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OSCILLATION OF A CIRCULAR CYLINDER SUBJECT TO WATER JET

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Oscillation of a circular cylinder subject to water jet is analyzed theoretically, taking into outgoing film flow followed by hydraulic jump, which is considered to be almost similar to oblique impingement of a circular jet to a flat surface. Observed translation and rotation of the cylinder is in good agreement with the analysis.

1. Introduction

A wooden substantially circular cylinder is subject to vertical water jet, where the cylinder is floating across nearly stationary water, keeping a part of its surface in contact with the edge of the plate with rotation of the cylinder, no slip. In case of laminar jet, periodic oscillation of the cylinder can be observed with hydraulic jump. Impingement of a circular liquid jet (unsubmerged, laminar or turbulent) on a plane surface (normal impingement or oblique impingement) is known to produce thin film flow followed by a hydraulic jump [1], [2].

The motion of a substantially circular cylinder subject to vertical laminar water jet is analyzed, taking into outgoing thin film flow of water jet.

2. Analysis

2.1. Measurement system

Measurement system consists of a substantially circular wooden cylinder (only both ends being slightly tapering such that I (inertial moment) $= 0.945 \times$

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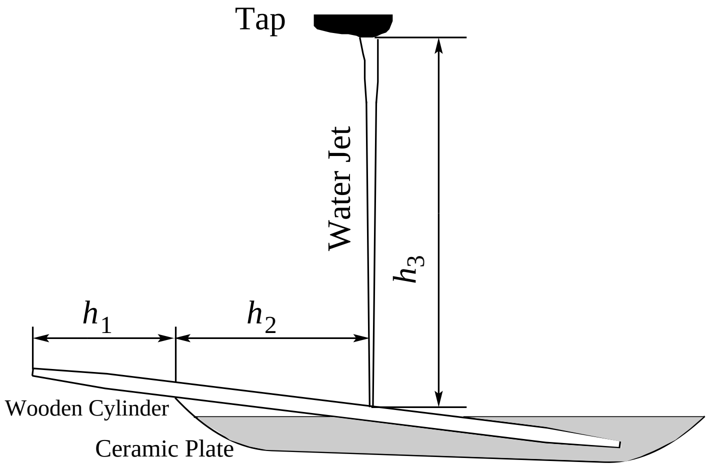

Figure 1: Configuration

Figure 2: Circular cylinder

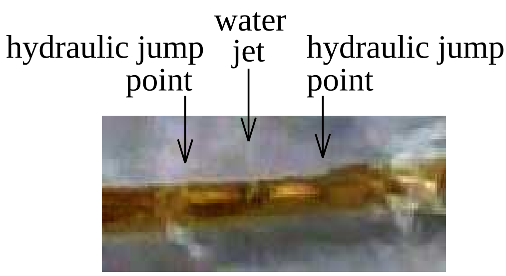

Figure 3: Film flow

$\left(\frac{1}{2}{r_0}^2 M\right)$, 240 mm in length, r_0 (radius of the cylinder) $=$ 3 mm, M (total mass) $=$ 4.0 g (wet state)) floating across substantially stationary water in a plate (overflowing) and contacting the edge of the plate (approx. 215 mm in diameter) as a point contact without slip (effects of surface tension between the cylinder and water being negligibly smaller than others), and a laminar water jet from a tap as shown in Fig. 1, where on an outgoing flow side of the water jet, film flow and hydraulic jump phenomenon can be found (Fig. 2, Fig. 3).

The motion of no-slip rotation of the cylinder (with parallel, normal, oblique lines pasted on its surface) was recorded with a video recorder (30 fps, frames

s^{-1}), the camera angle between the incident camera direction and the horizontal surface being (53 $\sim$ 63) degree. Examples of time sequential photographs are shown in Fig. 4 (a)–(e) at the flow rate Q of 9.53 cm^3/s, 11 °C, where where all the time interval between the preceding photos is (5/29.97) s. At this small flow rate impinging velocity, u_j, is nearly equal to free fall velocity (substantially independent of the time) $u_j = \sqrt{2\ gh_3} = 2.48$ m/s, $h_3 = 0.315$ m, water jet diameter, d, at the impingement location is defined by $d \equiv \sqrt{4Q/(\pi u_j)} = 2.2$ mm, and Reynolds number $Red(\equiv u_j d/\nu$, ν : kinematic viscosity) = 4300.

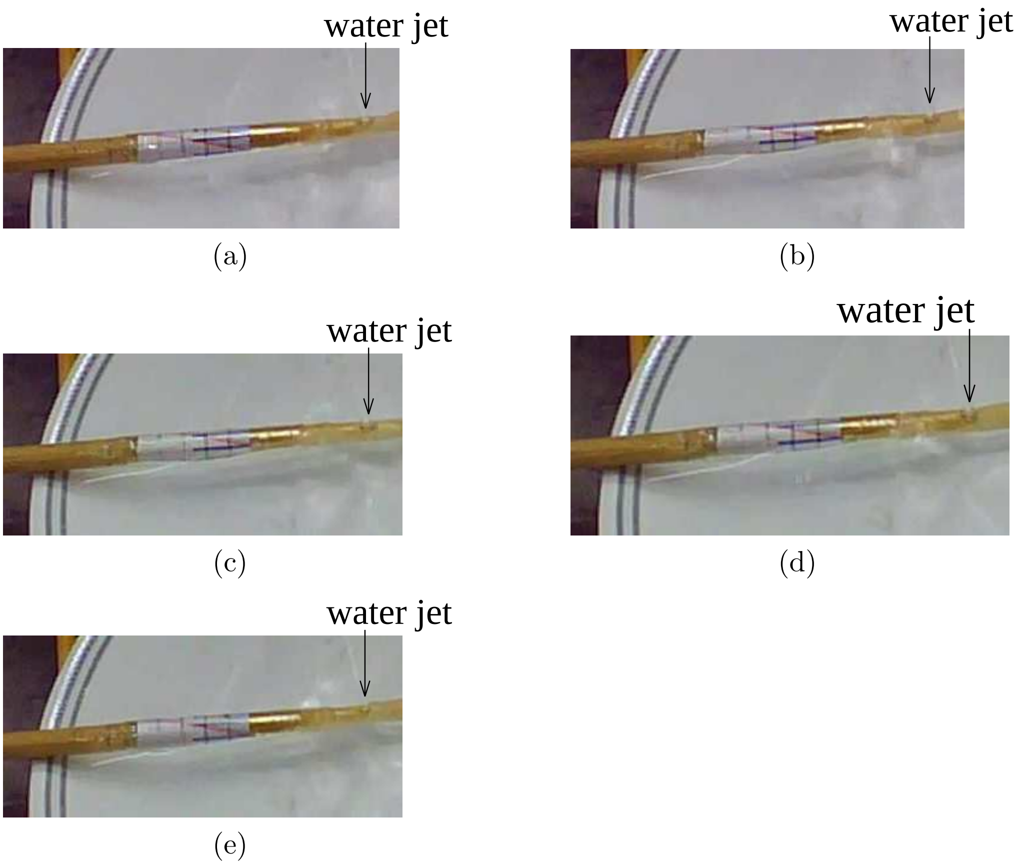

Figure 4: Time series

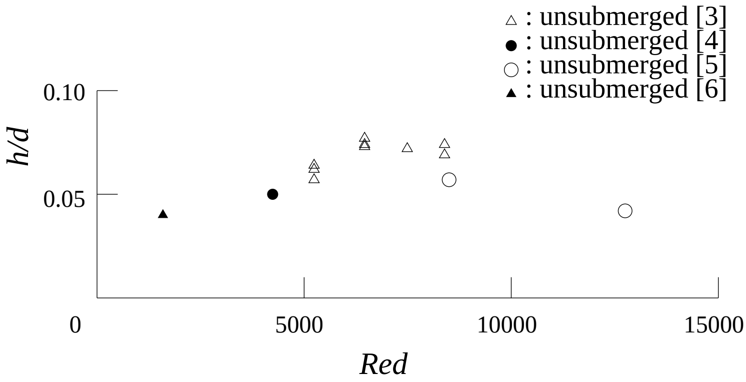

Figure 5: Film thickness

2.2. Estimation of film flow characteristics

In case of unsubmerged flow, the thickness h of film flow produced by oblique or normal impingement of circular liquid jet on a flat plane is roughly uniform and its dependency on a Reynolds number, $Red \equiv u_j\ d/\nu$, is shown in Fig. 5.

In case of submerged flow, as a measure of film thickness momentum thickness δ_2 can be used such that $\delta_2 \equiv \displaystyle\int_0^\infty \frac{u}{u_{\max}}\left(1-\frac{u}{u_{\max}}\right)dz$ (z: coordinate normal to the plane, u: velocity component parallel to the surface). Some experimental data computed from references are $\delta_2/d = 0.046$ ([7], air), 0.077 ([8], water), 0.051 ([9], water), 0.044 ([9], water), which is thicker than that in unsubmerged flow, $\delta_2/d \approx (2/15) \times (h/d)$.

Independent of unsubmerged or submerged impingement cases of circular jet for plane or cylindrical surfaces, the flow in the film flow region can be given by two-dimensional potential flow. Consider a coordinate (r, φ) mapped on a cylindrical surface from a plane polar coordinate as in Fig. 6.

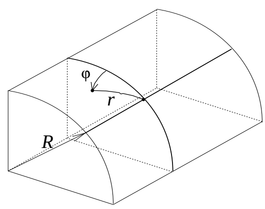

Figure 6: Coordinate system

The point $r = 0$ is the impinging point, and $R \to +\infty$ corresponds to a pure plane case. The governing equation of the potential function ϕ is

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{x}{R^2}\frac{\partial}{\partial x}\right)\phi = 0$$

$$x = r\cos\varphi, \quad y = r\sin\varphi$$

The zero-th general scalar potential solution, ϕ_0, and the first one, ϕ_1, as for even Fourier components of φ, i.e., the principal term of which corresponds to $\ln r$ and $r^{-1}\cos\varphi$ respectively, become

(1) $$\phi_0 = \ln r - \left(\frac{r}{R}\right)^2 \frac{1}{8}(1 + \varphi\sin 2\varphi) + O\left\{\left(\frac{r}{R}\right)^4\right\},$$

(2) $$\phi_1 = \frac{1}{r}\left[\cos\varphi + \left(\frac{r}{R}\right)^2 \frac{1}{4}\left(\varphi\sin\varphi - \frac{1}{4}\cos 3\varphi\right) + O\left\{\left(\frac{r}{R}\right)^4\right\}\right],$$

$$0 < r, \ |\varphi| < \pi.$$

Thus up to the first two components, the velocity potential function ϕ is $\phi = B_0\phi_0 + B_1\phi_1$. The coefficients B_0 and B_1 can be determined by impinging conditions and hydraulic jump conditions. For higher terms concerning the continuity of $\nabla\phi$ at $\varphi = \pm\pi$, treatment of secular terms is necessary. In case of plane flow cases, the ratio of mean radial component of velocity, u, ($= (\partial/\partial r)\phi$ in case of potential flow) to incoming jet velocity, u_j, is independent of u_j for the impingement angle $\psi = \pi/6$ (the angle between the jet direction and the normal of the plane such that $\psi = 0$ for normal impingement) and $\varphi = \pi/2$ in Fig. 7.

As for the coefficients B_0 and B_1, B_0 is independent of ψ, and B_1 is proportional to $\sin(2\psi)$ experimentally if $0 \le \psi \le \pi/4$ (unsubmerged or submerged) as in Figs 8 and 9.

In Fig. 8 theoretical curves are given by

$$u/u_j = 1.4 \times \frac{d}{r}\left(1 - 6 \times \frac{d}{r}\cos\varphi\sin 2\psi\right), \quad \varphi = \pi.$$

2.3. Relation between the velocity components

At a specified radius r for a plane impingement, let

$$u \equiv C_0 + C_1 \ \sin 2\psi \ \cos\varphi.$$

Figure 10 shows the relation between C_0 and C_1 computed from various sources ($\pi/12 \le \psi \le \pi/4$) (unsubmerged, submerged).

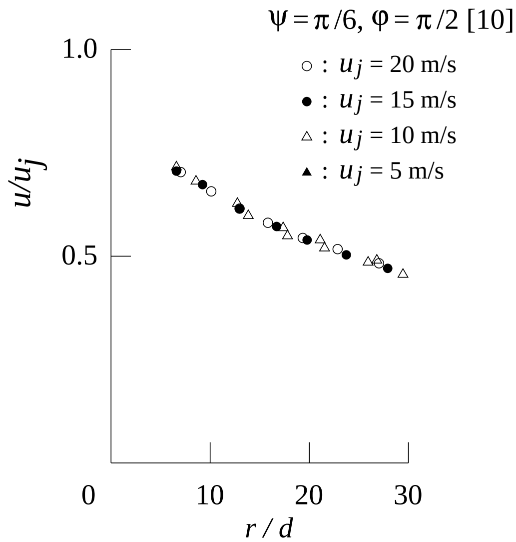

Figure 7: Velocity component

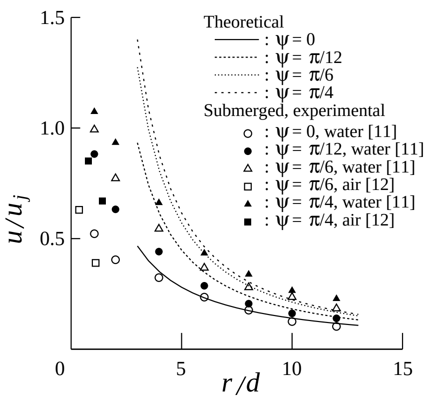

Figure 8: Velocity component, $\varphi = \pi$

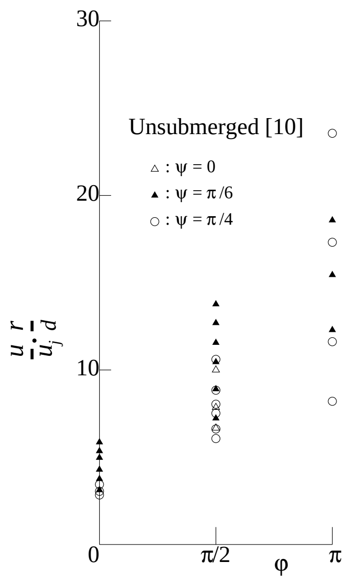

Figure 9: Velocity component

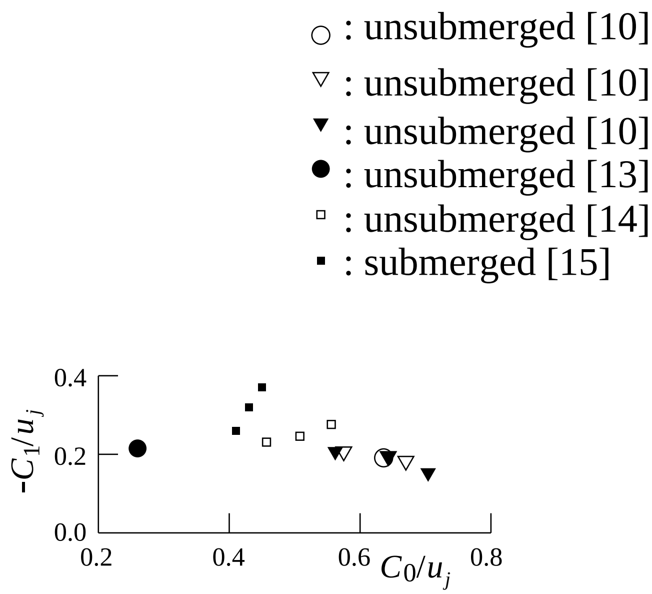

Figure 10: Fourier components

2.4. Torque contribution from outgoing film velocity

For the current case the jet diameter, d, at the impinging location satisfies $d \ll r_0$, so that the effect of the curvature at the surface can be ignored. Thus the torque contribution estimated at $r = r$ is

$$-\rho \, r_0 \, r \, h \int_0^{2\pi} u^2 \cos\varphi \, d\varphi \ ,$$

where u: radial component of velocity, h : uniform film thickness, ρ: density of water. Since in a good approximation,

$$ru = \text{const.} + A^* \cos\varphi \sin 2\psi \ ,$$

$$-\rho \, r_0 r h \int_0^{2\pi} u^2 \cos\varphi d\varphi = -\rho Q r_0 \frac{A^*}{r} \sin 2\psi,$$

where $Q = \text{const.} \times 2\pi h$.

2.5. Motion of the cylinder subject to water jet

Under a no-slip condition, rotation angle of the cylinder θ and impinging angle ψ is connected as $\theta = \sin\psi$, assuming $|\psi| < 1$ irrespective of time. Thus the equation of motion of the cylinder (t : time) is

$$I\,\frac{d^2\theta}{dt^2} = -P\sin\psi - A\sin 2\psi\ , \tag{3}$$

which is valid even if $\theta < 0$ through $\psi \equiv \sin^{-1}\theta$.

$$P = \rho Q r_0 u_j,$$
$$A = \rho Q r_0 A^*/r.$$

Equation (3) can be integrated once to give

$$I\left(\frac{d\theta}{dt}\right)^2 = -P\left(\theta^2 - {\theta_0}^2\right) + \frac{4}{3}A\left(\sqrt{1-\theta^2}^3 - \sqrt{1-{\theta_0}^2}^3\right), \tag{4}$$

which has a periodic solution if $0 < \theta_0 < 1$, and if $0 > A/P > -1/2$ and $\theta_0 \ll 1$, then the period T becomes

$$T \approx 4\sqrt{\frac{I}{P\,(1+2A/P)}} \qquad K\left(\frac{\theta_0}{2}\sqrt{\frac{-A/P}{1+2A/P}}\right), \tag{5}$$

where $K(\cdot)$: complete elliptic integral of the first kind. Actually

$$\frac{A}{P} = \frac{A^*}{r\,u_j} \approx \frac{\overline{u}}{u_j}\cdot\frac{C_1}{C_0} < 0,$$

$\overline{u}$: mean value of radial component of velocity (with respect to φ and film thickness). In case of $M = 4\times 10^{-3}$ kg, $r_0 = 3\times 10^{-3}$ m, $Q = 9.53\times 10^{-6}$ m^3/s, $u_j = 2.48$ m/s and $\rho = 10^3$ kg/m^3, $6\pi\sqrt{I/P} = 0.097$ s, $K(0) = \pi/2$ with $I \equiv 0.945\times\left(\frac{1}{2}Mr_0^2\right)$. Assuming $\overline{u}/u_j = 0.7$, $C_1/C_0 = -0.5$, $\theta_0 \ll 1$, then

$$T = 0.097\times\sqrt{\frac{1}{0.3}}\ (\mathrm{s}) = 0.177\ (\mathrm{s}).$$

Figures 11 and 12 show angle θ of rotation and horizontal translation z^* with time t respectively. Diameter of wooden cylinder : 6mm $\equiv 2\times r_0$; water flow rate Q : 9.53 cm^3 /s at 11 °C, $h_3 = 315$ mm, θ, z^*, t: inclusive of an arbitrary constant.

Figures 11 and 12 show approximate oscillation of the cylinder in cooperation with the assumption, $z^*/r_0 \approx \theta$ if the origin of the coordinates is selected suitably, which is valid for small amplitude. The period of oscillation observed is approximately 0.2 s in good agreement with the above computation.

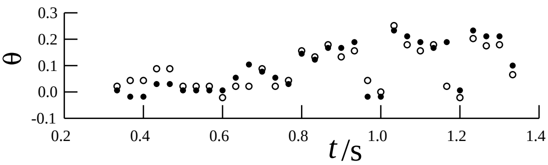

Figure 11: Angle of rotation with time

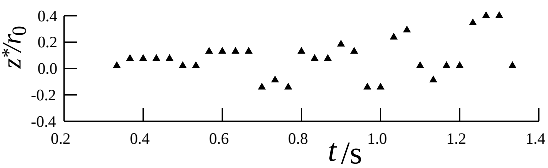

Figure 12: Translation with time

3. Conclusion

Oscillation of a circular cylinder subject to water jet is analyzed, taking into outgoing film flow. Observed translation and rotation of the cylinder is in good agreement with the analysis.

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