Web-Accessible Tools for Interval Linear Systems

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Interval webComputing is a collection of dynamic interactive pages designed to make specialized interval computations and visualization widely accessible through web browsers. Discussed are the general conception and two typical components: visualization of solution sets to interval linear systems and solvers of parametric interval linear systems.

1 Interval webComputing

Mathematical computing is inevitably becoming distributed over the Internet: for easy distribution of mathematical materials; to make specialized computations widely accessible; to allow easy interoperability; to aggregate functionalities from different systems; for web-based mathematical education involving active elements.

The research work directed towards developing techniques and algorithms for solving problems with result verification and problems involving uncertain input data by methods based on interval arithmetic can now be deployed by live interactive web sites, vastly increasing the number of people who can use and learn from these results. Interval webComputing is a service which aims to provide free remote access to interactive interval computations, dynamic graphics generation, online interval calculators, and active tutorials that make a good use of dynamic access to interval computing. The service is supplied by a collection of dynamic interactive web pages. Accessing these pages users enter or upload data, choose between different options, select input parameters and submit data to build up a sequence of results in a numeric, symbolic, graphics or combined form. Interval pages presently operate within a more general service framework, called webComputing, they are accessible at http://cose.math.bas.bg/webComputing/.

Interval webComputing is based on the computational and visualization power provided by the system *Mathematica* [6]. Some of the *Mathematica* code for interval computations, developed during the years and organized into packages, was used as a background of the developed dynamic interactive site to enable people to run this code and use the results for their regular work. The connection mechanism and technology empowering webComputing are built upon web*Mathematica*, an innovative product allowing *Mathematica* to run on a server to provide the necessary calculations and graphs [5]. web*Mathematica* is a server-based technology built on top of two standard Java technologies: Java Servlet and JavaServer Pages. It provides a collection of tools that allow *Mathematica* commands to be placed inside HTML pages. When a request is made for one of these pages, the *Mathematica* commands are evaluated on the server and the computed result is inserted into the page [5].

Most of the interval interfaces are designed to be multi-functional providing both computational power and interactive learning environment. Below we present two typical components of the interval webComputing: visualization of solution sets to interval linear systems and solvers of parametric interval linear systems.

2 Visualization of solution sets to interval linear systems

The learning environment of Interval webComputing can be demonstrated best by the visualization pages.

The solution sets of 2- and 3-dimensional interval linear systems, specified by the user, are generated dynamically. Changing data, you can explore different shapes of the solution sets, including unbounded set (drawn in a given region). Giving the solution set range to be a particular orthant, demonstrates the convexity property of the solution sets. Additionally, on the same graphics containing the solution set there can be also drawn: • the points which coordinates are solutions to point linear systems generated by all possible combinations of the interval end-points involved in the system; • the vertices of the solution set determining its exact hull, which demonstrates the efficiency of J. Rohn's sign-accord algorithm computing the coordinates of these points; • any number of interval boxes which may represent solution enclosures obtained by different methods.

Graphics is so useful for understanding the parametric solution set drawn by the corresponding webComputing page. Combining the parametric solution set and the solution set to the corresponding nonparametric system reveals the difference between both sets. All the above items, that can be represented for a nonparametric system, can be optionally drawn on the same graphics involving the parametric solution set. Additionally, plotting the Rohn's points, obtained for the parametric system, demonstrates the importance of checking monotonicity properties of the parametric solution set.

Some more interfacing features pertaining to the solution set pages are:

• A 3D graphics can be displayed either static or by an applet [1] allowing a real-time rotation and resizing of the image.

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• Additionally, the graphics image can be adjusted by a number of 2D/3D general graphics options: plot range, whether to draw axes or frame, axes/frame labels, thicks, colors, image size, aspect ratio, etc.

• The generated graphics image can be exported to the client machine in 12 graphics file formats.

All webComputing pages are self-contained having default values for the input and the optional parameters.

3 Web solvers of parametric linear systems

The computational power of the interval webComputing service is presently demonstrated by two solvers for parametric interval linear systems: one dealing with systems involving affine-linear dependencies between interval parameters, and another solving systems with nonlinear dependencies between the parameters. The corresponding web interfaces are based on the latest improved version of a *Mathematica* package IntervalComputations‘LinearSystems‘ [2].

Some functional and interfacing features are given below.

• Both solvers implement a recent generalization of S. Rump's parametric fixed-point iteration method [3] having an expanded scope of applicability. Since this is the only rigorous method with general purpose, the provided wide access to these unique by now solvers gives a suitable basis for comparison of different methods and studying their efficiency.

• Both solvers allow solving as square systems as over- or underdetermined parametric interval linear systems providing improved sharp solution enclosure for the nonsquare systems [4].

• Since traditional numerical systems do not have integrated symbolic capabilities with which to perform symbolic preprocessing, the role of computer algebra systems increase. Solving systems involving nonlinear dependencies applies algebraic simplification to get a sharp range enclosure of the quantities involved in the algorithm. Since algebraic computations are time consuming and web*Mathematica* applications have a fixed time limit for using the *Mathematica* kernel, this web solver is not suitable for large systems. On the contrary, the solver for systems with affine-linear dependencies is very fast being based on entirely numerical computations.

• Another unique feature provided by the web interface and the underlying software is the computing of component-wise inner approximation of the solution set hull [2] and gives a measure for the degree of sharpness of the outer enclosure.

• In order to serve real-life computational needs, both parametric solvers allow uploading data files from the client machine onto the server. For a parametric system, 3 data files (containing the matrix, the right-hand side vector and the rules for the parameters) are required. Present restriction to the maximum size of a data file is 4MB. Matrix/vector data in a file presently should be specified by *Mathematica* lists, or as sparse arrays [6]. Future enhancement of the solvers include downloading the generated results on the client machine and combining/reusing the results from different pages.

4 Perspectives

The interval webComputing service will expand by providing web interface to more interval tools and problem solvers, showcasing interval-related work in interactive web documents. From an application perspective the end-user benefits include: no need to buy, install, and maintain software; no need to develop user software; no need to learn different software applications training time being considerably reduced; certainty that use the most recent version. The technical professionals and interval researchers can: easily explore newly developed methods; compare the efficiency of different methods and software tools; demonstrate interval related work; teach interval methods involving students in an active exploration by doing. The remote Authoring Framework, provided by the wider webComputing service, allows development of highly interactive mathematical/interval web applications to be done easily and the applications to be powerful. The background code is not restricted to *Mathematica* code since the system has powerful connectivity technology that allows it to communicate with a large variety of external resources, provided e.g. by languages as Java, C, Fortran, etc. We hope to collaborate with more people interested in web accessible mathematics. The overall framework is intended to be open, flexible, and usable as a testbed for research or a platform for application developments.

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