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ABSTRACTS

# Fast domain decomposition solver for internal problems of 3D hierarchical hp-FEM

I. Anoufrieu, V. Korneev

Deriving fast domain decomposition (DD) preconditioners-solvers for *hp* discretizations of 2-nd order elliptic equations is still a challenge [1], although one pioneering solver of this type has been recently presented in [2] and generalized on adaptive *hp* discretizations in [3]. As it is well known, the most time consuming component of such solvers is the procedure for solving internal Dirichlet problems on subdomains of decomposition. For the discretizations generated by means of cubic reference elements with the hierarchical coordinate polynomials, the problem is equivalent to solving the finite difference (finite element with the first order finite elements) approximation of the 4-th order equation

$$z^2 u_{xxyy} + y^2 u_{xxzz} + x^2 u_{yyzz} = f(x, y, z), (x, y, z) \in \delta, u|_{\partial\delta} = 0(1)$$

on the uniform orthogonal grid in the unite cube  $\delta = (0,1)^3$ . We consider approach outlined in [4], which is based on DD for the problem (1) and employs the geometrically nonuniform in each variable decomposition mesh and approximation of the coefficients in each subdomain by constants. This allows us to design fast preconditioners-solvers for the local Dirichlet problems on subdomains of decomposition and their faces, and, therefore, to reduce the problem to solving the wire basket subproblem of a much smaller dimension.

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## On a parallel Maxwell eigensolver

Peter Arbenz, Martin Bečka, Roman Geus, U. Hetmaniuk

In the design of accelerator cavities a few eigenvalues of a large generalized matrix eigenvalue problem have to be computed. A matrix eigenvalue problem of the form

$$A\mathbf{x} = \lambda M\mathbf{x}, \quad C^T \mathbf{x} = \mathbf{0} \quad (1)$$

is obtained by the finite element discretization of the time-harmonic Maxwell equation in weak form by a combination of Nédélec (edge) and Lagrange (node) elements. Here,  $A$  and  $M$  are real symmetric.  $A$  is positive semidefinite and  $M$  is positive definite.

We found the Jacobi-Davidson (JD) method to be a very effective factorization-free algorithm to compute 5–10 eigenvalues at the low end of the spectrum of (1) provided that a good preconditioner is available for the correction equations that have to be solved in each step of the JD iteration.

The preconditioner of our choice is a combination of a hierarchical basis preconditioner and a smoothed aggregation AMG preconditioner. It is close to optimal regarding iteration count and scales with regard to memory consumption.

With our sequential code we managed to solve problems of size a few millions, however at a prohibitive execution time[1]. In this paper we present our parallel implementation of the previous sequential code. The parallel code makes extensive use of the Trilinos software framework.

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## Parallel PCG solver for non-conforming FEM elasticity problems

Gergana Bencheva, Svetozar Margenov

The recent efforts in development of efficient solution methods for non-conforming finite element systems are inspired by their importance for various applications in scientific computations and engineering. The non-conforming finite elements possess attractive properties as a stable discretization tools for ill-conditioned problems.

A new preconditioner for parallel solution of linear isotropic elasticity problem is proposed in this study. Rotated bilinear non-conforming finite elements on quadrilaterals are implemented to the Lamé system of PDEs. Two variants of the nodal basis functions corresponding to mid-point (MP) and integral mid-value (MV) interpolation operators are considered. The obtained discrete problem satisfies the Korn's second inequality. The linear system is solved using preconditioned conjugate gradient (PCG) iterative method based on the MIC(0) incomplete factorization.

Three steps are performed to construct the preconditioner. Isotropic displacement decomposition technique is applied at the beginning. A locally modified approximation of the obtained block-diagonal matrix is used at the second step giving possibility for: a) a stable MIC(0) factorization, and b) scalable parallel implementation. The obtained diagonal blocks are approximated by MIC(0) factorization at the end. The constructed preconditioner possess well parallelizable structure, preserving the robustness with respect to the local properties of the problem.

The related condition numbers are evaluated using local analysis for both MP and MV variants of the algorithm. The derived theoretical estimates for the parallel execution times as well as the presented set of numerical tests well illustrate the potential of the proposed approach.

## Multilevel preconditioners for the p-version of the finite element method

Sven Beuchler

In this talk, we discretize a uniformly elliptic second order boundary value problem in 2D by the  $p$ -version of the finite element method. An inexact Dirichlet-Dirichlet domain decomposition preconditioner for the system of linear algebraic equations is investigated. The ingredients of such a pre-conditioner are an pre-conditioner for the Schur complement, an pre-conditioner for the sub-domains and an extension operator operating from the edges of the elements into their interior. Using methods of multi-resolution analysis, we propose a new method in order to compute the extension efficiently. This type of extension is optimal, i.e. the  $H^1(\Omega)$ -norm of the extended function is bounded by the  $H^{0.5}(\partial\Omega)$ -norm of the given function. Numerical experiments show the optimal performance of the presented extension.

## Comparative numerical study to using TV regularization in image reconstruction

Gabriel Dimitriu

Total variation (TV) minimization represents an example of a class of recently introduced image processing techniques that use partial differential equations. Total variation has been shown to be

very successful in many image processing applications. The reason for using TV is that it has been shown to suppress noise effectively while capturing sharp edges. This characteristic makes it mainly attractive for those applications where the purpose is to identify the shape of objects (e.g. tumors in medical applications) that are distinguished from the background by sharp edges. The aim of the talk is to present some numerical results for the image reconstruction process by using an computational algorithm which incorporates the TV regularization.

## Harmonic and wavelet transform combination for fast content based image retrieval

Dimo T. Dimov

Content Based Image Retrieval (CBIR) is a relatively new area of Informatics and covers methods, techniques, and approaches for automatic

(or automated) retrieval of images by content description based on simple features as color, texture, shape, movement, etc., as well as on structures over them. Each CBIR system comprises a Database of Images (DBI), for which it should provide content-based image access methods being enough fast and reliable from a user viewpoint. From the other hand, the CBIR can be considered also a pattern recognition approach using a large dictionary of image examples, where the accents are put on

images preprocessing to adapt the appropriate recognition technique. A new heuristic approach to fast and reliable CBIR will be presented, namely arranged as a combination of Fourier transform and wavelet transform for images considered in a polar coordinate system. Experimental results will be also committed in comparison with other CBIR approaches experimented by the EIRS (Experimental Image Retrieval System of IIT) on a real DBI of hallmark images. Obviously, the approach proposed can be considered based on a linear 2D transform for global processing of initial images. The approach combination with differential filtering techniques of first and second order for possible image contouring will be also brought to discussion due to the Conference topic.

## Challenges in using splitting techniques for large-scale environmental modelling

Ivan Dimov, Tzvetan Ostromski, Zahari Zlatev

Splitting techniques are, to our knowledge, used in all operationally run large scale air pollution models with many scenarios. The modellers believe that really huge computational tasks can be made tractable on the available computers by dividing them into a sequence of smaller and simple sub-task. Probably, the first simple splitting procedure for partial differential equations was proposed by Bagrinovski and Godunov in 1957. Since that time many different splitting schemes were proposed and studied. A significant progress in splitting analysis was done during the last 3-4 years when the Laplace transformation technique was replaced by more powerful technique. Several splitting schemes for large scale air pollution models are analysed and some final (necessary and sufficient conditions) are formulated. The computational properties of splitting schemes are analysed from algorithmic point of view. Parallel properties of weighed splitting schemes are discussed.

## Wavelets in quantum chemistry

Heinz-Jrgen Flad, Wolfgang Hackbusch, Hongjun Luo, Dietmar Kolb

We present a multiscale method for electronic structure calculations based on the many-particle Schrödinger equation. Wavelets provide hierarchical basis sets that can be locally adapted to the length-

and energy-scales of physical phenomena. Taking a product ansatz for the wavefunction  $\Psi = \mathcal{F}\Phi$ , where  $\Phi$  corresponds to a given mean-field solution, we approximate the correlation factor  $\mathcal{F}$  in terms of hyperbolic wavelets. Due to the local character of electron correlations it is possible to get data sparse representations of the wavefunction. We discuss best N term approximations for the mean-field part and the correlation factor. The computational efficiency of wavelets in electronic structure calculations is demonstrated within the coupled electron-pair approximation. Based on a diagrammatic multiresolution analysis, we discuss various kinds of sparsity features.

## MIC(0) preconditioning of rotated bilinear FEM elliptic systems

Ivan Georgiev, Svetozar Margenov

New results about preconditioning of non-conforming FEM systems in the case of mesh anisotropy are presented. This study is focused on the implementations of rotated bilinear elements, where algorithms [MP] and [MV] stand for the variants of the nodal basis functions corresponding to midpoint and integral mid-value interpolation operators. The model elliptic problem under consideration is associated with the bilinear form

$$a_h(u, v) = \sum_{e \in \omega_h} \int_e a(e) \sum_{i=1}^2 u_{x_i} v_{x_i} de$$

where  $\omega_h$  is a decomposition of the computational domain  $\Omega$  into rectangles denoted by  $e$ . To get a stable  $MIC(0)$  factorization, in the general case, we first substitute the stiffness matrix by an auxiliary  $M$ -matrix. Then  $MIC(0)$  preconditioner is applied to this matrix. Two such approaches are presented where a local analysis is used to get estimates of the related condition numbers. Construction of the optimal  $M$ -matrix is very natural step of this study. For given SPD matrix  $K$  we want to find SPD  $M$ -matrix  $B$  such that the condition number of the generalized eigenvalue problem,

$$Ku = \lambda Bu$$

is as small as possible. The presented numerical tests well illustrate the behaviour of the theoretically studied algorithms.

## On the domain decomposition approach in some convection-diffusion-reaction problems

Krassimir Georgiev, Zahari Zlatev

The efficient numerical methods and algorithms for solving systems of convection–diffusion–reaction equations are of big priority due to numerous practically very much important problems. Prominent among this subject are simulations in air pollution modelling, pipe networks, acoustics, turbulent kinetic energy and its dispersion, extra stresses in non–Newton fluid flows, magneto–hydrodynamics, modelling of zeolite filters, etc.

We consider a system of convection–diffusion–reaction partial differential equations (PDE’s), which arise from the air pollution modelling and which is a very good test problem for studying the properties of the proposed numerical methods and algorithms. It is difficult to treat this system of PDE’s directly. Usually, different kinds of splitting are used in all known large-scale air pollution models. A splitting procedure, based on ideas proposed by Marchuk and McRae is used in this study. It leads to five sub-models, representing respectively the horizontal advection, the horizontal diffusion, the chemistry (together with the emission terms), the deposition and the vertical exchange. The submodels are discretized using different discretization techniques and treated successively at each time-step. This discretization leads to systems of ordinary differential equations (ODE’s) which are solved by using predictor-corrector schemes with several correctors. The size of any of the five ODE systems is equal to

the product of the number of the grid-points and the number of chemical species studied. It grows very quickly when the number of grid-points and/or the number of chemical species are increased dramatically - from 35 840 in the simplest case to 387 072 000 per time-step, and the typical number of time-steps is  $\approx 40\,000$ . It is very difficult to handle the arising huge computational tasks on the available computers. An elegant solution of this problem is to implement the model on a parallel computer architecture by dividing the computational load over a number of processors which are working in parallel. If we consider a parallel computer with  $p$  processors then the computational domain  $\Omega$  is divided in  $p$  subdomains. Different numerical algorithms are used in the different submodels. Therefore, two types of subdomains are appeared - *Nonoverlapping* subdomains for the chemistry and deposition submodels and *overlapping* subdomains for advection and diffusion submodels. For the communication part of the algorithm primitives from the Message Passing Interface (MPI) standard are used. The use of MPI is a key component to develop concurrent computing environment in which the application and tools can be transparently ported between different message-passing computers. Some numerical experiments performed on a Macintosh Linux cluster (four nodes MAC PowerPC; each node consists of two processors type G4) and on a SUN SunFire 6800 (24 processors) computers are reported and discussed.

## Stochastic optimization in finance

Krastyu Gummerov

Most financial activities involve optimal decision making under uncertainty. This paper is a survey concerning mainly two issues - stochastic control and stochastic programming. Their applications in finance are illustrated by Merton's problem and some financial planning systems. Some of the topics covered are: Hamilton-Jacobi-Bellman equation; two-stage and multistage linear recourse problems; decomposition and factorization methods. Decomposition methods are emphasized, because they are computationally most effective for large-scale problems.

## Canonical point sets in approximation by solutions of PDEs

Werner Haussmann

Starting with a classical result on best  $L^1$ -approximation by polynomials which says that the unique best  $L^1$ -approximation polynomial of degree  $\leq n$  to a function  $f$  with nonnegative  $n$ -th derivative on the interval  $[-1, 1]$  can be obtained by interpolation at  $n$  canonical points (namely the roots of the Chebyshev polynomial of second kind  $U_n$ ), we discuss how far results of this type are also true in the multivariate case.

We are lead to the questions, which function spaces play the role of the polynomials, what are the natural domains (replacing the interval  $[-1, 1]$ ), and how do canonical point sets look like.

The answer is that the functions spaces are defined as solutions of partial differential equations (e. g. harmonic functions, polyharmonic functions or blending functions). Natural domains of definitions are balls resp. cubes.

We give several results on the construction and characterization of best  $L^1$ -approximants by interpolation with respect to canonical point sets in the cases where the approximating functions are blending functions resp. polyharmonic functions which are defined by PDEs. Both, non-restricted and one-sided  $L^1$ -approximants are considered.

## On the performance of certain iterative solvers for saddle point problems arising in discretization of non-newtonian flow equations

Oleg Iliev, M.Moog, D.Niedziela, V.Starikovicus

Iterative solution of large scale systems arising after discretization and linearization of the unsteady non-Newtonian Navier–Stokes equations is studied. Carreau model is used to account for the non-Newtonian behavior of the fluid. Finite volume method is used to discretize the governing system of PDEs. Viscosity is treated explicitly (e.g., it is taken from the previous time step), while other terms are treated implicitly. Different preconditioners (block–diagonal, block–triangular, relaxed incomplete LU factorization, etc.) are used in conjunction with advanced iterative methods, namely, BiCGStab, CGS, GMRES. The action of the preconditioner in fact requires inverting different blocks. For this purpose, in addition to preconditioned BiCGStab, CGS, GMRES, we use also algebraic multigrid method (AMG). The performance of the iterative solvers is studied with respect to the number of unknowns, characteristic velocity in the basic flow, time step, deviation from Newtonian behavior, etc. Results from numerical experiments are presented and discussed.

## Numerical approximation of output functionals for Maxwell equations

Ferenc Izsak

In the analysis of boundary value problems for PDE’s we sometimes do not need to compute the solution on the whole domain, only a functional of this is required. For a special case of Maxwell equations we formalize a family of such problems and investigate the convergence of the computed functional values using standard edge elements. We use the concept of the dual problem and approximation theorems for the finite element space.

## A symmetric collocation method with fast evaluation

Michael J. Johnson

When applied to data arising from a partial differential equation, the radial basis function interpolation method is called *symmetric collocation*. This approach employs a basic function  $\phi$  and a certain space of functions formed by linear combinations of  $\phi$  and its partial derivatives. For example, if one considers Poisson’s equation with Dirichlet boundary conditions:

$$\Delta u = f \text{ on } \Omega \text{ with } u = g \text{ on } \partial\Omega,$$

then one would choose points  $\xi_i \in \Omega$  and  $\eta_j \in \partial\Omega$ , and then approximate the solution  $u$  by the function

$$s(x) = \sum_{i=1}^n \lambda_i \Delta\phi(x - \xi_i) + \sum_{j=1}^m \mu_j \phi(x - \eta_j),$$

where the coefficients  $\lambda_i$  and  $\mu_i$  are determined by the collocation equations

$$\Delta s(\xi_i) = f(\xi_i), \quad i = 1, 2, \dots, n, \quad s(\eta_j) = g(\eta_j), \quad j = 1, 2, \dots, m.$$

The major drawback to this approach is that each evaluation of the obtained function  $s$  typically requires  $O(m + n)$  floating point operations which is considered expensive. In order to reduce the evaluation costs, we suggest that the basic function  $\phi$  be chosen as a tensor product of univariate piecewise polynomials (in our experiments we have been using a tensor product of univariate Wendland functions). With such a choice of  $\phi$ , it is possible to efficiently evaluate  $s$  (and its partial derivatives) provided *most* of the points in  $\{\xi_1, \xi_2, \dots, \xi_n, \mu_1, \mu_2, \dots, \mu_m\}$  lie on a grid. If  $n$  is much larger than  $m$ , which is typical since  $n$  is the number of collocation points in  $\Omega$  and  $m$  is the number on the boundary, then this can be achieved by insisting that the points  $\{\xi_1, \xi_2, \dots, \xi_n\}$  lie on some grid.

In this talk we first give an elementary description of the theoretical foundations of symmetric collocation (ie minimal norm interpolation) and use the above example of Poisson’s equation to illustrate how the functions  $s$  is constructed. We then discuss the fast evaluation algorithm along with estimates on the number of floating point operations needed for a given set of evaluations. If time permits, we will present several numerical experiments which address the efficiency and accuracy of the method.

# A preconditioned iterative solution method for nonlinear elasticity systems

J. Kartson, O. Axelsson

A preconditioned outer-inner iterative solution method is proposed for a model in 3D nonlinear elasticity. The process involves an outer Newton iteration combined with a preconditioned inner CGM using separate displacement preconditioners. Such a preconditioner, already known to be efficient for linear models, arises as the discretization of three independent Laplacian operators. In this talk the resulting condition number is investigated with focus on independence of parameters. Estimates are given which show that the condition number is uniformly bounded w.r.t. both the studied Newton iterate and the chosen discretization.

# Schur complement preconditioning for highly orthotropic finite element problems

Vadim Korneev

We study fast preconditioners-solvers for Schur complements, which are used in the fast domain decomposition techniques suggested in [1,2] for solving the finite-difference (finite element with the first order elements) approximations of the elliptic equation in the unit square

$$y^2 \partial u^2 / \partial x^2 + x^2 \partial u^2 / \partial y^2 + (x^2 / y^2 + y^2 / x^2) = f, \quad (x, y) \in \Omega = (0, 1) \times (0, 1).$$

It has deteriorating coefficients before major terms and singular coefficients before the minor term. The preconditioning is based on the boundary norms for discrete "harmonic" functions, introduced for each subdomain of decomposition. The main tools for the derivation of these boundary norms are the boundary norms for harmonic functions in slim domains, designed by Maz'ya and Poborchi [3], and approximation results.

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# Radial basis functions with fast multipole method versus polysplines for structured data

Ognyan Kounchev

Both RBFs and polysplines belong to the PDE methods in Approximation and Spline theory. RBFs enhanced with the Fast Multipole Method have proved to be very satisfactory for wide classes of scattered data; consult [1], [2], [3] for RBFs and FMM. We consider a special class of data which is very important in practice: we assume that the data have some geometrical structure; in the two-dimensional case we assume that they lie on tracks and are relatively dense on such tracks; in higher dimensions we assume that the data lie on parallel hyperplanes and are dense there on the hypercube. We show that the interpolation polysplines are a "natural sparsifier" for such data, in the sense that the linear system necessary to be solved has a naturally sparse structure, and enjoy very high computational

efficiency for such data. We refer to the references [4], [5] on the theory of polysplines; consult also other references at the site [www.math.bas.bg/kounchev](http://www.math.bas.bg/kounchev). We compare the computational efficiency of both approaches. An essential advantage of the polysplines is that they are exact interpolator by construction and their efficiency is independent of the dimension - both these problems are partially solved and with extra effort for RBFs, see e.g. [2] and further interesting references at the site of Rick Beatson, <http://www.math.canterbury.ac.nz/mathrkb/beatson.html>.

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## Polysplines - a nonadaptive algorithm in image processing. Applications to astronomical images

Ognyan Kounchev, Damyan Kalaglarski

During the last decade there has been developed a number of approaches for providing a sparse image representation for images with "edge singularities", see [1], [2] and references therein. We present an application of the polysplines to the problems of Image Processing. We use some ideas of the previous authors to create a polyspline algorithm which provides a "natural sparsification" of images which are representable as piecewise analytic functions. Important features which distinguish our approach is that 1. it is fast 2. has a nice multiscale structure 3. it is non-adaptive in the process of "localization of the singularities" of the images by generating a multivariate pyramidal algorithm.

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## Polyspline surfaces - a new tool in CAGD

Ognyan Kounchev, Kiril Alexiev, Hermann Render

We present generalization of the classical B-spline curves. The polyspline surface is a surface of two variables in the three-dimensional space such that every component is a polyspline written by means of the "basic compactly supported polysplines". We use essentially the theory of compactly supported L-splines of special type, for the operator  $L = (d^2/dt^2 - k^2)^2$ , see Section 9.1 on polysplines on strips in the monograph [1] as well as the recent paper [2]. For the efficient computation of the polyspline surfaces we have proved a new type of recurrence relations for that class of special L-splines, which are 3-term and are with positive coefficients. Let us remark that they are the third recurrence relation ever known for L-splines, which has positive coefficients (the previous such relations, proved by T. Lyche and L. Schumaker, are for trigonometric and for hyperbolic L-splines, see e.g. [3]).

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## Comparative analysis of parameters obtained while simulating an air-pollution episode

Ana Lazarevska

This paper gives an outline to the influence of the geophysical, meteorological, emission and receptor input parameters quality when simulating an air pollution episode (APE). The parameters relevant for description, definition and development of an air pollution episode obtained with the non-steady state puff dispersion air quality model while simulating a short winter air pollution episode in the region City of Skopje are compared to the corresponding parameters resulting from the FLUENT simulation. The region comprised in this study is the City of Skopje and its surroundings, spreading over a territory of 40km by 60km. Skopje is the capitol of R. Macedonia, where over 30 Soundings from one upper-air and six surface stations used in the CALMET simulation of the meteorological conditions are employed as boundary conditions for the FLUENT simulation. Point air pollution sources are included accordingly to the Register of air-polluters supplemented by the Study of Japan International Cooperation Agency (JICA). Flow and geometry parameters of the point sources are in accordance with the Study of JICA. Due to the lack of detailed emission dynamics, total and constant emission rates are presumed. Emissions from domestic heating and traffic are excluded from the scope of this study. Ground concentrations of air pollutant species at discrete and gridded receptors resulting from the CALPUFF and FLUENT simulation are compared. The comparative analysis implies the possibility of supplementing the both software packages, aiming a better quality of the output. However, due to the poor quality of the input parameters, the comparison of the routinely measured 24hour ground concentrations with the simulated ones, shows that formation and development of an APE can be predicted only qualitatively, with a certainty as a function of the quality of the input parameters.

## Finite strip method for biharmonic equation

Ivan Lirkov, Svetozar Margenov

The small displacement plate bending is described by the biharmonic equation. The related mathematical model is described as a fourth order boundary value problem where the related boundary conditions correspond to clamped, joint and free edges. The finite strip method is used for numerical solution of the problem. This choice is mostly inspired by its computational efficiency. The efficient parallel implementation of the related algorithms is also a strong motivation for this study. The finite strips method can be derived applying a semi-discrete approach. The standard computational procedure reduces the problem to a set of linear systems of equations with seven-diagonal matrices. These systems are fully decoupled and hence, employing the generalized Thomas algorithm we obtain a total computational cost which is proportional to the number of unknowns. The communications of the parallel algorithm are restricted to the final step of the algorithm where the approximate solution is computed as a sum of the terms in the related sine expansion.

## Second order elliptic boundary value problems: two variations

W. R. Madych

The first concerns regularity of mixed boundary value problems and is a report of work done in collaboration with Moritz Kassman.

Consider the upper half space  $\Omega$  in  $R^n$ ,  $n \geq 2$ , with boundary  $\partial\Omega = R^{n-1} = D \cup N$ . In the case  $n - 1 = 1$  the sets  $D$  and  $N$  are the half lines  $D = R_+ = \{x : x > 0\}$  and  $N = R \setminus D$  while in the case  $n - 1 \geq 2$  the sets  $D$  and  $N$  are similarly defined half hyperplanes.

This variation concerns the  $H^1(\Omega)$  solutions  $u$  of the mixed Dirichlet-Neumann elliptic boundary value problem of second order

$$\operatorname{div} A \operatorname{grad} u = f \quad \text{on } \Omega \quad \text{with} \quad u|_D = g_D \quad \text{and} \quad u|_N = g_N$$

where  $f$  is in  $H^0(\Omega)$ ,  $g_D$  is in  $B_{2,1}^1(D)$ , and  $g_N$  is in  $B_{2,1}^0(N)$ .

We outline an argument which shows that such  $H^1(\Omega)$  solutions  $u$  are in  $B_{2,\infty}^{3/2}(\Omega)$ , in other words  $|u(x) - u(y)| \leq C|x - y|^{1/2}$  for all  $x$  and  $y$  in  $\Omega$ . Our development does not rely on interpolation theory and can be adapted to obtain regularity estimates for certain non-linear and higher order elliptic mixed boundary value problems.

The second variation concerns functions  $u$  which are harmonic in upper half spaces or slabs. The boundary values of such functions can be arbitrary members of fairly wide classes of distributions. We indicate in what sense such values are taken on and consider conditions which imply uniqueness of  $u$ .

## Aggregation-based multilevel preconditioning of non-conforming FEM elasticity problems

Svetozar Margenov

Preconditioning techniques based on various multilevel extensions of two-level splittings of finite element (FE) spaces lead to iterative methods which have an optimal rate of convergence and computational complexity with respect to the number of degrees of freedom. This article deals with the construction of algebraic two-level and multilevel preconditioning algorithms for Lamé equations of elasticity, which are discretized by Crouzeix-Raviart non-conforming linear finite elements on triangles. An important point to make is that in the non-conforming case the FE spaces corresponding to two successive levels of mesh refinements are not nested. To handle this, a proper aggregation-based two-level basis is considered, which enables us to fit the general framework for the two-level preconditioners and to generalize the method to the multilevel case. The derived estimate of the constant in the strengthened Cauchy-Bunyakowski-Schwarz (CBS) inequality is uniform with respect to both, mesh anisotropy and Poisson ratio, including in the *almost incompressible case*.

## Numerical approach in solving the pde for particular fluid dynamics cases

Zoran Markov

The solution of the fluid flow field requires solving the conservation equations for mass and momentum. For flows involving heat transfer or compressibility, an additional equation for energy conservation needs to be solved. Further transport equations also need to be solved when the flow is turbulent, which is the case in almost all practical fluid dynamic problems, beginning with relatively simple flows around a single airfoil to the complex three-dimensional flow through hydraulic turbomachinery. The difficulties in solving all of these equations have proved in the past as one of the factors, which slowed the faster development of numerical approach in the design of hydraulic turbomachinery. Lately, this approach is getting more applicable than ever, because of the extremely fast computers and their affordability, which was not the case until recently. Of course, some time will still pass until the reliability of the numerical solutions make the physical model testing, which is significantly more expensive and time demanding, completely obsolete. But, the greater speed of the computers and their value is only one side to the solution of the problem at hand. The other side was the development of reliable software for the solution

of complex fluid flow problems. When previously programming on the part of the scientist or designer was necessary for the calculation of the flow field, now these software packages give a great help in this respect. Of course, the user input is still an integral part of all the calculations of the PDE used to describe the fluid flow, because the calibration and verification of all numerical models is an interactive process, which gives at the end the desired results. The governing PDE for the conservation of mass and momentum, and (when appropriate) for energy and other scalars, such as turbulence, are solved based on a control-volume-based technique that consist of: division of the domain into discrete control volumes using a computational grid, integration of the governing equations on the individual control volumes to construct algebraic equations for the discrete dependent variables such as velocities, pressure and temperature and finally, linearization of the discretized equations and solution of the resultant linear equation system to yield updated values of the dependant variables. In general, two different numerical methods are used that employ a similar finite volume discretization process. The benefit of the results and data acquired using these methods, enable the development of hydraulic machinery that is more efficient, which is reflected on the power generation, as well on the environmental impact. So, having in mind all of the above, the objective of this presentation will be to demonstrate the accuracy of the numerical (CFD) analysis in predicting the experimentally acquired data, which will later be used as a powerful tool in the design process.

## A posteriori error estimations of some cell centered finite volume methods

Serge Nicaise

This talk will present the natural framework to residual based a posteriori error estimation of some cell centered finite volume methods for the Laplace or reaction-diffusion equations in  $\mathbb{R}^d$ ,  $d = 2$  or  $3$ . For that purpose we associate with the finite volume solution a reconstructed approximation, which is a kind of Morley interpolant. The error is then the difference between the exact solution and this Morley interpolant. The residual error estimator is based on the jump of normal and tangential derivatives of the Morley interpolant. We then prove the equivalence between the discrete  $H^1$  seminorm of the error and the residual error estimator. Numerical tests confirm our theoretical results.

## Algebraic multilevel preconditioning and aggregation

Yvan Notay

Algebraic multilevel techniques are increasingly popular to precondition very large sparse linear systems arising from the discretization of PDEs. Because they are *algebraic*, they can be applied to any sparse matrix without additional information from the underlying PDE. Because they are *multilevel*, they are potentially efficient, leading to fast convergence independently of the grid size.

In this talk, we first review and compare the different schemes that can be used once the basic algebraic ingredients have been defined. This includes algebraic multigrid (AMG) schemes, two-level approximate block factorizations, and several methods that exploit generalized hierarchical bases.

Next, we discuss more particularly the use of a simple aggregation-based scheme to build the coarse grid matrices. This choice is appealing because the number of nonzero entries per row is then approximately constant throughout all levels, whereas a significant increase is often observed with more sophisticated techniques.

However, it is generally difficult to obtain an efficient preconditioning on this basis. The main difficulty lies not in the coarse grid matrix itself, but rather in the piecewise constant interpolation that is naturally associated with the aggregation, and that is obviously poor. We show how this difficulty may be overcome with algebraic multilevel preconditioning based on a block ILU factorization of the matrix partitioned in hierarchical form.

### References

[1] Y. Notay, Algebraic multigrid and algebraic multilevel methods: a theoretical comparison, Tech. Rep. GANMN 04–01, Université Libre de Bruxelles, Brussels, Belgium, 2004.  
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## Mathematical modeling of large forest fires

Valeriy Perminov

A large technogeneous or space catastrophe, as a rule, is known to be accompanied by the initiation of big forest fires and the estimate of ecological and climatic impacts of severe fires, the prediction of the process influence on forest phytosenoses and ground layer state of the atmosphere is of interest. Considering that, natural investigations of these problems are merely impossible, methods of mathematical modeling are urgent. Mathematical model of forest fire was based on an analysis of known experimental data and using concept and methods from reactive media mechanics. Within the framework of this model, the forest and combustion products are considered as a homogeneous two temperatures, reacting, non - deformed medium. Temperatures of condensed (solid) and gaseous phases are separated out. The first includes a dry organic substance, moisture (water in the liquid-drop state), condensed pyrolysis and combustion products (coke, ash) and mineral part of forest fuels. In the gaseous phase we separate out only the components necessary to describe reactions of combustion (oxygen, combustible products of pyrolysis of forest fuels and the rest inert components). The solid phase constituting forest fuels has no intrinsic velocity, and its volumetric fractions, as compared to the gaseous phase, can be neglected in appropriate equations because a volume unit of wood. It is considered that 1) the flow has a developed turbulent nature, molecular transfer being neglected, 2) gaseous phase density doesn't depend on the pressure because of the low velocities of the flow in comparison with the velocity of the sound, 3) forest canopy is supposed to be non-deformed porous medium. The research is done by means of mathematical modeling of physical processes. It is based on numerical solution of Reynolds equations for chemical components and equations of energy conservation for gaseous and condensed (for canopy) phases. To describe the transfer of energy by radiation we use a diffusion approximation. To obtain discrete analogies a method of control volume is used. Calculation method and program have been checked. The boundary-value problem is solved numerically using the method of splitting according to physical processes. In the first stage, the hydrodynamic pattern of flow and distribution of scalar functions are calculated. The system of ordinary differential equations of chemical kinetics obtained as a result of splitting is then integrated. As a result of mathematical modeling the fields of temperatures, mass concentrations of components of gaseous phase, volume fractions of components of solid phase, as well as vectorial fields of velocity at different instants of time with taking into account mutual influence of the overterrestrial layer of atmosphere and a crown fire on each other will be obtained. It allows to investigate dynamics of forest fire spread under influence of various external conditions: a) meteorology conditions (air temperature, wind velocity etc.), b) terrain, c) type (various kinds of forest combustible materials) and their state (load, moisture etc.). A great deal of final and intermediate gaseous and dispersed combustion products of forest fuels is known to be exhausted into the atmosphere during forest fires: carbon monoxide, carbon dioxide, nitrogen oxide, soot, smoke, methane, other hydrocarbons and etc. The knowledge of these kinds of ejections enables a full estimate of the damage from forest fires to be made. In this paper attention is given to questions of description of the initial stage in the development of a mass forest fires initiated by high altitude radiant energy source (for example Tunguska celestial body fall and etc.). Ignition outlines, which fit well with the results of survey in the location of Tunguska celestial body fall, were obtained. Contours derived for collision catastrophes look like a circle arc in the neighbourhood of epicenter of the explosion and take the form of the ellipse extended in the flight trajectory projection direction of Tunguska celestial body. As distinct from collision catastrophes, ignition contours take the form of a circumference as illustrated as the result of numerical experiments for the ignition of a homogeneous vegetation layer by radiation from the air nuclear explosion.

# Non-classical receptive field inhibition and contour detection

Nicolai Petkov

Various visual effects show that the perception of an edge or line can be influenced by other such stimuli in the surroundings. Such effects can be related to non-classical receptive field (non-CRF) inhibition that is found in 80visual cortex.

A mathematical model of non-CRF inhibition is presented in which the response of an orientation selective cell is suppressed by the responses of other such cells beyond its classical (excitatory) receptive field. Non-CRF inhibition acts as a feature contrast computation for oriented stimuli: the response to an optimal stimulus over the receptive field is suppressed by similar stimuli in the surround. Consequently, it strongly reduces the responses to texture edges while scarcely affecting the responses to isolated contours. The biological utility of this neural mechanism might thus be that of contour (vs. texture) detection. Two types of inhibition are considered: isotropic and anisotropic that, respectively, do and do not depend on the orientation difference of center and surround stimuli.

The results of computer simulations based on the proposed model explain perceptual effects, such as orientation contrast pop-out, 'social conformity' of lines embedded in gratings, reduced saliency of contours surrounded by textures and decreased visibility of letters embedded in band-limited noise [Petkov and Westenberg, 2003 Biological Cybernetics 88: 236-246].

The insights into the biological role of non-CRF inhibition can be utilized in machine vision. The proposed model is employed in a contour detection algorithm that substantially outperforms previously known such algorithms in computer vision [Grigorescu et al, 2003 IEEE Transactions on Image Processing 12 729-739] [Grigorescu et al, 2004 Image and Vision Computing 22 609-622].

A demonstration will be given of a Gabor filter augmented with surround inhibition of texture edges. (The operator is available on-line at [www.cs.rug.nl/~petkov/](http://www.cs.rug.nl/~petkov/)).

The current mathematical model corresponds to a single time step development of a PDE. Transferring the current model to a PDE and experimenting with that PDE might be a fertile field for further research in this area.

Literature (downloadable from <http://www.cs.rug.nl/~petkov/publications/journals.html>):

N. Petkov and M. A. Westenberg: Suppression of contour perception by band-limited noise and its relation to non-classical receptive field inhibition, Biological Cybernetics, 88, 2003, 236-246.

C. Grigorescu, N. Petkov and M. A. Westenberg: Contour detection based on nonclassical receptive field inhibition, IEEE Trans. on Image Processing, 12 (7), 2003, 729-739.

C. Grigorescu, N. Petkov and M. A. Westenberg: Contour and boundary detection improved by surround suppression of texture edges, Image and Vision Computing, 22 (8) 2004, 609-622.

## PDE-methods in multivariate spline analysis

Hermann Render

In this talk we survey the theory of polysplines. Roughly speaking, polysplines of order  $p$  are functions on the euclidean space which are piecewise solutions of the polyharmonic equation and which obey certain matching conditions on given knot surfaces of codimension 1; for a precise definition and applications of polysplines see the monograph [2]. Special emphasis is given to cardinal polysplines  $u$  on strips: by definition,  $u$  is then  $(2p-2)$  times continuously differentiable, and  $\Delta^p u(x) = 0$  for all  $x$  in the open strips  $(j, j+1) \times \mathbb{R}^{n-1}$  for each  $j \in \mathbb{Z}$ . We present an interpolation theorem for polysplines in the flavor of the famous interpolation theorem of Schoenberg for univariate polynomial splines. A major result is the fact that interpolation with polyharmonic splines (introduced in [5] being a special case of interpolation with radial basis functions) on lattices of the form  $\mathbb{Z} \times a\mathbb{Z}^{n-1}$  for a real number  $a > 0$  leads in the limit  $a \rightarrow 0$  in a natural way to polysplines on strips. In the last part we present applications of polysplines in Wavelet Analysis. The talk is based joint work with A. Bejancu (University of Leeds, UK) and O. Kounchev (Bulgarian Academy of Sciences).

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## Corner sobolev spaces and asymptotics

B.-W. Schulze

The description of ellipticity, parametrices, and asymptotics of solutions in domains with corners requires specific notions that are adapted to the corner singularities. We discuss a category of weighted corner Sobolev spaces and subspaces with iterated asymptotics which is a natural framework for these problems. Parametrices of corner boundary value problems are formulated in terms of Mellin pseudo-differential operators that have a conormal symbolic structure. This is operator-valued, with values in boundary value problems on the cross sections that have conical singularities. The non-bijectivity points in the complex plane determine the asymptotics of solutions, which is in the corner case combined with edge asymptotics along the edges emanated from the corners.

## Schwarz-type methods and their application in geomechanics

Jiri Stary

The contribution describes parallel iterative solvers for the solution of linear systems arising from the finite element analysis of elasticity or thermo-elasticity problems in geomechanics. The solvers are based on the conjugate gradient method and domain decomposition techniques, which are used for both the parallelization and construction of effective preconditioners. Such techniques can use algorithms belonging to the class of Schwarz-type methods.

To demonstrate the behaviour and efficiency, the algorithms introduced in the contribution were implemented with the aid of PETSc library and tested at a solution of one benchmark problem on shared-memory and ditributed-memory machnies.

## Lattice-Boltzmann methods for complex fluids in complex structures

Konrad Steiner

During the last ten years, there are a large number of investigations done by mathematicians and physicists to understand the Lattice Boltzmann Method as a technique to solve the incompressible Navier-Stokes equation. The author will give a short overview about the concept of the method and will explain the advantages to model and simulate complex fluids in complex three-dimensional structures with Lattice Boltzmann. Furthermore some applications are shown, i.e. the production of fibre reinforced plastics by simulating Non-Newtonian flows through fibre-bundles.

# Multiresolution expansions of tempered distributions

Nenad Teofanov, Stevan Pilipovic

In this lecture we give properties of projections  $h_j$  of a tempered distribution  $h$  to the corresponding spaces  $V_j$ , in a regular multiresolution approximation of  $L^2(R)$ .

We show that the derivatives of  $h_j$  converge almost uniformly to the corresponding derivative of  $h$  as  $j$  tends to infinity, providing that  $h$  is smooth enough. Results related to tempered distributions are obtained by duality.

As an application, Abelian and Tauberian type theorems concerning the quasiasymptotic behavior of  $h$  at infinity via its multiresolution expansion and vice versa are given.

Math Subject Classifications. 42C15, 42C40, 46F12.

# Image denoising using PDE based algorithms

Cosmin Nicolae Varlan, Gabriel Dimitriu

By image reconstruction we understand obtaining the solution  $u$  of an operator equation of the form:  $Fu = z$ , where  $F$  is usually linear, and the data  $z$  is assumed to be contaminated by different levels of error. Two important special situations exist: 1) *Denoising*, or noise removal, where  $Fu = u$ ; and 2) *Deblurring*, where  $F$  is a Fredholm first kind integral operator which models "blurring" processes involved in image formation. The talk is focused on the first case and describes in details denoising algorithms based on nonlinear diffusion. Numerical results for image denoising are presented.

# On the convergence of a general class of finite volume methods

Holger Wendland

Finite volume methods are well-established tools for solving hyperbolic conservation laws numerically.

For discretizing in space, finite volume methods use cell average information. To be more precise, for a fixed time such cell averages are employed to reconstruct the unknown function  $u$  approximately. For a good reconstruction in regions where the solution of the conservation law is known or expected to be smooth a higher order reconstruction scheme is desirable.

In this talk I will describe how optimal recovery in particular by (conditionally) positive definite kernels can be employed to solve such reconstruction problems.

Moreover, I will give a general convergence result, which holds for *any* interpolatory and stable reconstruction process, in particular for those based upon optimal recovery.

# The use in various applications of an analytic solution method for the solution of the biharmonic problem

M.J. Wilson, M.I.G. Bloor

A method is presented for the solution of the biharmonic equation through the use of a combination of polynomial and eigen function solutions. Applications of the method are given in the areas of elasticity, fluid mechanics, and computer-aided geometric design (CAGD). The application of the method in CAGD, in particular the generation of a 4-sided surface patch, will be considered in detail, where the basic method must be modified in order to ensure that the boundary conditions on the problem are satisfied exactly.

# On the general solutions of the Brinkman and the Stokes stream function equation in spheroidal coordinates

Tomislav Zlatanovski

In connecting to our previous work, we are here concerned with boundary value solution to axisymmetric creeping flow past and through a porous spheroidal shell. The Brinkman model for the flow inside the porous media and the Stokes model in the free-fluid region(s) are used. The needed general solution for the Brinkman stream function equation for axisymmetric flow in spheroidal coordinates has been derived. Also, a novel, simpler form of the complete general solution for the limiting Stokes stream function equation for axisymmetric flow in spheroidal coordinates has been obtained. To solve the same problem in the case of arbitrary shaped porous body, a 3D numerical model on the base of an integral equation formulation has been implemented. Calculated results in comparison will be presented.

# Partitioning the chemical part of large air pollution models

Zahari Zlatev

Large air pollution models are described mathematically by systems of partial differential equations. The number of equations is normally equal to the number of chemical species studied by the model. The discretization of air pollution models leads to huge computational tasks. Most of the difficulties related to the numerical treatment of the discretized models are caused by the chemical part. There are three major sources of difficulties: (a) the mathematical terms by which the chemical reactions are described are non-linear, (b) the chemical terms are introducing stiffness and (c) the matrices, which have to be handled in the chemical part of a large air pollution model, are very badly scaled. This is why the treatment of the chemical sub-model is normally the most time-consuming part of the computational work (very often up to 90% of the total computational work is spent in the chemical part). This shows that one must be very careful when numerical methods for the chemical part of a large air pollution model are to be selected. The computational work can in many cases be reduced considerably when some kind of partitioning is used in the chemical sub-model. The conditions under which the partitioning procedures can successfully be used will be studied in this paper. Numerical examples, which illustrate the efficiency of the particular procedure used in the Unified Danish Eulerian Model (UNI-DEM), will be given.