Smooth Green's functions in $C^2$, we see that usually be an elliptic differential operator of order 2 with sufficiently

\[ \frac{\partial^2}{\partial x^2} u + \frac{\partial^2}{\partial y^2} u = f(x,y) \]

and the operator

let $D$ be a bounded domain in $\mathbb{R}^2$ with a regular boundary $\partial D$.

1. THE INVERSE POTENTIAL PROBLEM

Classical density result of Brezis [4] and Denny [5].

We choose these lemmas since they use similar techniques based on a

the function norm is unique.

that the elements of best approximation by harmonic functions in

1973 [2]. After that we describe some geometric properties of the

resolution of the inverse problem of potential theory [3].

First we prove a result about the uniqueness of the solution

and moment problem [1.8.15].

Let a linear function of potential theory [19]. To stress the analogy with the one-dimensional

we use the last name instead of the alternative one, in virtue of

distributed moment problem (DMP) see [11] and references there.

The present work continues the investigation of the socialled

INTRODUCTION

Bulgarian Academy of Science

Bulgarian Academy of Sciences

Institute of Mathematics

Ognian Kouneiher

THE DISTINGUISHED MOMENT PROBLEM

EXTERNSAL PROBLEMS
For a given \( \mu \in \mathbb{R} \), let \( (\lambda(\mu) : (x)) \) denote the solution to the following optimization problem:

\[
\max \{ \lambda(\mu) : (x) \} \text{ subject to } (x) \text{ is the vector-mass with } (x) \in \mathcal{X}, \quad \text{and } (\lambda(\mu) : (x)) \leq \mu.
\]

Then we have the following propositions:

(6.1) For \( \mu \geq 0 \), let \( (\lambda(\mu) : (x)) \) denote the optimal solution to the above optimization problem.

(6.2) For \( \mu < 0 \), let \( (\lambda(\mu) : (x)) \) denote the optimal solution to the above optimization problem.

In order to compute \( (\lambda(\mu) : (x)) \), we introduce the next step. In this step, we need to compute the dual problem. For this, we first need to formulate the dual problem. We formulate the dual problem as follows:

\[
(\lambda(\mu), (x)) = (\lambda(\mu) : (x))
\]

We will now introduce the following propositions:

(7.1) For \( \mu \geq 0 \), let \( (\lambda(\mu) : (x)) \) denote the optimal solution to the above optimization problem.

(7.2) For \( \mu < 0 \), let \( (\lambda(\mu) : (x)) \) denote the optimal solution to the above optimization problem.

We will now introduce the following propositions:
In order to prove the existence of a point $x$ such that $f(x) = 0$, we proceed as follows:

1. Assume that $f: D \rightarrow \mathbb{R}$ is a continuous function on a compact set $D$. Let $g: D \rightarrow \mathbb{R}$ be defined by $g(x) = f(x) - m(x)$, where $m(x)$ is a continuous function on $D$.

2. Since $D$ is compact and $f$ and $m$ are continuous, $g$ is also continuous.

3. By the Extreme Value Theorem, there exists a point $x_0 \in D$ such that $g(x_0) = \min_{x \in D} g(x)$.

4. If $g(x_0) = 0$, then we are done. Otherwise, suppose $g(x_0) < 0$.

5. Since $g(x)$ approaches 0 as $x$ approaches the boundary of $D$, there exists a point $x_1 \in D$ such that $g(x_1) > 0$.

6. Hence, $g(x)$ changes sign on $D$, and by the Intermediate Value Theorem, there exists a point $x_2 \in D$ such that $g(x_2) = 0$. Therefore, $f(x_2) = m(x_2)$.

This proves the existence of at least one point $x_2$ in $D$ such that $f(x_2) = 0$.
the following holds: for any (finite) subset set $S$ of $\mathbb{C}$, 

$$\frac{1}{n^2} \sum_{z \in S} \frac{1}{|z|^2} = \frac{1}{n} \sum_{z \in S} \frac{1}{|z|}$$

Furthermore, the sequence of rational functions $f_n(z)$ converges uniformly on compact subsets of $\mathbb{C}$ to the function $f(z)$.

The rational functions $f_n(z)$ are examples of rational approximations to the function $f(z)$.

Theorem (Riemann) states that if a function $f(z)$ is bounded and continuous on a connected set $G$, then $f(z)$ is holomorphic on $G$.

Theorem (Weierstrass) states that for any sequence of rational functions $f_n(z)$ that converges uniformly on compact subsets of $\mathbb{C}$ to a function $f(z)$, then $f(z)$ is holomorphic on $\mathbb{C}$.

Theorem (Mergelyan) states that for any bounded and continuous function $f(z)$ on a compact subset of $\mathbb{C}$, there exists a sequence of rational functions $f_n(z)$ that converges uniformly on compact subsets of $\mathbb{C}$ to $f(z)$.

Theorem (Stone) states that for any compact set $K$ in $\mathbb{C}$, there exists a sequence of rational functions $f_n(z)$ that converges uniformly on compact subsets of $\mathbb{C}$ to a function $f(z)$.

Theorem (Runge) states that for any compact set $K$ in $\mathbb{C}$, there exists a sequence of rational functions $f_n(z)$ that converges uniformly on compact subsets of $\mathbb{C}$ to a function $f(z)$.

Theorem (Beurling) states that for any compact set $K$ in $\mathbb{C}$, there exists a sequence of rational functions $f_n(z)$ that converges uniformly on compact subsets of $\mathbb{C}$ to a function $f(z)$.

Theorem (Mergelyan) states that for any bounded and continuous function $f(z)$ on a compact subset of $\mathbb{C}$, there exists a sequence of rational functions $f_n(z)$ that converges uniformly on compact subsets of $\mathbb{C}$ to $f(z)$.


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