$B(\nu_2-\nu_1)$ is not empty.

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DUALITY PROPERTIES FOR THE EXTREME VALUES OF INTEGRALS IN DISTRIBUTED MOMENTS

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Here we investigate the general properties of the extremal problems for the so-called distributed moment problem, arising in optimal control [5]. Our considerations concern only the case of elliptic partial differential equations but the same theorems can be proved for arbitrary linear p.d.e.

Let G be a bounded domain in R^n and the operator $L(x,D)=\sum_{|k|\le 2m} a_k D^k$ be an elliptic differential operator of order $2m$ with sufficiently smooth coefficients in the closed hull $\bar{G}$ [1].

We introduce the set of functions [1,2]

$$N(L) = \left\{ h(x);\ h(x)\in N(L),\ Lh(x)=0,\ \ x\in\bar{G},\ h(x)\in C^{m-1}(\bar{G})\cap C^{2m}(G)\right\}$$

$$N_+(L) = \left\{ h(x);\ h(x)\in N(L),\ h(x)\ge 0,\ x\in\bar{G}\right\} .$$

The following boundary operators are given [3]

$$B_j(x,D) = \sum_{|k|\le m_j} b_{jk} D^k \quad ,\quad j=1,\dots,m .$$

We suppose that the region G and the operators L, B_j satis fy the conditions studied in [2,3] which provide solvability of the boundary value problem

$$(1)\qquad \begin{aligned} &L(x,D)h(x) = 0 \;, \quad x\in\bar{G} \\ &B_j(x,D)h(x) = g_j(x) \;, \quad x\in\partial G,\; j=1,\dots,m, \end{aligned}$$

for sufficiently regular functions $g_j(x)$ on the boundary ∂G.

Through $\mathcal{M}(\bar{G})$ and $\mathcal{M}(\partial G)$ we denote the set of all Radon measures (mass distributions) in $\bar{G}$, G respectively [2]

The following generalization of the sweeping-out method of Poincaré [4] will be used [2] :

<u>Lemma 1.</u> For any measure $\mu\in\mathcal{M}(\bar{G})$ there exists a vector measure $\nu = (\nu_j)\in[\mathcal{M}(\partial G)]^m$ such that the following equality holds

$$(2)\qquad \int_{\bar{G}} h(x)\,d\mu(x) = \sum_{j=1}^{m}\int_{G} B_j h(y)\,d\nu_j(y)\;, \quad \forall h(x)\in N(L)\;.$$

We shall use the abbreviations of (2):

$$(3)\qquad \mu(h) = \nu(Bh) \quad \text{or} \quad \Pi\mu = \nu \quad .$$

For a given vector measure $\nu\in[\mathcal{M}(\partial G)]^m$ consider the set of all mass distributions (positive measures) [5]

$$(4)\qquad B(\nu) = \left\{\, \mu\in\mathcal{M}(\bar{G});\; \Pi\mu = \nu,\; \mu\ge 0 \,\right\}$$

For a given continuous function $w(x)$, $x\in\bar{G}$, let us introduce the following sets:

$$\underline{P}(w) = \left\{\, h(x)\in N(L);\; h(x)\le w(x),\; x\in\bar{G} \,\right\} \quad ,$$

$$\bar{P}(w) = \left\{\, h(x)\in N(L);\; h(x)\ge w(x),\; x\in\bar{G} \,\right\} \quad .$$

The sets $\underline{P}$ and $\bar{P}$ are not empty since every constant is in $N(L)$.

<u>Theorem 1.</u> Let the vector measure $\nu\in[\mathcal{M}(\partial G)]^m$ be given.

Then

a) The equalities hold

$$(6)\qquad \min\left\{ \int_{\bar{G}} w(x)\,d\mu(x) \;;\; \mu\in B(\nu) \right\} = \max\left\{ \nu(h);\; h\in\underline{P} \right\}$$

$$(7)\qquad \max\left\{ \int_{\bar{G}} w(x)\,d\mu(x) \;;\; \mu\in B(\nu) \right\} = \min\left\{ \nu(h);\; h\in\bar{P} \right\}$$

b) The integral

$$(8)\qquad \int_{\bar{G}} w(x)\,d\mu(x) \;, \quad \mu\in B(\nu)$$

takes its minimal(maximal) value for $\mu = \mu_0\in B(\nu)$ if and only if there exists a function $h_0(x)\in\underline{P}(w)$ (resp. $h_0(x)\in\bar{P}(w)$) which coincides with $w(x)$ in those points $x\in\bar{G}$ where $d\mu_0\ne 0$.

The proof is similar to the one for the one-dimensional case in [7, p.175] .

Let us introduce the notations:

$$\underline{I}(\nu,w) = \min\left\{ \int_{\bar{G}} w(x)\,d\mu(x);\; \mu\in B(\nu) \right\}$$

$$\bar{I}(\nu,w) = \max\left\{ \int_{\bar{G}} w(x)\,d\mu(x);\; \mu\in B(\nu) \right\}$$

From Theorem 1 easily follows

<u>Corollary 1.</u> For a given measure $\mu\in\mathcal{M}(\bar{G})$ the deviations

$$\underline{D}(\mu) = \int_{\bar{G}} w(x)\,d\mu(x) - \underline{I}(\Pi\mu,w) \;;\; \bar{D}(\mu) = \bar{I}(\Pi\mu,w) - \int_{\bar{G}} w(x)\,d\mu(x)$$

re isotonely nondecreasing functions of μ i.e. $\underline{D}(\mu 8)$ (μ'') and $\bar{D}(\mu')\le\bar{D}(\mu'')$ if $d\mu'\le d\mu''$.

<u>orollary 2.</u> The dispersion $D(\nu) = \bar{I}(\nu,w) - \underline{I}(\nu,w)$ is an isoonic increasing function of ν i.e. $D(\nu_1)\le D(\nu_2)$ if the set