth**, Metric embeddings of Laakso graphs into Banach spaces
11:00–11:30	**Coffee Break**
Chairman:	**Alexander Ioffe**
11:30–12:00	**Vladimir Veliov**, Sufficient optimality conditions and strong subregularity OPE and PDE optimal control
12:00–12:30	**Pierpaolo Soravia**, Almost everywhere local Lipschitz continuity of the minimum time function and smooth Lyapunov functions for a class of step two Carnot groups
12:30–14:00	**Lunch**
Chairman:	**Helene Frankowska**
14:00–14:30	**Nadezhda K. Ribarska**, A sufficient condition for tangential transversality
14:30–15:00	**Mikhail Krastanov**, On the basic problem of calculus of variations with pure state constraints
15:00–15:30	**Coffee Break**
Chairman:	**Milen Ivanov**
15:30–16:00	**Maria Tasheva**, Transversality and strong tangetial transversality
16:00–16:30	**Stoyan Apostolov**, Nonuniform convergence on any open subset
20:30	**Official Dinner**

July 7, Friday

Chairman:	**Nadia Zlateva**
9:30–10:00	**Hristo Sendov**, Polar convexity and a refinement of the Gauss-Lucas theorem
10:00–10:30	**Alona Mokhov**, Older and recent developments in set-valued approximation
10:30–11:00	**Pando Georgiev**, Variational principles for monotone variational inequalities
11:00–11:30	**Coffee Break**
Chairman:	**Mayer Aladjem**
11:30–12:00	**Vera Roshchina**, Faces of convex sets in high dimensions: an overview
12:00–12:30	**Matey Konstantinov**, Epigraphical characterization of uniformly lower regular functions in Hilbert spaces
12:30–14:00	**Lunch**
Chairman:	**Hristo Sendov**
14:00–14:30	**Detelina Kamburova**, Saddle points in completely regular topological spaces
14:30–15:00	**Hristina Topalova**, Perturbation method in Orlicz sequence spaces
15:00–15:30	**Coffee Break**
Chairman:	**Robert Baier**
15:30–16:00	**Mihail Hamamdjiev**, Rolle type inequality
16:00–16:30	**Boyan Stefanov**, A sufficient optimality condition for a constrained infinite-time horizon linear-quadratic game
19:00–20:30	**Dinner**

July 8, Saturday

Departure

Abstracts

Nonuniform Convergence on Any Open Subset

Stoyan Apostolov*, *Zhivko Petrov

We consider the classical problem of finding sequence of functions which converges pointwise to 0, but not uniformly on any compact subset. We introduce the so called height function in order to study the problem systematically. We examine some properties restricting what a height function associated to a continuous function might be. Next we show that some of these properties could not be made more restrictive, and in a sense show that a large set of sequences satifies an extreme form of them.

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(Strengthened) One-Sided Lipschitz Set-Valued Maps: I. Examples and Applications

Robert Baier*, *Elza Farkhi

The one-sided Lipschitz (OSL) condition is a notion for single-valued functions and set-valued maps to weaken the Lipschitz condition and even includes certain discontinuous maps. Applications of the OSL condition can be found in approximation results of reachable sets of nonlinear control problems resp. differential inclusions. In contrast to the Lipschitz modulus the OSL constant in the (one-sided) estimate can be negative allowing a boundedness or damping effects of errors even for infinite time horizon.

We collect several examples of single-valued functions and set-valued maps for which the OSL condition hold. Among them are Lipschitz, monotone decreasing or dissipative set-valued monotone maps, linear functions (with possible negative OSL constants) and the well-known negative sign function. OSL functions can be smooth, Lipschitz continuous, they may have infinite slopes of the derivative or may be discontinuous. Many activation functions in machine learning as sigmoidal or the saturation function, the Heaviside function or the ReLU one (rectified linear unit) are OSL.

The (pointwise) negative of maximal monotone maps and the Filippov regularization of discontinuous OSL right-hand sides are further interesting examples. In applications outer and inner perturbations of OSL maps play an important role. In contrast to monotone or dissipative systems, OSL right-hand sides of differential inclusions allow non-uniqueness of solutions. The talk discusses several variants of (strengthened) OSL conditions, explain their advantages and limitations with the help of model problems in mechanical systems with Coulomb and dry friction, in electric circuits or with linear feedback together with the convergence order of Euler's method.

Lagrangian Approach to Totally Dissipative Evolutions in Wasserstein Spaces

Giulia Cavagnari*, *Giuseppe Savaré*, *Giacomo Enrico Sodini

Aim of the talk is to study well-posedness for evolution equations/inclusions in the Wasserstein space $\mathcal{P}_2$ of probability measures driven by *totally dissipative* multivalued probability vector fields (MPVF). For this purpose, we lift the problem to an Hilbert space L^2 of random variables and we show that maximal totally dissipative MPVFs are in one-to-one correspondence with maximal dissipative operators in L^2 that are law invariant. This allows us to study the problem in an Hilbertian setting where the implicit Euler scheme can be applied to generate the unique Lagrangian flow for the differential inclusion of the corresponding Cauchy problem. This machinery is then imported in the Wasserstein framework where we obtain, as a byproduct, a Lagrangian characterization for the (unique) corresponding evolution of probability measures. This turns out to be also the unique *EVI* solution of the MPVF.

We also analyze the case of MPVFs satisfying only a (weaker, not total) dissipativity condition as in our previous work [1], as for e.g. the (opposite of) the subdifferential of convex functionals in $\mathcal{P}_2$. In this case, we provide sufficient conditions (related to a discrete approximation property) in order to obtain the well-posedness and the Lagrangian characterization results previously mentioned.

Determination of functions by Metric Slopes

Aris Daniilidis

Two smooth, convex and bounded from below functions in a Hilbert space are equal up to a constant if and only if their derivatives have the same norm everywhere. We shall give an analogous determination property for the class of continuous, coercive functions in compact metric spaces using the notion of metric slope and discuss extensions in a more general case.

Talk based on several works in collaboration with:

T. M. Le (TU Wien, Austria), L. Miclo (TSE, France) and D. Salas (UOH, Chile).

Recurrent points in linear dynamics

Robert Deville

Phenomena that look non linear appear in linear dynamics whenever the underlying Banach space is infinite dimensional. We exhibit several such phenomena, for instance there exists a linear operator on a Banach space X such that both the set of points such that $(\|T^n x\|)$ converges and its complement are dense in X. We also show that on every separable, infinite dimensional Banach space X, it is possible to construct a linear operator T such that there is no x in X such that $(\|T^n x\|)$ tends to infinity, but both the set of recurrent points for T and its complement have non empty interior.

Metric embeddings of Laakso graphs into Banach spaces

Stephen J. Dilworth*, *Denka Kutzarova*, *Svetozar Stankov

Let X be a Banach space which is not super-reflexive, i.e., which does not admit an equivalent uniformly convex norm. Then, for each $n \geq 1$ and $\varepsilon > 0$, we exhibit metric embeddings of the Laakso graph $\mathcal{L}_n$ into X with distortion less than $2+\varepsilon$ and into $L_1[0,1]$ with distortion $4/3$. These results improve previous estimates although we do not know whether they are optimal. However, we show that the distortion of an embedding of $\mathcal{L}_2$ (respectively, the diamond graph D_2) into $L_1[0,1]$ is at least $9/8$ (respectively, $5/4$).

Conformable Evolution Inclusions With Causal Operators

T. Donchev*, *A.I. Lazu

In this paper we study local and nonlocal problems of conformable semilinear evolution inclusions with causal operators. The class of problems studied here includes time lag conformable system and a large classes of conformable parabolic multivalued differential and integro-differential equations. We first investigate this problems with the help of measure of noncompactness and afterwards with the help of compact semigroup in order to prove existence of solutions. The advantages and disadvantages of both approaches are then pointed out.

Acknowledgements. The work was supported by the Bulgarian National Science Fund under Project KP-06-N32/7.

Continuous and Discrete Filippov-Type Stability for One-Sided Lipschitzian Inclusions

Robert Baier, Elza Farkhi

The *one-sided Lipschitz (OSL)* regularity of set-valued maps weakens the Lipschitz continuity and allows discontinuities. It is applied e.g., in the analysis of approximations of reachable sets of nonlinear control problems. In this field, Filippov theorems in continuous and discrete time are important tools for stability analysis of control systems or differential inclusions and for convergence proofs for their discretizations.

We study the sensitivity of continuous and discrete time (difference) inclusions with respect to perturbations in the initial set, in the right-hand side set (outer perturbation) and in the state variable argument of the right-hand side (inner perturbation). While in the classical Filippov theorems for Lipschitz right-hand sides, the distance between the trajectory sets of the perturbed and the original system is Lipschitz (of first order) with respect to the perturbations, in the OSL case this distance typically involves the square root of the inner perturbations (and the square root of the step size for the discretizations). These estimates are improved to the first order for *strengthened one-sided Lipschitz (SOSL)* right-hand sides both for differential and inclusions and their Euler discretizations. Thus the Lipschitz rates in the Filippov theorem are extended from the classical Lipschitz set-valued maps to SOSL ones.

We discuss some applications related to infinite time horizon problems, estimates for reachable sets of the (set-valued) explicit Euler method and discrete relaxation results: reachable sets of nonconvex difference inclusions are compared to the ones with the convexified right-hand sides in the case of OSL maps.

Hamilton-Jacobi-Bellman equation on the Wasserstein Space $\mathcal{P}_2(\mathbb{R}^d)$

H. Frankowska*, *Z. Badreddine

This talk is devoted to the Hamilton-Jacobi-Bellman (HJB) equation associated to the Mayer type optimal control problem on the Wasserstein space $\mathcal{P}_2(\mathbb{R}^d)$ of Borel probability measures:

$$\text{minimize } g(\mu(1))$$

over all the solutions defined on the time interval $[0, 1]$ of the continuity equation

$$\partial_t \mu(t) + \operatorname{div}\left(f(\mu(t), u(t)) \cdot \mu(t)\right) = 0,\ \mu(0) = \mu_0,\ \ u(t) \in U$$

with μ_0 having a compact support. In the above $f : \mathcal{P}_2(\mathbb{R}^d) \times U \to Lip(\mathbb{R}^d, \mathbb{R}^d)$, U is a compact metric space and $Lip(\mathbb{R}^d, \mathbb{R}^d)$ denotes the space of bounded Lipschitz functions from $\mathbb{R}^d$ into itself.

Solutions to (HJB) are defined in terms of the Hadamard type sub/superdifferentials and, under Lipschitz-like assumptions, the value function of the Mayer problem is such solution of (HJB). Continuous solutions are unique whenever we focus our attention on solutions defined on explicitly described time dependent compact valued tubes of probability measures.

We also discuss some viability and invariance theorems in the Wasserstein space and introduce a new notion of proximal normal.

Rolle Type Inequality

Mihail Hamamdjiev*, *Milen Ivanov*, *Nadia Zlateva

We establish a multivalued Rolle type inequality using the Long Orbit or Empty Value (LOEV) principle. We also discuss extensions to the second order case.

Variational Principles For Monotone Variational Inequalities

Pando Gr. Georgiev

We consider a parameterized variational inequality (A, Y) in a Banach space E defined on a closed, convex and bounded subset Y of E by a monotone operator A depending on a parameter. We prove that under suitable conditions, there exists an arbitrarily small monotone perturbation of A such that the perturbed variational inequality has a solution which is a continuous function of the parameter, and is near to a given approximate solution. In the nonparametric case this can be considered as a variational principle for variational inequalities, an analogue of the Borwein-Preiss variational principle.

Some applications are given: 1) an analogue of the Nash equilibrium problem, defined by a partially monotone operator, when one of the domain is not a strong compact set. In the case of two functions, this result can be considered as a Sion's type minimax theorem for monotone operators; 2) a variant of the parametric Borwein-Preiss variational principle for Gâteaux differentiable convex functions under relaxed assumptions; 3) a generic result in sense of porosity stating that the most variational inequalities considered here are well posed (which means that the complement of the set of the well posed variational inequalities is σ-porous).

The tool for proving the main result is a useful lemma about existence of continuous ε-solutions of a variational inequality depending on a parameter. It has an independent interest and allows a direct proof of an analogue of Ky Fan's type inequality for monotone operators, introduced here, which leads to a new proof of the Schauder fixed point theorem.

On the strong minimum in the classical problem of calculus of variations

Alexander Ioffe

The main result to be presented at the talk contains a second order necessary condition for a strong minimum in the standard problem of calculus of variations. Novelty of this result is emphasized by the fact that no idea of a possibility of such a condition has ever appeared in classical theory. A simple example shows that the theorem can work when other known necessary conditions fail. Originally the theorem was obtained as a consequence of a corresponding result for a sufficiently general class of optimal control problems (Calculus of Variations and PDEs (2020), 59:83). But an independent proof of the theorem is noticeably simpler and will be described in some details.

Hadamard Inverse Function Theorem Proved by Variational Analysis

Milen Ivanov*, *Nadia Zlateva

We present a proof of Hadamard Inverse Function Theorem by the methods of Variational Analysis, adapting an idea of I. Ekeland and E. Séré [1].

Recall that Hadamard Inverse Function Theorem states:

Let $f \in C^1$, $f'(x)$ be invertible for all x and satisfying

$$\|[f'(x)]^{-1}\| \leq M, \quad \forall x \in X, \tag{0.1}$$

for some $M > 0$.

Then f is C^1 invertible on X.

In other words, there is $g \in C^1$ such that

$$g(f(x)) = f(g(x)) = x, \quad \forall x \in X.$$

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Saddle Points in Completely Regular Topological Spaces

Detelina Kamburova*, *Rumen Marinov*, *Nadia Zlateva

We give a characterization of completely regular topological spaces. Applying some recent results for supinf problems in completely regular topological spaces we establish a variational principle for saddle point problem. Well-posedness of saddle point problems is studied as well.

Epigraphical Characterization Of Uniformly Lower Regular Functions In Hilbert Spaces

Matey Konstantinov*, *Nadia Zlateva

We provide a characterization of uniformly lower regular functions defined on a Hilbert space. To this end we introduce and study a property we call epi prox-regularity of an epigraph set which slightly differs from the well-known prox-regularity property of a set.

The Solution of the Most Ill- of All Ill-Posed Inverse Problems - Dimiter Zidarov's Legacy in Potetial theory, Gravimetry and Magnetometry in Bulgaria

Ognyan Kounchev

The mathematical theory of the solution of the Inverse problems in Potential theory starts with Newton's theorem. Tikhonov (1943) has invented his regularization method motivated by the Inverse Gravimetric problem. In the present lecture we will tell the story of the solution of Inverse potential problems in Gravimetry in Bulgaria, which is based on the theory of mother bodies and partial bubbling of Dimiter Zidarov. The reader may consult the two monographs, of 1968 and 1982, [1],[2]. The final numerical algorithm based on Zidarov's approach has been completed in 1986 by Stoyan Avdev [3]. One has to emphasize that without the theory of D. Zidarov about Graviequivalent bodies the numerical solution would not be possible. Just the general regularization approach of A. Tikhonov is not enough to have the complete solution. One of the biggest discoveries of Zidarov is the notion of Mother body, which is the basis for a well-posed "sub-problem" thanks to the method of partial balayage. His examples of mother bodies have become classical in the area, cf. [4], [5], [6].

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On the basic problem of calculus of variations with pure state constraints

Mikhail I. Krastanov, Nadezhda K. Ribarska

The basic problem of calculus of variations with locally Lipschitz criterion is considered in the presence of pure state constraints of equality type. A necessary optimality condition is obtained under suitable assumptions. An example illustrating the applicability of the obtained results is presented.

From Property (β) of Rolewicz to Metric Geometry

Denka Kutzarova

In connection to well-posedness of optimization problems, in 1987 Stefan Rolewicz introduced a new geometric property of the norm of a Banach space and called it (β). The spaces which admit an equivalent norm with property (β) formed a new isomorphic class between super-reflexive and reflexive spaces.

This area of research got a new life in a paper of Baudier, Kalton and Lancien from 2010 where they gave a metric characterization of asymptotic super-reflexive spaces, which turned out to be exactly the new isomorphic class of spaces with an equivalent norm with property (β). In a paper from 2012, Lima and Andrianarivony showed the importance of property (β) for solving a ten-year old question of Bates, Johnson, Lindenstrauss, Preiss and Schechtman about uniform (nonlinear) quotients. Independently, also in 2012, Revalski and Zhivkov defined the notion of compact uniform convexity in connection to the study of metric projections, and that turned out to be (isometrically) the same as the property (β) of Rolewicz. That led to the introduction of asymptotic midpoint uniform convexity (AMUC), and later it was proved that there are no uniform bi-Lipschitz embeddings of some types of countably branching graphs(for example, countably branching diamond and Laakso graphs) into Banach spaces with an equivalent AMUC norm.

The lecture will be a short survey of these results, including several papers of the author with different sets of coauthors, both from the first and the second period of this research.

Invertibility Of Weakly Gâteaux Differentiable Maps Between Banach Spaces

Milen Ivanov, Jesús A. Jaramillo, Sebastián Lajara, Nadia Zlateva

We provide several criteria for the existence of continuous selections and invertibility of continuous weakly Gâteaux differentiable maps between Banach spaces.

Characterizing a class of social ranking functions

Roberto Lucchetti, Monica Milasi, Stefano Moretti

In Social Choice a problem of great interest is to define a ranking for a set of alternatives, given rankings provided by a fixed number n of agents. Thus, we fix a finite set $A = \{a, b, c, \dots\}$ and a positive integer n; to each alternative $i \in A$ we associate an n-vector $V(i) := (v_1, \dots, v_m)$, $v_s \in I_s$, where I_s, $s = 1, \dots, n$, is the interval $I_s = [\frac{n-s}{n}, \frac{n-s+1}{n}]$. $V(i)$ is called a
textit valuation of i and we shall denote by $V_s(i)$ the s-th valuation v_s of i. $T := (V(i))_{i \in A}$ is a set of valuations, one for each alternative; we write $V(i, T)$ whenever we need to specify that the evaluation of the alternative i is part of the set T of evaluations. Finally, denote by $\mathcal{T}(A)$ the set of all evaluations of dimension n over the set of alternatives A. Observe that each single vector of valuations of every alternative is arranged in decreasing order.

Aim of this paper is the study of a class of social ranking Functions. A social ranking solution is a function $F : \mathcal{T}(A) \to \mathcal{R}(A)$. More precisely, our main result provides a small meaningful set of properties, connected to classical ones in Social Choice, to guarantee that the social ranking function is lexicographic. This means that there exists a linear order L on the columns such that in a first step a ranking is made by looking at the evaluations provided by the first column in the order L; since in many cases several alternatives can be indifferent, then the above ranking is refined by looking at the second column in the order L: this can break some of the previous ties. And so on. We further provide an algorithm that, given a lexicographic F, allows finding the linear order L in the above Definition. Finally, we provide a characterization for three specific lexicographic social ranking functions well known in the literature of Social Choice and Voting.

Some uniqueness results for minimizers of classical integral functionals

Luca Lussardi

The existence of Lipschitz minimizers of classical integral functionals with prescribed boundary conditions dates back to '70 (Hartmann and Stampacchia) and nowdays it is a classical result. In the absence of strict convexity of the integrand, the uniqueness of minimizers is not in general guaranteed. Nevertheless, Parks in '80 proved, by means of a minimal surfaces argument, that the total variation has a unique Lipschitz minimizer. Together with Elvira Mascolo (University of Florence), in 2017 we proved the uniqueness of Lipschitz minimizers for more general functionals in dimension 2. When the dimension is bigger than 2 the result is still true but it requires a more careful analysis of the level sets of Lipschitz functions, and this is contained in a work I'm writing together with Giovanni Alberti (University of Pisa).

On Locally Uniformly Rotund Renorming of the Space of Continuous Functions on a Compact Admitting a Fully Closed Projection

***Todor Manev*, Nadezhda Ribarska, Stanimir Troyanski**

We show that if the compact space X admits a fully closed projection onto a compact Y such that $C(Y)$ admits an equivalent locally uniformly rotund (LUR) norm, as do the spaces $C(\pi^{-1}(y))$ for all y in Y, then $C(X)$ is also LUR renormable. A continuous map $\pi : X \to Y$ between Hausdorff compacta is called fully closed if the intersection $\pi(A) \cap \pi(B)$ is finite whenever A and B are closed disjoint subsets of X. As a main corollary, we prove that $C(K)$ admits an equivalent LUR norm if K is a Fedorchuk compact of finite spectral height.

Dynamical Systems in the Wasserstein Space and Their L^2 Representation

***Antonio Marigonda*, Marc Quincampoix**

S everal optimal control problems in the Euclidean space, like systems with uncertainty, control of flock dynamics, or control of multiagent systems, can be naturally formulated in the space of probability measures on the Euclidean space. This leads to the study of dynamics and viscosity solutions to the Hamilton-Jacobi-Bellman equation satisfied by the value functions of those control problems, both stated in the Wasserstein space of probability measures. Since this space can be also viewed as the set of the laws of random variables in a suitable L^2 space, in this talk we aim to study such control systems in the Wasserstein space and to investigate the relations between dynamical systems in Wasserstein space and their representations by dynamical systems in L^2, both from the points of view of trajectories and of (first order) Hamilton-Jacobi-Bellman equations.

Older and Recent Developments in Set-Valued Approximation

Nira Dyn, Elza Farkhi, *Alona Mokhov*

Set-valued functions (functions mapping a closed real interval to general compact sets in R^n) find applications in different fields such as economy, optimization, dynamical systems, control theory, game theory, differential inclusions, geometric modeling. In the recent decades, analysis along with the approximation of set-valued functions (SVFs, multifunctions) have been developing rapidly. Older approaches investigate mostly multifunctions with convex compact images. The standard tools here are the Minkowski linear combinations, the support functions and the Aumann integral. Yet, if the images of set-valued functions are not necessarily convex, then the techniques based on these tools may fail. Design and analysis of approximation methods for SVFs with general compact sets it is more challenging task and needs different approaches.
Metric approximations allow to approximate set-valued functions with compact, not necessarily convex values. This talk surveys the progress on the metric approximation of set-valued functions. We will focus on some new developments as the Fourier approximation and integral operators on multifunctions of bounded variation. The results extend classical theorems for real functions of bounded variation to set-valued ones.

On The Small-Time Local Controllability

Mikhail Iv. Krastanov, Margarita N. Nikolova

A class of polynomial control systems is considered. The local properties of the corresponding reachable sets are studied by using a general differential geometrical approach based on the Campbell-Baker-Hausdorff formula. Two sufficient conditions as well as a necessary condition for small-time local controllability are presented. Suitable examples illustrate the existing gap between the presented necessary and sufficient conditions.

Weak* Closures and Derived Sets for Convex Sets in Dual Banach Spaces

Mikhail Ostrovskii

Weak* sequential closures play an important role in many problems. For example, the conditions for regularizability of unbounded linear operators which are inverses to bounded linear operators can be stated using weak* sequential closures of images of dual operators.

S. Banach and S. Mazurkiewicz started to develop the theory of weak* sequential closures (weak* derived sets) in 1929-1932. I plan to describe the history of the topic and its applications. The main new result: For every nonreflexive Banach space X and every countable successor ordinal α, there exists a convex subset A in X^* such that α is the least ordinal for which the weak* derived set of order α coincides with the weak* closure of A. This result extends the previously known results on weak* derived sets by Ostrovskii (2011) and Silber (2021).

An Hamiltonian Approach To State Constrained Optimal Control Problems

Laura Poggiolini*, *Francesca C. Chittaro

We establish sufficient optimality conditions for strong-local optimality of Pontryagin extremals for some single-input control-affine problems. More precisely, we consider the control problem

$$\begin{aligned}
&\dot{\xi}(t) = f_0(\xi(t)) + u(t) f_1(\xi(t)) \quad \text{a.e. } t \in [0,T],\\
&\xi(0) = x_0, \quad \xi(T) \in \mathcal{N}_f,\\
&c(\xi(t)) \leq 0 \quad \forall t \in [0,T], \qquad\qquad |u(t)| \leq 1 \quad \text{a.e. } t \in [0,T],
\end{aligned}$$

where the state space is a smooth manifold M, the function $c\colon M \to \mathbb{R}$ defining the state constraint is assumed to be smooth on a neighborhood of its zero-level set; f_0, f_1 are smooth vector fields on M and $\mathcal{N}_f$ is a submanifold of M. We associate with the problem above a cost J to be minimized, that can be either in Mayer form or the minimum time to reach $\mathcal{N}_f$, i.e. we deal with both the following problems

$$\text{minimize } \psi(\xi(T)), \quad T > 0 \text{ fixed},$$

$$\text{minimize } T, \quad T > 0 \text{ free}.$$

We study extremals containing a bang arc, a boundary arc, followed by a finite number of bang arcs. The sufficient conditions are given by some regularity conditions on the boundary arc, together with a strengthened version of the necessary conditions, and the coerciveness of a suitable finite-dimensional quadratic form. The sufficiency of the provided conditions is proven via Hamiltonian methods. We are currently studying the case when also an internal singular arc is present.

The Use Of Banach-Mazur Game In Variational Analysis: Some Recent Results

Julian P. Revalski

In this talk we will present some recent results which demonstrate the use of the well-known Banach-Mazur game to tackle problems from variationl analysis. The first example is related to variational principles for **maximization** of lower semicontinuous functions (not for minimization of such functions which is the usual case). It turns out that the existence of a winning strategy for one of the players in the Banach-Mazur game characterizes the validity of a generic variational principle for maximization of continuous bounded perturbations of a fixed lower semicontunuous function which is bounded from above.

The second example is related to the existence of residually defined selections of a certain class of set-valued mappings. Here again, the existence of special type of winning strategies for one of the players in the Banach-Mazur game characterizes the existence of such selections. The results are based on the two papers given in the references.

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A sufficient condition for tangential transversality

Nadezhda K. Ribarska, Mikhail I. Krastanov

A sufficient condition for tangential transversality involving measures of non-compactness as well as a Lagrange multiplier theorem for the infinite-dimensional optimization problem are obtained. The relation of the obtained results to the basic problem of calculus of variations is discussed. Also, an application to the problem of calculus of variations in the presence of pure state constraints of inequality type is presented.

Faces of convex sets in high dimensions: an overview

Vera Roshchina

In this talk I will cover some recent results on the facial structure of convex sets in high dimensions, focussing on the significant differences between what we can imagine happens in two- and three-dimensional spaces, and the complexity of facial structure of convex sets in dimensions four and higher, including infinite-dimensional vector spaces.

Polar Convexity and a Refinement of the Gauss-Lucas Theorem

Hristo Sendov

We will introduce the notion of polar convexity, which extends the usual notion of convexity. We will give examples, explain its basic properties, and show how it arises in various situations. Then, we will use it to give a new refinement of the classical Gauss-Lucas theorem for complex polynomials. The Gauss-Lucas theorem states that the critical points of a polynomial are in the convex hull of its zeros.

Small Time Local Attainability for Nonlinear Systems: a Hamiltonian Approach

Pierpaolo Soravia

A classical problem in optimal control is small time attainability of sets, as it also has important consequences in the study of boundary value problems for partial differential equations of Hamilton-Jacobi type. We will discuss a new approach to small time local attainability of smooth manifolds of any dimensions, possibly with a boundary. We give explicit pointwise conditions which use derivatives of the controlled vector field and of the functions determining the equations of the target. Our sufficient conditions are of any order and they involve the use of recursive higher order hamiltonians. Our conditions are equivalent to usual ones using Lie brackets instead in the case of second order conditions for symmetric or affine systems, but in general in our opinion are more natural, easier to find and less involved to discuss, especially for higher order conditions.

A Sufficient Optimality Condition for a Constrained Infinite-Time Horizon Linear-Quadratic Game

Mikhail I. Krastanov, Rossen Rozenov, Boyan K. Stefanov

A problem of a constrained infinite-time horizon linear quadratic game for continuous systems is considered. Using a suitable transformation, this problem is reduced to a finite-time horizon game. The main result is a sufficient condition for the existence of a saddle point equilibrium. To illustrate this result, our approach is applied to a basic monetary policy model.

Transversality and Strong Tangetial Transversality

Nadezhda K. Ribarska, Maria Tasheva

The relation between the properties transversality and strong tangential transversality of sets in a Banach space is explored. It is obtained that for two sets in a Banach space strong tangential transversality implies transversality and the reverse implication fails. An application to Banach space theory is presented. A definition of strong tangential transversality for a finite number of sets is proposed which preserves the main properties of the notion.

Perturbation Method in Orlicz Sequence Spaces

Hristina Topalova*, *Nadia Zlateva

We develop a new perturbation method in Orlicz sequence spaces ℓ_M with Orlicz function M satisfying Δ_2 condition at zero. This result allows one to support from below any bounded below lower semicontinuous function with bounded support, with a perturbation of the defining function σ_M.

We give few examples how the method can be used for determining the type of the smoothness of certain Orlicz spaces.

Sufficient Optimality Conditions and Strong Subregularity for ODE and PDE Optimal Control

Vladimir M. Veliov

Strong metric subregularity (SMsR) of mappings is a fundamental property in the analysis of approximation methods for Variational Inequalities (VIs), such as finite-dimensional approximations, gradient projection methods, Newton-type methods, etc. VIs arise, in particular, when considering the system of first-order optimality conditions for control-constrained optimal control problems. There is a relation (sometimes even coincidence) between the known sufficient optimality conditions and the conditions for SMsR. The talk will present such conditions for VIs associated with ODE or PDE optimal control problems. The focus will be on problems that are affine with respect to the control.

The talk is based on joint papers with A. Corella, N. Jork, and N. Osmolovskii.

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