We suppose that the region \( D \) and the operator \( T \) are given.

The boundary operator are given

\[
\left\{ \begin{array}{l}
\Gamma_1 \in \mathbb{R},
\Gamma_2 \in \mathbb{R},
\end{array} \right.
\]

Let \( D \) be an arbitrary differential operator of order \( 2 \).

* Let \( D \) be a bounded domain in \( \mathbb{R}^n \) and the operator \( T(x) \) be the general boundary operator.

** Let \( D \) be an arbitrary differential equation.

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** Notes:**

1. *Boundary* conditions.
2. *Differential* boundary conditions.
5. *Applications* of boundary conditions.
8. *Approximation* methods.

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**References:**

5. *Applications* of boundary conditions. 19.
The set \( \mathcal{V} = \{ v \mid v = (v_1, \ldots, v_n) \} \) is the vector space of all \( n \)-dimensional vectors.

Theorem 1

Let the vector measure \( \mathcal{P} \) be given.

Theorem 2

Let \( \mathcal{V} \) be an \( n \)-dimensional vector space and \( \mathcal{P} \) a vector measure on \( \mathcal{V} \).

For a given continuous function \( f : \mathcal{V} \to \mathcal{R} \),

\[ \int f d\mathcal{P} = \sum_{i=1}^{n} f(v_i) \mathcal{P}(v_i) \]

for any \( v \in \mathcal{V} \).

Example

Let \( \mathcal{P} \) be the Lebesgue measure on \( \mathcal{V} \).

Then

\[ \int f d\mathcal{P} = \sum_{i=1}^{n} f(v_i) \mathcal{P}(v_i) \]

for any \( v \in \mathcal{V} \).

Note

The following theorems are special cases of the above theorems.

Theorem 3

Let \( \mathcal{P} \) be a vector measure on \( \mathcal{V} \).

Then

\[ \int f d\mathcal{P} = \sum_{i=1}^{n} f(v_i) \mathcal{P}(v_i) \]

for any \( v \in \mathcal{V} \).

Proof

The proof is similar to the one for the one-dimensional case.