The Effect of the Elliptic Polarization on the Quasi-Particle Dynamics of Linearly Coupled Systems of Nonlinear Schrödinger Equations

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Outline

- Problem Formulation
- Numerical Method
- Generation of Approximate Initial Conditions
- Linear Coupling and Polarization Dynamics
- Conclusions
- References

Problem Formulation: Equations

CNLSE is system of nonlinearly coupled Schrödinger equations (called the Gross-Pitaevskii or Manakov-type system):

$$
\begin{aligned}
\mathrm{i}\psi_t &= \beta\psi_{xx} + \left[\alpha_1|\psi|^2 + (\alpha_1 + 2\alpha_2)|\phi|^2\right]\psi, \\
\mathrm{i}\phi_t &= \beta\phi_{xx} + \left[\alpha_1|\phi|^2 + (\alpha_1 + 2\alpha_2)|\psi|^2\right]\phi,
\end{aligned}
\tag{1}
$$

where:
β is the dispersion coefficient;
α_1 describes the self-focusing of a signal for pulses in birefringent media;
α_2 (called cross-modulation parameter) governs the nonlinear coupling between the equations. When $\alpha_2 = 0$, no nonlinear coupling is present despite the fact that "cross-terms" proportional to α_1 appear in the equations. For $\alpha_2 = 0$, the solutions of the two equations are identical, $\psi \equiv \phi$, and equal to the solution of single NLSE with nonlinearity coefficient $\alpha = 2\alpha_1$.

Problem Formulation: Equations

Now we concentrate on linearly coupled system of NLSE.
Obviously this system is Manakov type and the magnitude of linear coupling generates breathing the solitons although noninteracting

$$
\begin{aligned}
\mathrm{i}\Psi_t &= \beta\Psi_{xx} + \alpha_1\left[|\Psi|^2 + |\Phi|^2\right]\Psi - \Gamma\Phi, \\
\mathrm{i}\Phi_t &= \beta\Phi_{xx} + \alpha_1\left[|\Phi|^2 + \Psi|^2\right]\Phi - \Gamma\Psi,
\end{aligned}
\tag{2}
$$

with initial conditions

$$
\Psi = \psi\cos(\Gamma t) + \mathrm{i}\phi\sin(\Gamma t), \quad \Phi = \phi\cos(\Gamma t) + \mathrm{i}\psi\sin(\Gamma t), \tag{3}
$$

where ϕ and ψ are assumed to be sech-solutions of (1).
$\Gamma = \Gamma_r + \mathrm{i}\Gamma_i$ is the magnitude of linear coupling. Γ_r governs the oscillations between states termed as breathing solitons, while Γ_i describes the gain behavior of soliton solutions.
Hence (2) posses solutions, which are combinations of interacting solitons oscillating with frequency Γ_r.

Problem Formulation: Choice of Initial Conditions

These solutions are pulses whose modulation amplitude is of general form (non-sech) and their polarization rotates with time. This determines the choice of initial conditions for numerical investigation of temporal evolution of interacting solitons. In the present paper we concern ourselves with the soliton solutions which are localized envelops on a propagating carrier wave.

Problem Formulation: Choice of Initial Conditions

We assume that for each of the functions ϕ, ψ the initial condition has the general type

$$\begin{aligned}\psi &= A_\psi(x+X-c_\psi t)\exp\left\{\mathrm{i}\left[n_\psi t - \tfrac{1}{2}c_\psi(x-X-c_\psi t)+\delta_\psi\right]\right\}\\ \phi &= A_\phi(x+X-c_\phi t)\exp\left\{\mathrm{i}\left[n_\phi t - \tfrac{1}{2}c_\phi(x-X-c_\phi t)+\delta_\phi\right]\right\},\end{aligned} \tag{4}$$

where c_ψ, c_ϕ are the phase speeds and X's are the initial positions of the centers of the solitons; n_ψ, n_ϕ are the carrier frequencies for the two components; δ_ψ and δ_ϕ are the phases of the two components. Note that the phase speed must be the same for the two components ψ and ϕ. If they propagate with different phase speeds, after some time the two components will be in two different positions in space, and will no longer form a single structure. For the envelopes (A_ψ, A_ϕ),
$\theta \equiv \arctan(\max|\phi|/\max|\psi|)$ is a polarization angle.

Problem Formulation: Choice of Initial Conditions

Generally the carrier frequencies for the two components $n_\psi \neq n_\phi$ – elliptic polarization. When $n_\psi = n_\phi$ – circular polarization. If one of them vanishes – linear polarization (sech soliton).
In general case the initial condition is solution of the following system of nonlinear conjugated equations

$$
\begin{aligned}
A''_\psi + \left(n_\psi + \tfrac{1}{4}c_\psi^2\right) A_\psi + \left[\alpha_1 A_\psi^2 + (\alpha_1 + 2\alpha_2) A_\phi^2\right] A_\psi = 0 \\
A''_\phi + \left(n_\phi + \tfrac{1}{4}c_\phi^2\right) A_\phi + \left[\alpha_1 A_\phi^2 + (\alpha_1 + 2\alpha_2) A_\psi^2\right] A_\phi = 0.
\end{aligned}
\qquad (5)
$$

The system admits bifurcation solutions since the trivial solution obviously is always present.

Problem Formulation: Choice of Initial Conditions

For the linearly coupled initial conditions we have explicitly the same conjugate bifurcation system but with trivial nonlinear coupling, $\alpha_2 = 0$,

$$\begin{aligned} A''_\Psi + \left(n_\Psi + \tfrac{1}{4}c_\Psi^2\right)A_\Psi + \alpha_1\left(A_\Psi^2 + A_\Phi^2\right)A_\Psi &= 0 \\ A''_\Phi + \left(n_\Phi + \tfrac{1}{4}c_\Phi^2\right)A_\Phi + \alpha_1\left(A_\Phi^2 + A_\Psi^2\right)A_\Phi &= 0. \end{aligned} \tag{6}$$

Our aim is to understand better the influence of the initial polarization on the particle-like behavior of the localized waves. We call a localized wave a quasi-particle (QP) if it survives the collision with other QPs (or some other kind of interactions) without losing its identity.

Initial Conditions

We solve the auxiliary conjugated system (5) or (6) with asymptotic boundary conditions using Newton method and the initial approximation of sought nontrivial solution is sech-function. The final solution, however, is not obligatory sech-function. It is a two-component polarized soliton solution.

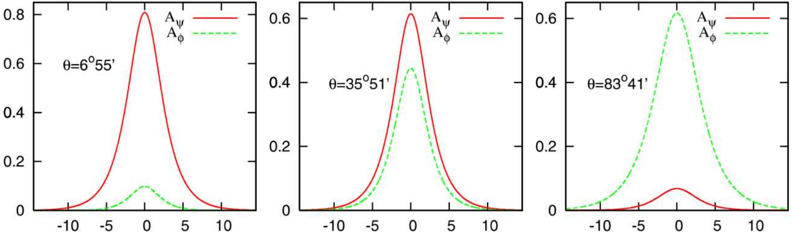

Figure: 1. Amplitudes A_ψ and A_ϕ for $c_l = -c_r = 1$, $\alpha_1 = 0.75$, $\alpha_2 = 0.2$. Left:$n_\psi = -0.68$; middle: $n_\psi = -0.55$; right: $n_\psi = -0.395$.

Initial Conditions

Another dimension of complexity is introduced by the phases of the different components. The initial difference in phases can have a profound influence on the polarizations of the solitons after the interaction. The relative shift of real and imaginary parts is what matters in this case.

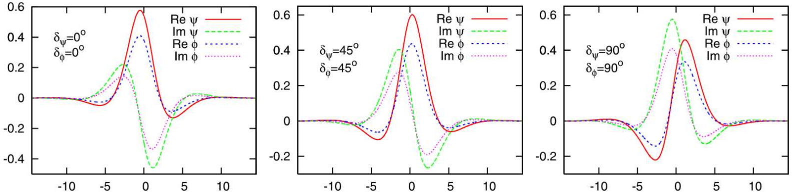

Figure: 2. Real and imaginary parts of the amplitudes from the case shown in the middle panel of Figure 1 and the dependence on phase angle.

Problem Formulation: Conservation Laws

Define "mass", M, (pseudo)momentum, P, and energy, E:

$$M \stackrel{\text{def}}{=} \frac{1}{2\beta}\int_{-L_1}^{L_2}(|\psi|^2+|\phi|^2)\mathrm{d}x, \quad P \stackrel{\text{def}}{=} -\int_{-L_1}^{L_2}\mathcal{I}(\psi\bar{\psi}_x+\phi\bar{\phi}_x)\mathrm{d}x,$$

$$E \stackrel{\text{def}}{=} \int_{-L_1}^{L_2}\mathcal{H}\mathrm{d}x, \quad \text{where} \tag{7}$$

$$\mathcal{H} \stackrel{\text{def}}{=} \beta\left(|\psi_x|^2+|\phi_x|^2\right)-\tfrac{1}{2}\alpha_1(|\psi|^4+|\phi|^4)$$
$$-(\alpha_1+2\alpha_2)\left(|\phi|^2|\psi|^2\right)-2\Gamma[\Re(\bar{\psi}\bar{\phi})]$$

is the Hamiltonian density of the system. Here $-L_1$ and L_2 are the left end and the right end of the interval under consideration. For the linear coupling case, $\alpha_2=0$ and $\Gamma\neq 0$ the functions ψ and ϕ correspond to notations in (3). The following conservation/balance laws hold, namely

$$\frac{dM}{dt}=0, \qquad \frac{dP}{dt}=\mathcal{H}\big|_{x=L_2}-\mathcal{H}\big|_{x=-L_1}, \qquad \frac{dE}{dt}=0, \tag{8}$$

Numerical Method

To solve the main problem numerically, we use an implicit conservative scheme in complex arithmetic

$$
\begin{aligned}
&\mathrm{i}\frac{\psi_i^{n+1}-\psi_i^n}{\tau} = \frac{\beta}{2h^2}\left(\psi_{i-1}^{n+1}-2\psi_i^{n+1}+\psi_{i+1}^{n+1}+\psi_{i-1}^{n}-2\psi_i^{n}+\psi_{i+1}^{n}\right)\\
&+\frac{\psi_i^{n+1}+\psi_i^n}{4}\left[\alpha_1\left(|\psi_i^{n+1}|^2+|\psi_i^n|^2\right)+(\alpha_1+2\alpha_2)\left(|\phi_i^{n+1}|^2+|\phi_i^n|^2\right)\right]\\
&\quad-\tfrac{1}{2}\Gamma\left(\phi_i^{n+1}+\phi_i^n\right),
\end{aligned}
$$

$$
\begin{aligned}
&\mathrm{i}\frac{\phi_i^{n+1}-\phi_i^n}{\tau} = \frac{\beta}{2h^2}\left(\phi_{i-1}^{n+1}-2\phi_i^{n+1}+\phi_{i+1}^{n+1}+\phi_{i-1}^{n}-2\phi_i^{n}+\phi_{i+1}^{n}\right)\\
&+\frac{\phi_i^{n+1}+\phi_i^n}{4}\left[\alpha_1\left(|\phi_i^{n+1}|^2+|\phi_i^n|^2\right)+(\alpha_1+2\alpha_2)\left(|\psi_i^{n+1}|^2+|\psi_i^n|^2\right)\right]\\
&\quad-\tfrac{1}{2}\Gamma\left(\psi_i^{n+1}+\psi_i^n\right).
\end{aligned}
$$

Numerical Method: Conserved Properties

It is not only convergent (consistent and stable), but also conserves mass and energy, i.e., there exist discrete analogs for (8), which arise from the scheme.

$$M^n = \sum_{i=2}^{N-1} \left(|\psi_i^n|^2 + |\phi_i^n|^2 \right) = \mathrm{const},$$

$$E^n = \sum_{i=2}^{N-1} \frac{-\beta}{2h^2} \left(|\psi_{i+1}^n - \psi_i^n|^2 + |\phi_{i+1}^n - \phi_i^n|^2 \right) + \frac{\alpha_1}{4} \left(|\psi_i^n|^4 + |\phi_i^n|^4 \right)$$
$$+ \frac{1}{2}(\alpha_1 + 2\alpha_2) \left(|\psi_i^n|^2 |\phi_i^n|^2 \right) - \Gamma \Re[\bar{\phi}_i^n \psi_i^n] = \mathrm{const},$$
$$\text{for} \quad \text{all} \quad n \geq 0.$$

These values are kept constant during the time stepping. The above scheme is of Crank-Nicolson type for the linear terms and we employ internal iterations to achieve implicit approximation of the nonlinear terms, i.e., we use its linearized implementation.

Circularly Polarized Solitons ($\theta_{\rm in} = 45^\circ$)

This is a special elliptic polarization with $\theta_{\rm in} = 45^\circ$. It can be generated from the auxiliary bifurcation system. Because the parametric space of the problem is too big to be explored in full, we choose $n_{l\psi} = n_{r\psi} = n_{l\phi} = n_{r\phi} = -1.5$, $c_l = -c_r = 1$, $\alpha_1 = 0.75$, $\Gamma = 0.175$ and focus on the effects of $\vec{\delta}$.

- We have found that the phases of the components play an essential role on the full energy of QPs. The magnitude of the latter essentially depends on the choice of initial phase difference (Figure 7);
- The pseudomomentum is also conserved and it is trivial due to the symmetry (Figure 6);

Circularly Polarized Solitons ($\theta_{\text{in}} = 45^{\circ}$)

- The individual masses, however, breathe together with the individual (rotational) polarizations. Their amplitude and period do not influenced from the initial phase difference (Figure 6);
- The total mass is constant while the total polarization oscillates and suffers a 'shock in polarization ' when QPs enter the collision. The polarization amplitude evidently depends on the initial phase difference (Figures 8,9,10);
- Due to the real linear coupling the polarization angle of QPs can change independently of the collision.

Initial Circular Polarizations: $\theta = 45^\circ$, $\alpha_2 = 0$, $c_l = -c_r = 1$, $n_\psi = n_\phi = -1.5$, $\Gamma = 0.175$

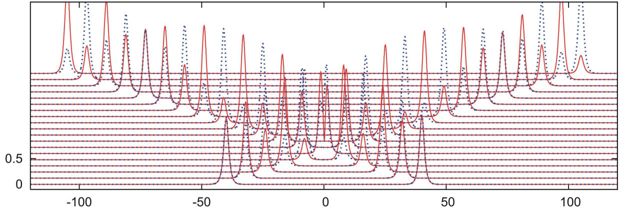

Figure: 3. $\delta_l = 0^\circ$, $\delta_r = 0^\circ$

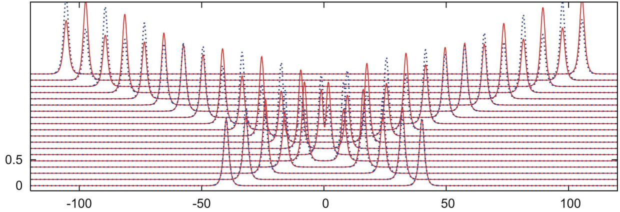

Figure: 4. $\delta_l = 0^\circ$, $\delta_r = 90^\circ$

Initial Circular Polarizations: $\theta = 45^\circ$, $\alpha_2 = 0$, $c_l = -c_r = 1$, $n_\psi = n_\phi = -1.5$, $\Gamma = 0.175$

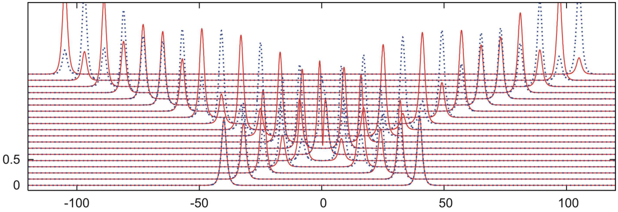

Figure: 5. $\delta_l = 0^\circ$, $\delta_r = 180^\circ$

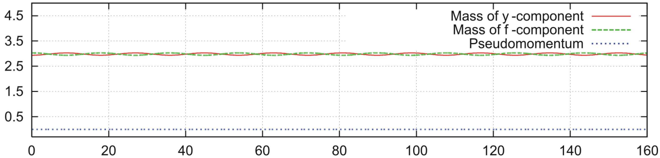

Figure: 6. $\delta_l = 0^\circ$, $\delta_r = 0^\circ; 90^\circ; 180^\circ$, $P = 10^{-3} \div 10^{-11}$

Initial Circular Polarizations: $\theta = 45^\circ$, $\alpha_2 = 0$, $c_l = -c_r = 1$, $n_\psi = n_\phi = -1.5$, $\Gamma = 0.175$

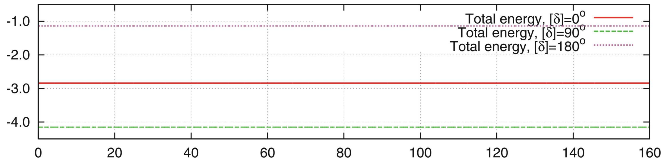

Figure: 7. Influence of the initial phase difference on the energy:

$\delta_l = 0^\circ$, $\delta_r = 0^\circ$ – $E = -2.842$;

$\delta_l = 0^\circ$, $\delta_r = 90^\circ$ – $E = -4.15$;

$\delta_l = 0^\circ$, $\delta_r = 180^\circ$ – $E = -1.139$

Initial Circular Polarizations: $\theta = 45^\circ$, $\alpha_2 = 0$, $c_l = -c_r = 1$, $n_\psi = n_\phi = -1.5$, $\Gamma = 0.175$

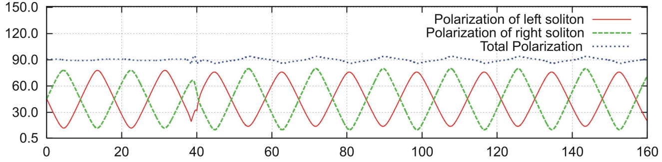

Figure: 8. $\delta_l = 0^\circ$, $\delta_r = 0^\circ$

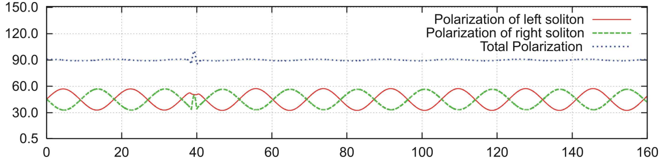

Figure: 9. $\delta_l = 0^\circ$, $\delta_r = 90^\circ$

Initial Circular Polarizations: $\theta = 45^\circ$, $\alpha_2 = 0$, $c_l = -c_r = 1$, $n_\psi = n_\phi = -1.5$, $\Gamma = 0.175$

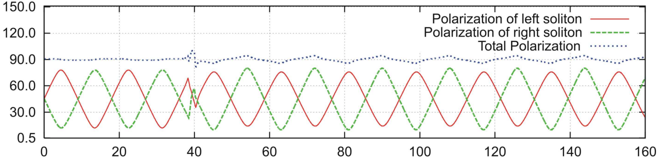

Figure: 10. $\delta_l = 0^\circ$, $\delta_r = 180^\circ$

Elliptically Polarized Solitons ($\theta_{\rm in} = 23^\circ 44'$)

We consider two solitons with equal initial elliptic polarizations. The initial configuration is generated from the auxiliary bifurcation system. Because the parametric space of the problem is too big to be explored in full, we choose $n_{l\psi} = n_{r\psi} = -1.1$, $n_{l\phi} = n_{r\phi} = -1.5$, $c_l = -c_r = 1$, $\alpha_1 = 0.75$, $\Gamma = 0.175$ and focus on the effects of $\vec{\delta}$.
Because the results are qualitatively the same as in the previous case of circular polarization we skip their discussion.

Initial Elliptic Polarizations: $\theta = 23^\circ 44'$, $\alpha_2 = 0$, $c_l = -c_r = 1$, $n_{l\psi} = n_{r\psi} = -1.1$, $n_{l\phi} = n_{r\phi} = -1.5$, $\Gamma = 0.175$

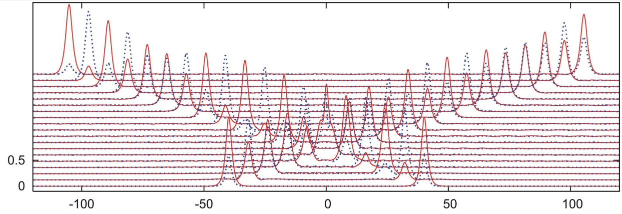

Figure: 11. $\delta_l = 0^\circ$, $\delta_r = 0^\circ$

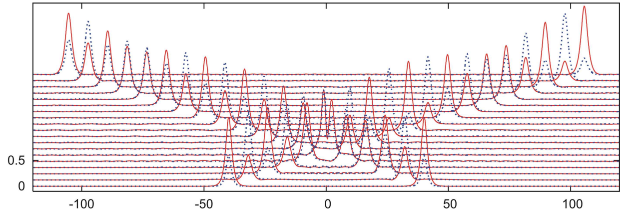

Figure: 12. $\delta_l = 0^\circ$, $\delta_r = 90^\circ$

Initial Elliptic Polarizations: $\theta = 23^\circ 44'$, $\alpha_2 = 0$, $c_l = -c_r = 1$, $n_{l\psi} = n_{r\psi} = -1.1$, $n_{l\phi} = n_{r\phi} = -1.5$, $\Gamma = 0.175$

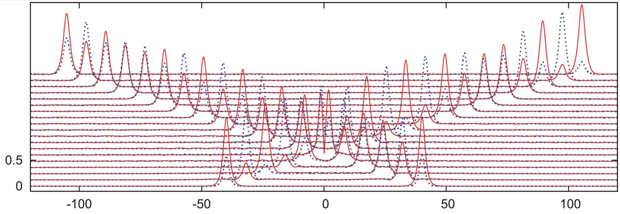

Figure: 13. $\delta_l = 0^\circ$, $\delta_r = 135^\circ$

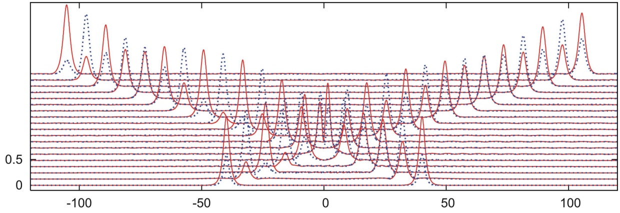

Figure: 14. $\delta_l = 0^\circ$, $\delta_r = 180^\circ$

Initial Elliptic Polarizations: $\theta = 23°44'$, $\alpha_2 = 0$, $c_l = -c_r = 1$, $n_{l\psi} = n_{r\psi} = -1.1$, $n_{l\phi} = n_{r\phi} = -1.5$, $\Gamma = 0.175$

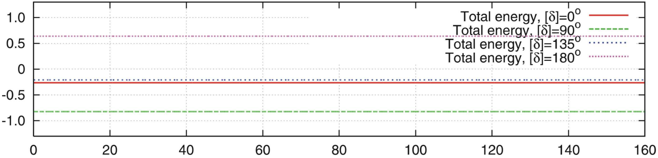

Figure: 15. Influence of the initial phase difference on the total energy:
$\delta_l = 0°$, $\delta_r = 0°$ – $E = -0.262$;
$\delta_l = 0°$, $\delta_r = 90°$ – $E = -0.821$;
$\delta_l = 0°$, $\delta_r = 135°$ – $E = -0.206$;
$\delta_l = 0°$, $\delta_r = 180°$ – $E = 0.640$

Initial Elliptic Polarizations: $\theta = 23°44'$, $\alpha_2 = 0$, $c_l = -c_r = 1$, $n_{l\psi} = n_{r\psi} = -1.1$, $n_{l\phi} = n_{r\phi} = -1.5$, $\Gamma = 0.175$

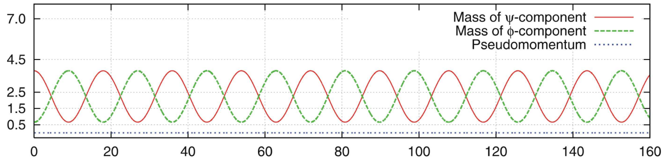

Figure: 16. $\delta_l = 0°$, $\delta_r = 90°$, $P = 10^{-3} \div 10^{-5}$

Initial Elliptic Polarizations: $\theta = 23°44'$, $\alpha_2 = 0$, $c_l = -c_r = 1$, $n_{l\psi} = n_{r\psi} = -1.1$, $n_{l\phi} = n_{r\phi} = -1.5$, $\Gamma = 0.175$

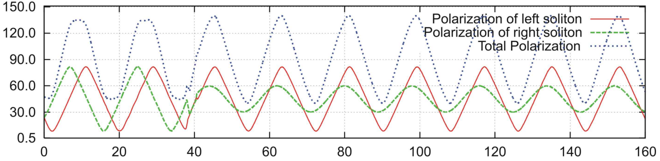

Figure: 17. $\delta_l = 0°$, $\delta_r = 0°$

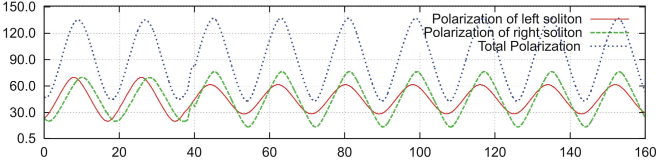

Figure: 18. $\delta_l = 0°$, $\delta_r = 90°$

Initial Elliptic Polarizations: $\theta = 23°44'$, $\alpha_2 = 0$, $c_l = -c_r = 1$, $n_{l\psi} = n_{r\psi} = -1.1$, $n_{l\phi} = n_{r\phi} = -1.5$, $\Gamma = 0.175$

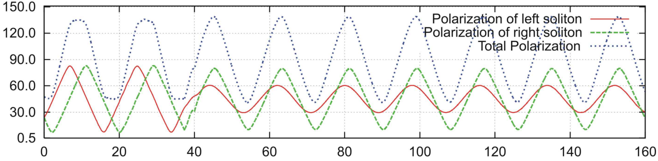

Figure: 19. $\delta_l = 0°$, $\delta_r = 135°$

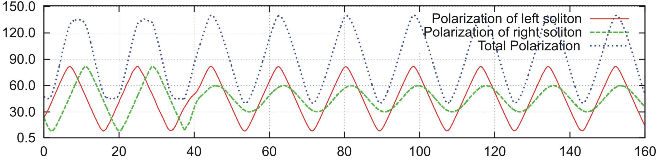

Figure: 20. $\delta_l = 0°$, $\delta_r = 180°$

Polarized Solitons with Different Polarization Angles ($\theta_{l\mathrm{in}} = 23^\circ 44'$, $\theta_{r\mathrm{in}} = 25^\circ 23'$)

In this case the solitons start with different elliptic polarizations. It is generated from the auxiliary bifurcation system. Because the parametric space of the problem is too big to be explored in full, we choose $n_{l\psi} = n_{r\psi} = -1.1$, $n_{l\phi} = n_{r\phi} = -1.5$, $c_l = 1$, $c_r = 0.8$, $\alpha_1 = 0.75$, $\Gamma = 0.175$ and focus on the effects of $\vec{\delta}$.

- We have found that the phases of the components play an essential role on the full energy of QPs. The magnitude of the latter essentially depends on the choice of initial phase difference (Figure 30);
- The pseudomomentum is also conserved and does not depend on the initial phase difference. It is not trivial. (Figure 24,25,26);

Polarized Solitons with Different Polarization Angles ($\theta_{l\mathrm{in}} = 23^\circ 44'$, $\theta_{r\mathrm{in}} = 25^\circ 23'$)

- The individual masses, however, breathe together with the individual (rotational) polarizations. Their amplitude and period do not influenced from the initial phase difference (Figure 24,25,26) and are conserved within one full period of the breathing. The total mass is constant;
- Both the individual and total polarizations breathe and suffers a 'shock in polarization ' when QPs enter the collision. The polarization amplitude evidently depends on the initial phase difference (Figures 27,28,29). The above quantities are conserved within one full period of the breathing;
- Due to the real linear coupling the polarization angle of QPs can change independently of the collision.

Overtake. Different Elliptic Polarizations: $\theta_l = 23°44'$, $\theta_r = 25°23'$, $\alpha_2 = 0$, $c_l = 1$, $c_r = 0.8$, $n_{l\psi} = n_{r\psi} = -1.1$, $n_{l\phi} = n_{r\phi} = -1.5$, $\Gamma = 0.175$

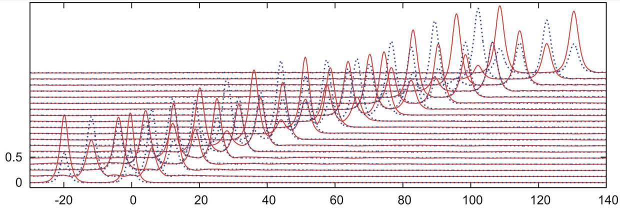

Figure: 21. $\delta_l = 0°$, $\delta_r = 0°$

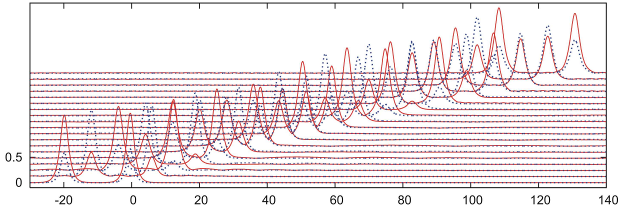

Figure: 22. $\delta_l = 0°$, $\delta_r = 90°$

Overtake. Different Initial Elliptic Polarizations: $\theta_l = 23°44'$, $\theta_r = 25°23'$, $\alpha_2 = 0$, $c_l = 1$, $c_r = 0.8$, $n_{l\psi} = n_{r\psi} = -1.1$, $n_{l\phi} = n_{r\phi} = -1.5$, $\Gamma = 0.175$

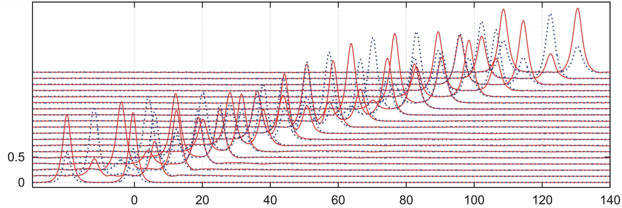

Figure: 23. $\delta_l = 0°$, $\delta_r = 180°$

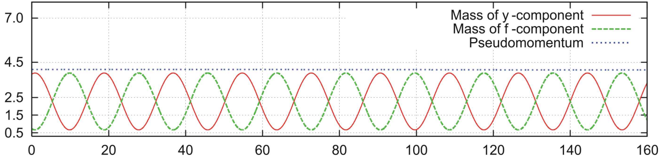

Figure: 24. $\delta_l = 0°$, $\delta_r = 0°$, $P = 4.08 \div 4.05$

Overtake. Different Initial Elliptic Polarizations: $\theta_l = 23°44'$, $\theta_r = 25°23'$, $\alpha_2 = 0$, $c_l = 1$, $c_r = 0.8$, $n_{l\psi} = n_{r\psi} = -1.1$, $n_{l\phi} = n_{r\phi} = -1.5$, $\Gamma = 0.175$

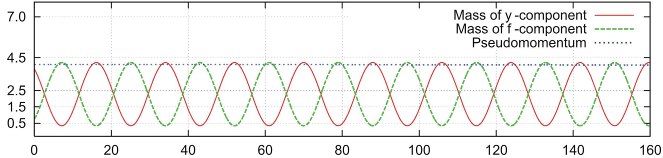

Figure: 25. $\delta_l = 0°$, $\delta_r = 90°$, $P = 4.08 \div 4.05$

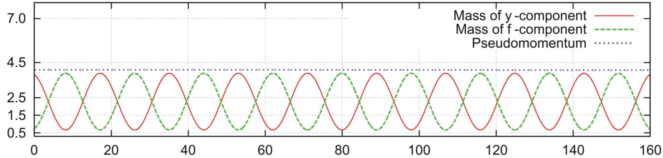

Figure: 26. $\delta_l = 0°$, $\delta_r = 180°$, $P = 4.08 \div 4.05$

Overtake. Different Initial Elliptic Polarizations: $\theta_l = 23°44'$, $\theta_r = 25°23'$, $\alpha_2 = 0$, $c_l = 1$, $c_r = 0.8$, $n_{l\psi} = n_{r\psi} = -1.1$, $n_{l\phi} = n_{r\phi} = -1.5$, $\Gamma = 0.175$

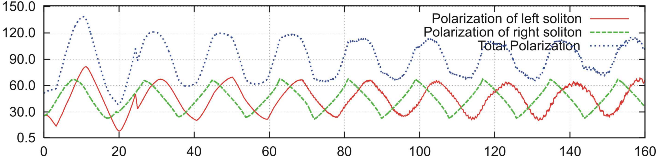

Figure: 27. $\delta_l = 0°$, $\delta_r = 0°$

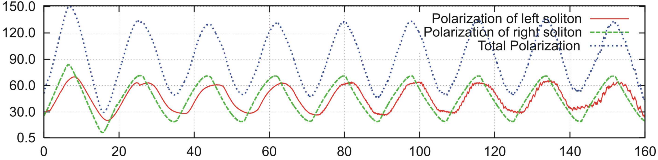

Figure: 28. $\delta_l = 0°$, $\delta_r = 90°$

Overtake. Different Initial Elliptic Polarizations: $\theta_l = 23°44'$, $\theta_r = 25°23'$, $\alpha_2 = 0$, $c_l = 1$, $c_r = 0.8$, $n_{l\psi} = n_{r\psi} = -1.1$, $n_{l\phi} = n_{r\phi} = -1.5$, $\Gamma = 0.175$

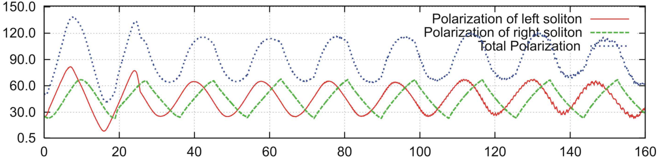

Figure: 29. $\delta_l = 0°$, $\delta_r = 180°$

Overtake. Different Initial Elliptic Polarizations: $\theta_l = 23°44'$, $\theta_r = 25°23'$, $\alpha_2 = 0$, $c_l = 1$, $c_r = 0.8$, $n_{l\psi} = n_{r\psi} = -1.1$, $n_{l\phi} = n_{r\phi} = -1.5$, $\Gamma = 0.175$

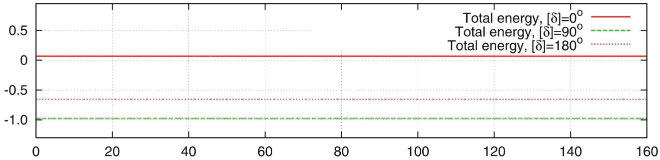

Figure: 30. Influence of the initial phase difference on the total energy:
$\delta_l = 0°$, $\delta_r = 0°$ – $E = 0.0673$;
$\delta_l = 0°$, $\delta_r = 90°$ – $E = -0.976$;
$\delta_l = 0°$, $\delta_r = 180°$ – $E = -0.657$

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Thanks

Thank you for your kind attention !