Multivariate Polysplines

Dedicated to the memory of Tseni

Multivariate Polysplines: Applications to Numerical and Wavelet Analysis

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Preface

In the present the theory of Partial Differential Equations (PDEs) is so overwhelmed by the study of Boundary Value Problems that one can hardly believe that from a global perspective these are no more than a modest part of the properties of the differential equations. Apparently, the Qualitative theory of PDEs is a lot more difficult. This may be understood by using an analogy with the one-dimensional case: the boundary value problems on a compact interval are hardly a topic to discuss for the algebraic polynomials when we consider the last as solutions of ordinary differential equations. Topics of interest are the *Descartes*' rule of signs or the *Budan-Fourier* theorem for the number of sign changes (or zeros) in a compact interval, and other lot deeper properties[1]. On the other hand we are quite far from proving analogs of the Descartes' rule and the Budan-Fourier theorem for polyharmonic functions; even the formulation of the proper analogs is a problem. Similar questions for arbitrary higher-order elliptic equations or for nonlinear equations seem to be rather advanced.

The main message of the present book is that the solutions of higher-order elliptic equations, in particular, the polyharmonic functions, may be used as building blocks of multivariate splines – which we call *polysplines* – in much the same way as the one-dimensional polynomials are used to build the one-dimensional splines. We study *cardinal polysplines* and *polyharmonic wavelets* in a complete analogy with the one-dimensional polynomial *cardinal splines* and *cardinal spline wavelets*. All these results may be considered as a step in the direction of qualitative theory of elliptic PDEs.

The reader should not be scared by the big volume of the present book. It has become bigger for reasons of readability. Another reason for the increase of the volume is that the book is intended for readers with varied backgrounds. The *primary purpose* was to provide readers having a modest (or no) background in PDEs, and more interests in CAGD, spline and wavelet analysis, with an exposition of the theory of polysplines at least in special domains. Thus the biggest Part I has appeared. Once such reader has overcome the initial Chapters of Part I he/she might be willing to see the new developments in *cardinal polysplines* and *polyharmonic wavelet analysis* in Part II and Part III. The *secondary purpose* was to provide readers having more considerable background in PDEs with a proper introduction to the basics of the one-dimensional spline theory and wavelet analysis and the smooth transition to the theory of polysplines in Part IV.

In the present volume we were able to cover only some part of the topics of Numerical Analysis: interpolation by polysplines, cardinal interpolation for special break-surfaces,

[1] Consult the first part of the famous book of problems in analysis of Polya and Szegö, or [50, p. 89] (this reference is found at the end of Part I.

convergence of the polyspline interpolation in special cases. The polyharmonic wavelet analysis has outweighed the very interesting topics as

- “Polyharmonic” Euler-Maclaurin formulas and Bernoulli polysplines,
- Optimal recovery and polysplines,
- Peano kernels and mean-value properties for polyharmonic functions, and
- Approximation and interpolation theory by polyharmonic functions and polysplines.

They are left for a next volume.

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