Cyclic Histogram Thresholding and Multithresholding (*)

Dimo Dimov, and Lasko Laskov

Abstract: The paper concerns the problem of thresholding of an integer domain of 1D cyclic histogram (periodic function) resulting in two or more consecutive regions (classes). An optimal solution is searched for in the terms of the statistical criterion well known in the pattern recognition area as Fisher’s LDA (Linear Discriminant Analysis) and also successfully applied for image binarization by Otsu (1979). An effective (quadratic complexity) extension of the Otsu’s method is also known, which segments the image by respective thresholding of the image intensity histogram into arbitrary number of classes. We propose one more extension of this approach for the case of the cyclic histograms. Similar problem can be brought by the optimal segmentation of color images based on their HSV histogram, and more general in all problems which try to approximate a given periodic function with a predefined number of Gaussians. The paper describes the theoretical basis and the experimental evaluation of the proposed approach.

Key words: Cyclic histogram thresholding and multithresholding, Periodic function approximation by Gaussians, Image processing.

1. INTRODUCTION

The examined problem usually is a result of so called histogram approaches to image binarization [1, 2, 3]. The histogram that is a statistical function of the image intensity, is being divided according to an optimality criterion into two compact parts, and during the binarization, one of the parts is labeled as background (e.g. white), the other – as object of interest (e.g. black). Analogously, one can binarize also color images, for example using the corresponding Hue-histogram of the HSV color scheme of the image, but in this case the applied histogram is a cyclic (periodic) one, [4].

More generally, if we exclude the physical meaning of the term histogram, the considered problem can be brought by attempts to approximate a periodic function (e.g. statistical) by a given number of simple functions, e.g. statistical distributions like Gaussians.

Our approach to the examined problem is an extension of the classical approaches for thresholding (and multi-level thresholding) which divide a histogram in two or more compact sequential parts, but in the case of a cyclic histogram. As a base approach to this extension we adopt the Otsu’s method [2].

2. BACKGROUND

For thresholding of a given histogram \( H(i), \) \( i = 0,1,...,(T-1) \) into two Gaussian components, Otsu [2] applies an approach frequently associated with the name of Fisher in the Linear Discriminant Analysis (LDA), see [5]. More precisely, Otsu searches a discriminant point (a threshold) \( t, \) \( 0 \leq t < T, \) via the criterion \( \lambda = \sigma_{Wh}^2 / \sigma_{Btw}^2 = \max, \) where \( \sigma_{Wh}^2 \) is the so called within-class variance, and \( \sigma_{Btw}^2 \) the between-class variance
\[
\sigma_{Wh}^2 = \sigma_0^2 + \sigma_1^2, \quad \sigma_{Btw}^2 = \omega_0 (\mu_0 - \mu_{img})^2 + \omega_1 (\mu_1 - \mu_{img})^2.
\]
Here \( (\mu_0, \sigma_0^2), \) \( (\mu_1, \sigma_1^2) \) and \( (\mu_{img}, \sigma_{img}^2) \) are the parameters (mean and variance) of the corresponding Gaussian models for the object class, for the background class and for the whole image, and \( \omega_0, \omega_1, \omega_0 + \omega_1 = 1 \) play the role of normalizing coefficients.

Since the two classes are initially unknown the preliminary statistics accumulation using Fisher’s LDA would lead to an inefficient procedure. That is why, Otsu proposes an

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equivalent but more efficient in terms of calculations criterion $\eta = \sigma^2_{\text{Brew}}/\sigma^2_{\text{img}} \sim \sigma^2_{\text{Brew}}$, $\sigma^2_{\text{img}} = \text{cte}$, which is maximized by an item-by-item examination:

$$t_{\text{opti}} = \arg \max_{0 \leq t < T} \eta(t) = \arg \max_{0 \leq t < T} \left( \omega_i(t) \omega_j(t) \left( \mu_i(t) - \mu_j(t) \right)^2 \right),$$

(1)

$$\omega_i(t) = \frac{1}{\Omega} \sum_{i=0}^{T-1} H(i), \quad \omega_j(t) = \frac{1}{\Omega} \sum_{i=0}^{T-1} H(i), \quad \mu_i(t) = \frac{1}{\Omega} \sum_{i=0}^{T-1} iH(i), \quad \mu_j(t) = \frac{1}{\Omega} \sum_{i=0}^{T-1} iH(i), \quad \Omega = \sum_{i=0}^{T-1} H(i).$$

This method is of complexity $~T$, where in the case of gray-level images usually $T = 256$. The iterative extension of the method for $M$ classes $(M > 2)$ leads to exponential complexity $~T^{M-1}$, [2], see also [6]. But, few years before the efforts in [6], an effective algorithm was already proposed, based on dynamic programming and having a complexity of $~T^2M$, see Kurita, Otsu, and Abdelmalek [3].

3. CYCLIC THRESHOLDING IN TWO CLASSES

The approach towards the problem solution can be iterative like the proposed solution in [2] for three $(M = 3)$ classes.

3.1. Intuitive approach towards the solution. Algorithm $A_0$.

We can take as a solution the couple $\left(t_0, t_1\right)$, which maximizes the criterion $\eta = \eta(t \mid t_0)$, introduced for the cyclic histogram $H(t) = H(t + T)$, $t = 0, 1, \ldots, (T - 1)$ by analogy with (1). Thus, for all the possible starting points $t_0$, $t_0 = -1, 0, 1, \ldots, (T - 2)$, we can define the threshold $t_1$ as:

$$t_1 = \arg \max_{0 \leq t < T, t \neq t_0 \leq T} \left( \eta(t \mid t_0) \right).$$

The following are the considerations on which the intuitive approach $(A_0)$ has been designed in [4], on the example of the HSV scheme interpretation for a given image:

- There are two thresholds, $t_0$ and $t_1$, to separate two classes (continuous areas) in the HS-histogram (Fig.1) resulting in a periodic H-histogram (Fig.2).
- Let us suppose that the histogram start-point coincides with the threshold $t_0$. Then we have to calculate the threshold $t_1$ to maximize the criterion $\eta(t \mid t_0)$.
- As $t_0$ is a priori unknown we have to repeat the above procedure for each $t_0$, $t_0 = -1, 0, 1, \ldots, (T - 2)$, and to get as result this couple $\left(t_0, t_1\right)$ which maximizes $\eta(t \mid t_0)$.

The intuitive approach $A_0$ is implausible, for example considering the results of binarization of sequential frames of a video-clip [7], because:

![Fig.1. The HS color histogram of an image; both H-thresholds, $t_0 = 110^\circ$, and $t_1 = 291^\circ$, are outlined.](image1.png)

![Fig.2. An optimal couple of thresholds $(t_0, t_1)$, $t_0$=110, $t_1$=291, that is equivalent to $t_0$=470 = $\left(291\right)$=110 mod$(T)$, $t_1$=651 = $\left(470\right)$=291 mod$(T)$, because of periodicity of the histogram $H(t)$=H$(t+T)$, T=360.](image2.png)
In the case of smooth changes in two sequential frames, we expect that the corresponding smooth changes will occur in the averaged intensities $\mu_0$, $\mu_1$ as well as in the other statistical characteristics $\sigma_0$, $\sigma_1$, $\omega_0$, $\omega_1$ of the two classes (object and background). The intuitive $\mathcal{A}_0$ is not proved to fulfill this requirement. In contrary:

The experiment with binarization of a large number of video-clips from the type “face on blue background shot by moving camera” [7], shows obvious leaps of the averaged intensities $\mu_0$ and $\mu_1$ of the two classes.

Interpretation of the corresponding histograms: The criterion function $\eta = \eta(t | t_0)$ is not guaranteed to be one-modal and frequently it has two (or more) well distinguishable local maxima. For example, as the video clip advances, a given local maximum can grow up exceeding the current global maximum. Then the position of the global maximum catastrophically changes, reflecting in the choice of $\mu_1$ depending on $\mu_0$, see also Fig 3.

![Fig.3. A catastrophic jump of the optimal decision $(t_0, t_1)$ is shown when the “maximum of $\eta(t | t_0)$” criterion is used. On the right of two consecutive images from a video the respective graphics are shown – the H-histogram bars (each below), both the classes (marked artificially in the middles) and the $\eta(t | t_0)$ function (each above). The means $\mu_0$ and $\mu_1$ corresponding to the found thresholds $t_0$ and $t_1$ are also shown.](image)

### 3.2. New idea for the problem solution

Let us denote by $\mathcal{A}$ the algorithm for optimal solution $t_1$, $t_1 \in \{-1,0,1,\ldots,(T-2)\}$ of the described problem following (1). Let us denote by $\mathcal{A}(t_0)$ the extension of $\mathcal{A}$ for some initial value $t_0$: $t_0 \in \{-1,0,1,\ldots,(T-2)\}$, $t_1 = \mathcal{A}(t_0)$. Then, by considerations of symmetry it must be also fulfilled:

$$t_0 \cong \mathcal{A}(t_1) \equiv \mathcal{A}(\mathcal{A}(t_0)) \mod(T), \quad t_0 \in \{-1,0,1,\ldots,(T-2)\}.$$  

(2)
Moreover, the above must be true for each of the threshold values \( t_0 \) and \( t_1 \) of the optimal pair \((t_0, t_1)\). If we consider that \( t_0 \) and \( t_1 \) are integer values, and that \( t_1 \neq t_0 \), then we can propose the following sequence of actions to determine the optimal pair \((t_0, t_1)\):

- For a given \( t_0, t_1 \in [-1,0,1,...,(T-2)] \) we have: \( t_1 = A(t_0) \), \( t_1 \in \{t_0 + t \mid t = 1,2,...,(T-1)\} \).
- From the threshold \( t_1 \) we calculate a new start position \( \tilde{t}_0 \), \( \tilde{t}_0 \equiv (t_1 + t_0 + 1) \bmod(T) \).
- For \( \tilde{t}_0, \tilde{t}_1 \in [-1,0,1,...,(T-2)] \) we get the new threshold \( \tilde{t}_1 = A(\tilde{t}_0) \), \( \tilde{t}_1 \in \{\tilde{t}_0 + t \mid t = 1,2,...,(T-1)\} \).
- From \( \tilde{t}_1 \) we calculate the next start position \( \tilde{\tilde{t}}_0 \), \( \tilde{\tilde{t}}_0 \equiv (\tilde{t}_1 + \tilde{t}_0 + 1) \bmod(T) \).
- The new start position \( \tilde{\tilde{t}}_0 \) must coincide with the initial one \( t_0 \) in the frames of periodicity \( T \), i.e.:

\[
\tilde{\tilde{t}}_0 \equiv t_0 \bmod(T).
\]

Or putted together we obtain:

\[
A((A(t_0) + t_0 + 1) \bmod(T)) + A(t_0) + t_0 + 2 \equiv t_0 \bmod(T),
\]

or equivalently:

\[
A((t_1 + t_0 + 1) \bmod(T)) + t_1 + 2 \equiv 0 \bmod(T), \quad t_1 = A(t_0).
\]

Unlike (2), the equation (3a) requires the minimal (only single) extension (see also Fig.2) of the base histogram \( H(t) \) to \( \tilde{H}(t) \):

\[
\tilde{H}(t) = \begin{cases} H(t), & t \in \{0,1,...,(T-1)\} \\ H(t-T), & t \in \{T,(T+1),...,2T-1\} \end{cases}
\]

that has been implemented in the next algorithm \( \text{AL1} \) for the case of two classes:

3.3. Algorithm \( \text{AL1} \):

- \( \text{AL1}.\text{step 1} \): For all \( t_0 \), \( t_0 = -1,0,1,...,(T-2) \), find the corresponding \( t_1 = A(t_0) \).

- \( \text{AL1}.\text{step 2} \): Separate all pairs \((t_0, t_1)\), for which the equation (3a) is fulfilled. The number of the found couples is even, i.e. there exists at least two couples \((t_0, t_1)\) and \((t_1, t_0)\), which are symmetric.

- \( \text{AL1}.\text{step 3} \): If the number of the found couples is greater than two, choose the couple that maximizes the criterion function \( \eta(t \mid t_0) \) from (1). (End of \( \text{AL1} \)).

The complexity of the algorithm \( \text{AL1} \) is evaluated to \( \sim T^2 \).

4. CYCLIC MULTI-LEVEL THRESHOLDING

Suppose we have an algorithm \( B \) for optimal segmentation of \((M+1)\) successive regions (classes) of a conventional histogram \( H \), by \( M \), \( M \geq 1 \) thresholds \((t_1, t_2,..., t_M)\), \( 0 \leq t_1 < t_2 < ... < t_M < T \), where \( T \) defines the domain of the histogram \( H(t) \), \( t \in \{0,1,...,(T-1)\} \).

Let us denote by \( B(t_0) \) the extension of the algorithm \( B \) for a starting value \( t_0 \), namely: \((t_1, t_2,..., t_M) = B(t_0), t_0 \in \{-1,0,1,...,(T-2)\}\). In this way, the base algorithm \( B \) is represented as \( B(-1) \). The complexity of \( B \), is evaluated to \( \sim (T-M)^2 M \), see also [3].

The complexity of \( B(.) \) is similar, since it can be implemented from \( B \) by simple readdressing of the extended \( \tilde{H} \) from (4), because of the \( H \) periodicity, see also Fig.2.

For the sake of concreteness of \( B(.) \), we will examine in analogy with (1) the following extended criterion function:

\[
\eta = \eta(t_M, t_{M-1}... t_1 \mid t_0) = \frac{\sigma_{\text{brw}}^2(t_0)}{\sigma_{\text{img}}^2(t_0)}
\]

where
\[ \sigma_{\text{Brw}}^2(t_0) = \sum_{i=0}^{M} \omega_i(t_0) \left( \mu_i(t_0) - \mu_{\text{img}}(t_0) \right)^2, \]

\[ \omega_i(t_0) = \frac{1}{\Omega} \sum_{j=1}^{l_{\text{tot}}} \tilde{H}(j), \quad \mu_i(t_0) = \frac{1}{\Omega \omega_i(t_0)} \sum_{j=1}^{l_{\text{tot}}} j \tilde{H}(j), \quad i = 0, 1, 2, \ldots, M; \]

\[ \sigma_{\text{img}}^2(t_0) = \frac{1}{\Omega} \sum_{j=0}^{T-1} (j - \mu_{\text{img}}(t_0))^2 \tilde{H}(j), \quad \mu_{\text{img}}(t_0) = \frac{1}{\Omega} \sum_{j=0}^{T-1} j \tilde{H}(j), \quad t_0 = -1, 0, 1, 2, \ldots (T-2); \]

\[ \Omega = \sum_{j=0}^{T-1} H(j), \quad t_{M+1} = T-1. \]

These considerations allow the extension of the algorithm \( A1 \) to the following algorithm \( A2 \) for periodic histogram thresholding in \( M+1 \) classes, \( M \geq 1 \):

### 4.1. Algorithm \( A2 \):

**A2. step 1**: For each starting value \( t_0 \), \( t_0 = -1, 0, 1, \ldots, (T-2) \), calculate \( M \) optimal thresholds \( (t_1, t_2, \ldots, t_M) \) using the algorithm \( B(\cdot) \), applied on the extended histogram \( \tilde{H}(t) \) from (4). Place the results in an array \( \text{Mot} \) (Matrix of optimal thresholds) of size \( T \times (M+1) \) and description as follows:

- **row 0**: \( (t_0, t_1, t_2, \ldots, t_M) \) , \( -1 = t_{0,0} < t_{0,1} < \ldots < t_{0,M} < (2T-1) \)
- **row 1**: \( (t_{1}, t_{1,1}, t_{1,2}, \ldots, t_{1,M}) \) , \( 0 = t_{1,0} < t_{1,1} < \ldots < t_{1,M} < (2T-1) \)
- **row \( k \)**: \( (t_{k,0}, t_{k,1}, t_{k,2}, \ldots, t_{k,M}) \) , \( (k-1) = t_{k,0} < t_{k,1} < \ldots < t_{k,M} < (2T-1) \)
- **row \( (T-1) \)**: \( (t_{T-1,0}, t_{T-1,1}, \ldots, t_{T-1,M}) \) , \( (T-2) = t_{T-1,0} < t_{T-1,1} < \ldots < t_{T-1,M} < (2T-1) \),

where each of the sequences \( (t_{k,0}, t_{k,1}, t_{k,2}, \ldots, t_{k,M}) \), \( k = 0, 1, \ldots, (T-1) \) corresponds to some concrete solution \( t_0 < t_1 < \ldots < t_M < (2T-1) \), i.e. \( (t_1, t_2, \ldots, t_M) = B(t_0) \), for \( t_0 = k-1 \).

**A2. step 2**: Extend each row \( k \), \( k = 0, 1, \ldots, (T-1) \) of \( \text{Mot} \) to the matrix \( \text{Mrot}(k) \) (Matrix of rotated optimal thresholds) with dimension \( (M+1) \times (M+1) \), where its rows \( (m) \), \( m = 1, 2, \ldots, M \), are the corresponding rows \( (t_{m,0}) \) of \( \text{Mot} \), chosen by the rule \( t_{m,0} = t_{k,m} \), \( t_{k,m} \in \text{Mrot}(k) \), and after that respectively “shifted cyclically to right” with \( m \) positions:

\[
\begin{align*}
    m = 0 & : \quad (t_{0,0}) \quad t_{0,1} \quad t_{0,2} \quad \ldots \quad t_{0,M-1} \quad t_{0,M} \\
    m = 1 & : \quad t_{1,0} \quad t_{1,1} \quad t_{1,2} \quad \ldots \quad t_{1,M-1} \quad t_{1,M} \\
    m = 2 & : \quad t_{2,0} \quad t_{2,1} \quad t_{2,2} \quad \ldots \quad t_{2,M-1} \quad t_{2,M} \\
    \vdots & \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\
    m = M & : \quad t_{M,0} \quad t_{M,1} \quad t_{M,2} \quad \ldots \quad t_{M,M-1} \quad t_{M,M} 
\end{align*}
\]

Extend \( \text{Mot} \) to \( \text{Marot} \) (Matrix of all rotated optimal thresholds), using vertical concatenation of the corresponding matrices \( \text{Mrot}(k) \), \( k = 0, 1, \ldots, (T-1) \). The resulting \( \text{Marot} \):

\[
\text{Marot} = \text{Mrot}(0) \land \text{Mrot}(1) \land \ldots \land \text{Mrot}(k) \land \ldots \land \text{Mrot}(T-1),
\]

is a three-dimensional matrix with size \( T \times (M+1) \times (M+1) \).

**A2. step 3**: Recalculate the elements of \( \text{Marot} \) according to the beginning \((t = 0)\) of the original histogram \( H \), considering also its cyclic recurrence \( H(t) = H(t+T) \):

\[
t_{k,m,j} \equiv t_{k,m,j} \mod(T), \quad 0 \leq t_{k,m,j} < T, \quad i \in \{0, 1, \ldots, M\}, \quad m \in \{0, 1, \ldots, M\}, \quad k \in \{0, 1, \ldots, (T-1)\}.
\]
The row content \((m), m=1,2,...,M\) of a given \(\mathsf{Mot}(k)\), represents all possible cyclic sequences of solutions \((t_{l1}, t_{l2},..., t_{lM}) = \mathsf{B}(t_{l,k}), S = t_{l,m}\), for a given solution \((t_{k1}, t_{k2},..., t_{km},..., t_{kM}) = \mathsf{B}(t_{k,0})\), recorded in the row \((k)\), \(k = 0,1,...,(T-1)\) of \(\mathsf{Mot}\).

The column content \((i), i = 0,1,2,...,M\) of a given \(\mathsf{Mot}(k)\), represents all possible values of the \(i\)-th threshold, which can occur in the solutions \((t_{l1}, t_{l2},..., t_{l,j},..., t_{lM}) = \mathsf{B}(t_{l,k}), S = t_{l,m}, m = 1,2,...,M\), recorded in \(\mathsf{Mot}(k), k = 0,1,...,(T-1)\). Because of symmetry reasons, for the optimal solution \(k_{\text{opti}}\) (i.e. one of the \(\mathsf{Mot}\) rows), we expect that the deviations in the columns between the thresholds \((i), i = 0,1,2,...,M\) of the corresponding \(\mathsf{Mot}(k_{\text{opti}})\) to be minimal.

**A2. step 4:** For each one \(\mathsf{Mot}(k), k = 0,1,...,(T-1)\) calculate the average values \(E_{k,i}\) of the thresholds in the rows \((i), i = 0,1,2,...,M\) and recalculate (center) the elements of \(\mathsf{Mot}(k)\) according to these average values:

\[
\Delta_{k,m,i} = \tau_{k,m,i} - E_{k,i}, m \in \{0,1,...,M\}, E_{k,j} = \frac{1}{M+1} \sum_{m=0}^{M} \tau_{k,m,j}, i \in \{0,1,...,M\},
\]

where the new values \(\Delta_{k,m,i}\) of \(\mathsf{Mot}(k)\) represent the relative deviations of the old \(\tau_{k,m,i}\) from the corresponding centers \(E_{k,j}\).

**A2. step 5:** For each one \(\mathsf{Mot}(k), k = 0,1,...,(T-1)\) calculate the averaged absolute deviations \(\epsilon_{k,m}\) in its rows \((m)\):

\[
\epsilon_{k,m} = \frac{1}{M+1} \sum_{i=0}^{M} |\Delta_{k,m,i}|, m = 0,1,...,M,
\]
as well as the value of the possible minimum \(\epsilon_{\text{min}}(k) = \min_{0 \leq k \leq M} \epsilon_{k,m}\). Obviously, \(\epsilon_{\text{min}}(k) \geq 0\).

The numbers \(\epsilon_{\text{min}}(k), k = 0,1,...,(T-1)\) are regarded as a measure of the closeness of the corresponding solutions \((t_{k1}, t_{k2},..., t_{km},..., t_{kM}) = \mathsf{B}(t_{k,0})\), recorded in \(\mathsf{Mot}\), to the optimal solution \((k_{\text{opti}})\), which corresponds to the minimum \(\epsilon_{\text{min}}\), \(\epsilon_{\text{min}} = \min_{0 \leq k < T} (\epsilon_{\text{min}}(k))\), \(k_{\text{opti}} = \arg \min_{0 \leq k < T} (\epsilon_{\text{min}}(k))\).

**A2. step 6:** Calculate the set \(K\) from the rows numbers \((k)\), for which the corresponding \(\epsilon_{\text{min}}(k)\) reaches the minimum \(\epsilon_{\text{min}}\). Thus, each row \((k), k \in K\) of the initial matrix \(\mathsf{Mot}\) (step S1), which takes part by definition in the recalculated \(\mathsf{Mot}(k)\) (steps S2–S4), can be an optimum solution candidate.

Apparently, because of considerations for symmetry, for the size of \(K\) we have that \(|K| \geq M\). Moreover: \(|K| = nM, n \geq 1, n\) is integer, and \(K\) is divided in \(n\) classes of equivalence \(K_j, j = 1,2,...,n\), where each of the rows \((k), k \in K_j\), is obtained from one of the others \((r), r \in K \setminus K_j\) by a cyclic shift to the left by \((k-r)\) positions.

**A2. step 7:** If \(|K| = M\), for definiteness, for optimal decision we choose only one of the rows \((k), k \in K\), for example the one for which \(\tau_{k,0,0} = \min\) (see Step 3).

Otherwise, if \(|K| = nM\), \(n > 1\), we define additionally the class \(K_j, j \in \{1,2,...,n\}\) of optimal solutions, using the maximum of the extended base criterion \(\eta = \eta(t_{m1}, t_{M-1},..., t_1 | t_0)\) from (5). Again, for definiteness, for optimal decision we choose only one of the rows \((k), k \in K_j\), for example the one for which \(\tau_{k,0,0} = \min\). (End of A2).
4.2. Additional explanations for the algorithm \( A_2 \):

- General optimizations of the program structures for implementation of matrices \( \text{Mot} \), \( \text{Mrot} \), and \( \text{Marot} \). We will note that the matrices \( \text{Mrot}(k), k = 0, 1, \ldots, (T - 1) \) can be reduced to a single working matrix \( \text{Mrot} \) with dimensions \((M + 1) \times (M + 1)\), which reduces the “big” matrix \( \text{Marot} \) to the base matrix \( \text{Mot} \).

- At the same time, in Step 5 the classical “least square method” can be used, which will be a slight drawback in the terms of processing speed.

- The complexity of \( A_2 \) is determined mainly by the complexity of its Step 1 and is calculated to \(~ T(T - M)^2 M\), \( M \) the number of classes, i.e. the number of cyclic thresholds.

- Apparently, for the case of two classes \( (M = 2) \), it is more efficiently to use the algorithm \( A_1 \) (complexity \(~ T^2 \)) instead of \( A_2 \) (complexity \(~ T^3 \)).

4. EXPERIMENTS AND RESULTS

We carried out an experimental analysis of the proposed approach through an arbitrary picture of outdoor view (Fig.4) to assure a larger spectrum of colors most of all for the tests of algorithm \( A_2 \). The experimental software is a C++ written Windows-XP application operating on an IBM compatible PC: Intel Pentium 4 CPU 2.8GHz, MM 2.0GB.

The results of the experiments (execution times) of the proposed algorithms are represented in Table 1, where the discrete Hue-histogram is considered a priori calculated for \( T = 360 \) (angular degrees).

![Fig.4. An arbitrary picture of outdoor view (left above), its Hue-histogram optimal thresholding in 8 levels (on the right), and its segmentation in corresponding 8 color means (left down) are shown. Regularly increasing gray intensities are used to represent the respective Hue-means, see also row \((A_2, M=8)\) in Table 1.](image)

<table>
<thead>
<tr>
<th>Algorithm &amp; number ((M)) of classes</th>
<th>Processing speed ((S)) ([s])</th>
<th>Processing speed per class ((S/M)) ([s])</th>
<th>Threshold series found (by (T=360))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_0 ), ( M=2 )</td>
<td>0.007</td>
<td>0.003</td>
<td>126, 313</td>
</tr>
<tr>
<td>( A_1 ), ( M=2 )</td>
<td>0.006</td>
<td>0.003</td>
<td>135, 315</td>
</tr>
<tr>
<td>( A_2 ), ( M=2 )</td>
<td>0.656</td>
<td>0.328</td>
<td>135, 315</td>
</tr>
<tr>
<td>( A_2 ), ( M=3 )</td>
<td>1.062</td>
<td>0.354</td>
<td>63, 154, 305</td>
</tr>
<tr>
<td>( A_2 ), ( M=4 )</td>
<td>1.422</td>
<td>0.356</td>
<td>46, 93, 167, 300</td>
</tr>
<tr>
<td>( A_2 ), ( M=8 )</td>
<td>2.641</td>
<td>0.330</td>
<td>29, 55, 85, 123, 178, 228, 284, 349</td>
</tr>
<tr>
<td>( A_2 ), ( M=16 )</td>
<td>4.562</td>
<td>0.285</td>
<td>12, 27, 39, 52, 65, 78, 93, 111, 137, 176, 205, 219, 237, 262, 301, 344</td>
</tr>
</tbody>
</table>
The measured times $S$ correspond to the theoretical valuations of the algorithms:

$S(A_{10}) \sim O(A_{10}) \sim T^2$, $S(A_{11}) \sim O(A_{11}) \sim T^2$ and $S(A_{2}, M) \sim O(A_{2}, M) \sim T(T - M)^2 M$, demonstrated by the ratio $S/M$ (column 3 of the table) that slightly diminish with $M$ at $A_{2}$. $A_{2}$ should be preferred in all cases of $M > 3$, while $A_{1}$: for $M = 2$. Only for $M = 3$ classes, experiments show that the inner $b(.)$ of $A_{2}$ is better to perform in a classical way, e.g. [6].

5. CONCLUSION

In this paper, following the Otsu’s criterion [2], a new method is proposed for optimal thresholding and multithresholding of cyclic histograms. The method preserves the symmetry of the solution for the expense of insignificant (in most cases) deviations from the optimum of an intuitive iterative extension of the Otsu’s criterion, cf. [4]. The method can be modified for other approaches for thresholding and multithresholding of histograms, e.g. entropy approach mentioned in [1, 3].

Two algorithms are proposed, $A_{1}$ and $A_{2}$. $A_{1}$ concerns thresholding in 2 classes and has quadratic complexity, while $A_{2}$ concerns multithresholding in more than 2 classes and has cubic complexity (per class). This speed up is provided by dynamic programming for the base algorithms (i.e. for the non-cyclic case) in analogy with the idea from [3].

The motivation for the proposed approach development is connected with discovering of effective and statistically optimal method for color images segmentation based on the HSV or HLS color schemes, which histograms become cyclic. Unlike the intuitive approach proposed in [4], marked here as $A_{10}$, the current method ($A_{11}, A_{2}$) is optimized for color segmentation of image sequences (video-clips). The method provides a smooth change of the parameters of approximating Gaussians with the smooth change of the position of the camera, which is usually connected with unwanted change of illumination, see [7].

The method is applicable in the multiple cases of discrete approximation of a function by Gaussians and particularly when a stress is put on the symmetry (periodicity) of the approximation. In this prospective as future work the authors intend to investigate a similar approach based on the wavelet and Fourier analysis.

REFERENCES


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