$B(\nu_2-\nu_1)$ is not empty.

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DUALITY PROPERTIES FOR THE EXTREME VALUES OF INTEGRALS IN DISTRIBUTED MOMENTS

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Here we investigate the general properties of the extremal problems for the so-called distributed moment problem, arising in optimal control [5] . Our considerations concern only the case of elliptic partial differential equations but the same theorems can be proved for arbitrary linear p.d.e.

Let G be a bounded domain in R^n and the operator $L(x,D)=\sum_{|k|\le 2m} a_k D^k$ be an elliptic differential operator of order $2m$ with sufficiently smooth coefficients in the closed hull $\bar{G}$ [1] .

We introduce the set of functions [1,2]

$$N(L) = \left\{ h(x);\ h(x)\in N(L),\ Lh(x)=0,\ \ x\in\bar{G},\ h(x)\in C^{m-1}(\bar{G})\cap C^{2m}(G)\right\}$$

$$N_+(L) = \left\{ h(x);\ h(x)\in N(L),\ h(x)\ge 0,\ x\in\bar{G}\right\} .$$

The following boundary operators are given [3]

$$B_j(x,D) = \sum_{|k|\le m_j} b_{jk} D^k \ , \quad j=1,\dots,m \ .$$

We suppose that the region G and the operators L, B_j satis fy the conditions studied in [2,3] which provide solvability of the boundary value problem

$$(1)\qquad \begin{aligned} &L(x,D)h(x) = 0 \ , \quad x \in \overline{G} \\ &B_j(x,D)h(x) = g_j(x) \ , \quad x \in \partial G, \ j=1,\dots,m, \end{aligned}$$

for sufficiently regular functions $g_j(x)$ on the boundary ∂G.

Through $\mathcal{M}(\overline{G})$ and $\mathcal{M}(\partial G)$ we denote the set of all Radon measures (mass distributions) in $\overline{G}$, G respectively [2]

The following generalization of the sweeping-out method of Poincaré [4] will be used [2] :

__Lemma 1.__ For any measure $\mu \in \mathcal{M}(\overline{G})$ there exists a vector measure $\nu = (\nu_j) \in [\mathcal{M}(\partial G)]^m$ such that the following equality holds

$$(2)\qquad \int_{\overline{G}} h(x)\,d\mu(x) = \sum_{j=1}^{m} \int_{G} B_j h(y)\,d\nu_j(y) \ , \quad \forall h(x) \in N(L) \ .$$

We shall use the abbreviations of (2):

$$(3)\qquad \mu(h) = \nu(Bh) \qquad \text{or} \qquad \Pi\mu = \nu \ .$$

For a given vector measure $\nu \in [\mathcal{M}(\partial G)]^m$ consider the set of all mass distributions (positive measures) [5]

$$(4)\qquad B(\nu) = \{ \mu \in \mathcal{M}(\overline{G});\ \Pi\mu = \nu,\ \mu \geq 0 \}$$

For a given continuous function $w(x)$, $x \in \overline{G}$, let us introduce the following sets:

$$\underline{P}(w) = \{ h(x) \in N(L);\ h(x) \leq w(x),\ x \in \overline{G} \} \ ,$$

$$\overline{P}(w) = \{ h(x) \in N(L);\ h(x) \geq w(x),\ x \in \overline{G} \} \ .$$

The sets $\underline{P}$ and $\overline{P}$ are not empty since every constant is in $N(L)$.

__Theorem 1.__ Let the vector measure $\nu \in [\mathcal{M}(\partial G)]^m$ be given.

Then

a) The equalities hold

$$(6)\qquad \min \Big\{ \int_{\overline{G}} w(x)\,d\mu(x) \ ;\ \mu \in B(\nu) \Big\} = \max \{ \nu(h);\ h \in \underline{P} \}$$

$$(7)\qquad \max \Big\{ \int_{\overline{G}} w(x)\,d\mu(x) \ ;\ \mu \in B(\nu) \Big\} = \min \{ \nu(h);\ h \in \overline{P} \}$$

b) The integral

$$(8)\qquad \int_{\overline{G}} w(x)\,d\mu(x) \ , \quad \mu \in B(\nu)$$

takes its minimal(maximal) value for $\mu = \mu_0 \in B(\nu)$ if and only if there exists a function $h_0(x) \in \underline{P}(w)$ (resp. $h_0(x) \in \overline{P}(w)$) which coincides with $w(x)$ in those points $x \in \overline{G}$ where $d\mu_0 \neq 0$.

The proof is similar to the one for the one-dimensional case in [7, p.175] .

Let us introduce the notations:

$$\underline{I}(\nu,w) = \min \Big\{ \int_{\overline{G}} w(x)\,d\mu(x);\ \mu \in B(\nu) \Big\}$$

$$\overline{I}(\nu,w) = \max \Big\{ \int_{\overline{G}} w(x)\,d\mu(x);\ \mu \in B(\nu) \Big\}$$

From Theorem 1 easily follows

__Corollary 1.__ For a given measure $\mu \in \mathcal{M}(\overline{G})$ the deviations

$$\underline{D}(\mu) = \int_{\overline{G}} w(x)\,d\mu(x) - \underline{I}(\Pi\mu,w) \ ;\ \overline{D}(\mu) = \overline{I}(\Pi\mu,w) - \int_{\overline{G}} w(x)\,d\mu(x)$$

re isotonely nondecreasing functions of μ i.e. $\underline{D}(\mu 8)$ (μ'') and $\overline{D}(\mu') \leq \overline{D}(\mu'')$ if $d\mu' \leq d\mu''$.

__orollary 2.__ The dispersion $D(\nu) = \overline{I}(\nu,w) - \underline{I}(\nu,w)$ is an isoonic increasing function of ν i.e. $D(\nu_1) \leq D(\nu_2)$ if the set