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THE MULTIPLIER EXTENSIONS OF ADMISSIBLE VECTOR MODULES AND THE MIKUSIŃSKI-TYPE CONVERGENCES

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This paper is devoted primarily to initiating an abstract theory for the convolution calculus For this, the concept of quotient algebras and the type I convergence of Mikusiński are generalized. Thus, a complete algebraic foundation and some preliminary considerations to a topological foundation of an abstract convolution calculus are obtained.

1. The multiplier extensions of admissible vector modules.

Definition 1.1. *Suppose that*

(I) $\mathcal{A}$ *is a commutative algebra over* $\mathbf{K}$ $(=\mathbf{R}$ *or* $\mathbf{C})$;

(II) $\mathcal{B}$ *is a vector space over* $\mathbf{K}$ *and an* $\mathcal{A}$*-module such that* $\alpha(f*\varphi)=(\alpha f)*\varphi=f*(\alpha\varphi)$ *for all* $\alpha\in\mathbf{K}$, $f\in\mathcal{B}$ *and* $\varphi\in\mathcal{A}$;

(III) $\mathcal{A}$ *is a subspace and a submodule of* $\mathcal{B}$ *such that for every* $0\neq f\in\mathcal{B}$ *there exists* $\varphi\in\mathcal{A}$ *such that* $f*\varphi\neq 0$.

Remark 1.2. Since $\mathcal{A}$ is commutative, we may use the usual convention that $\varphi*f=f*\varphi$ if $\varphi\in\mathcal{A}$ and $f\in\mathcal{B}$.

We shall call such modules admissible vector modules. Examples for admissible convolution vector modules can be found in [10, 13, 14, 15, 3, 8].

Definition 1.3. $D\subset\mathcal{A}$ *is said to be a divisor of zero in* $\mathcal{B}$ *if there exists* $0\neq f\in\mathcal{B}$ *such that* $f*\varphi=0$ *for all* $\varphi\in D$.

If $\varphi\in\mathcal{A}$ *and* $\{\varphi\}$ *is a divisor of zero in* $\mathcal{B}$, *then we simply say that* φ *is a divisor of zero in* $\mathcal{B}$.

Definition 1.4. *Let* $\mathfrak{M}=\mathfrak{M}(\mathcal{A},\mathcal{B})$ *be the set of all functions*

$$F: D_F\subset\mathcal{A}\to\mathcal{B}$$

such that D_F *is not a divisor of zero in* $\mathcal{B}$ *and* $F(\varphi)*\psi=\varphi*F(\psi)$ *for all* φ, $\psi\in D_F$. *Moreover, for* $F\in\mathfrak{M}$ *let*

$$\overline{F}=\{(\varphi,f)\in\mathcal{A}\times\mathcal{B}:\forall\sigma\in D_F: f*\sigma=\varphi*F(\sigma)\}.$$

Lemma 1.5. *Let* $F\in\mathfrak{M}$. *Then*

(i) $\overline{F}\in\mathfrak{M}$;

(ii) $D_{\overline{F}}$ *is an ideal in* $\mathcal{A}$;

(iii) $\overline{F}$ *is a vector space and a module homomorphism of* $D_{\overline{F}}$ *into* $\mathcal{B}$.

Proof. If (φ,f_1), $(\varphi,f_2)\in\overline{F}$, then $f_1*\sigma=\varphi*F(\sigma)$ and $f_2*\sigma=\varphi*F(\sigma)$, i. e.

$$(f_1-f_2)*\sigma=0$$

for all $\sigma\in D_F$. Hence, since D_F is not a divisor of zero in $\mathcal{B}$, it follows that $f_1=f_2$. Consequently, $\overline{F}$ is a function.

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If $\varphi, \psi \in D_{\overline{F}}$, then $\overline{F}(\varphi) * \sigma = \varphi * F(\sigma)$ and $\overline{F}(\psi) * \sigma = \psi * F(\sigma)$, and hence

$$(\overline{F}(\varphi) * \psi - \varphi * \overline{F}(\psi)) * \sigma = 0$$

for all $\sigma \in D_F$. This implies that $\overline{F}(\varphi) * \psi = \varphi * \overline{F}(\psi)$. Now, since $D_F \subset D_{\overline{F}}$, it is clear that (i) holds.

Similar calculations can be used to prove (ii) and (iii). For example, if $\varphi \in D_{\overline{F}}$ and $\psi \in \mathcal{A}$, then we have

$$(\overline{F}(\varphi) * \psi) * \sigma = (\varphi * \psi) * F(\sigma)$$

for all $\sigma \in D_F$, which implies that $(\varphi * \psi, \overline{F}(\varphi) * \psi) \in \overline{F}$, i. e. $\varphi * \psi \in D_{\overline{F}}$ and $\overline{F}(\varphi * \psi) = \overline{F}(\varphi) * \psi$.

Definition 1.6. *Let* $\mathfrak{M} = \mathfrak{M}(\mathcal{A}, \mathcal{B}) = \{\overline{F} : F \in \mathfrak{M}\}$ *and*

$$\mathfrak{N} = \mathfrak{N}(\mathcal{A}, \mathcal{B}) = \{\overline{\Phi} : \Phi \in \mathfrak{M}, \Phi(D_\Phi) \subset \mathcal{A}\}.$$

Moreover, for $F, G \in \mathfrak{M}$ *and* $\Phi \in \mathfrak{N}$, *let* $F \oplus G = \overline{F + G}$ *and* $F * \Phi = \overline{F \circ \Phi}$.

Theorem 1.7. $\mathfrak{N}$ *is a commutative ring with unity, and* $\mathfrak{M}$ *is a unitial* $\mathfrak{N}$*-module.*

Proof. In the proof we shall often use the following obvious facts:

(a) If $D_1, D_2 \subset \mathcal{A}$ are not divisors of zero in $\mathcal{B}$, then $D_1 * D_2$ is not a divisor of zero in $\mathcal{B}$. Moreover, if in addition $D_1 * \mathcal{A} \subset D_1$ and $D_2 * \mathcal{A} \subset D_2$, then $D_1 \cap D_2$ is also not a divisor of zero in $\mathcal{B}$.

(b) If $\Phi \in \mathfrak{N}$, then $\Phi^{-1}(\mathcal{A})$ is an ideal in $\mathcal{A}$ which is not a divisor of zero in $\mathcal{B}$.

(c) If $F, G \in \mathfrak{M}$, $D \subset D_F \cap D_G$ is not a divisor of zero in $\mathcal{B}$ and $F(\varphi) = G(\varphi)$ for all $\varphi \in D$, then $F = G$.

The proof of the theorem may be carried out in three steps:

The first step is to prove that if $F, G \in \mathfrak{M}$ and $\Phi, \Psi \in \mathfrak{N}$, then $F \oplus G$, $F * \Phi \in \mathfrak{M}$ and $\Phi \oplus \Psi, \Phi * \Psi \in \mathfrak{N}$. For example, we prove that $F * \Phi \in \mathfrak{M}$. Clearly, we have $(F \circ \Phi)(\varphi * \psi) = F(\Phi(\varphi * \psi)) = F(\varphi * \Phi(\psi)) = F(\varphi) * \Phi(\psi)$ for all $\varphi \in D_F$ and $\psi \in \Phi^{-1}(\mathcal{A})$. Hence, it follows that $D_F * \Phi^{-1}(\mathcal{A}) \subset D_{F \circ \Phi}$. Consequently, $D_{F \circ \Phi}$ is not a divisor of zero in $\mathcal{B}$. Moreover, we have $(F \circ \Phi)(\varphi) * \psi = F(\Phi(\varphi)) * \psi = F(\Phi(\varphi) * \psi) = F(\varphi * \Phi(\psi)) = \varphi * F(\Phi)(\psi)) = \varphi * (F \circ \Phi)(\psi)$ for all $\varphi, \psi \in D_{F \circ \Phi}$. Thus $F \circ \Phi \in \mathfrak{M}$ and so $F * \Phi = \overline{F \circ \Phi} \in \mathfrak{M}$.

The second step is to prove the required commutative, associative and distributive laws for $\oplus$ and $*$. For example, we prove that if $F \in \mathfrak{M}$ and $\Phi, \Psi \in \mathfrak{N}$, then $F * (\Phi \oplus \Psi) = F * \Phi \oplus F * \Psi$. Clearly we have $(F * (\Phi \oplus \Psi))(\varphi) = F((\Phi \oplus \Psi)(\varphi)) = F(\Phi(\varphi) + \Psi(\varphi)) = F(\Phi(\varphi)) + F(\Psi(\varphi)) = (F * \Phi)(\varphi) + (F * \Psi)(\varphi) = (F * \Phi \oplus F * \Psi)(\varphi)$ for all $\varphi \in D_F * (\Phi^{-1}(\mathcal{A}) \cap \Psi^{-1}(\mathcal{A}))$. Hence, since $D_F * (\Phi^{-1}(\mathcal{A}) \cap \Psi^{-1}(\mathcal{A}))$ is not a divisor of zero in $\mathcal{B}$, it follows that $F * (\Phi \oplus \Psi) = F * \Phi \oplus F * \Psi$.

The third step is to prove the existence of certain special elements in $\mathfrak{M}$. Let 0 and 1 be the functions defined on $\mathcal{A}$ by $0(\varphi) = 0$ and $1(\varphi) = \varphi$. Then $0, 1 \in \mathfrak{N}$ and $F \oplus 0 = \overline{F + 0} = \overline{F} = F$, $F \oplus (-F) = \overline{F + (-F)} = \overline{0 \mid D_F} = 0$, $F * 1 = \overline{F \circ 1} = \overline{F} = F$, for all $F \in \mathfrak{M}$.

Definition 1.8. *For* $\alpha \in \mathbf{K}$, *let* F_α *be the function defined on* $\mathcal{A}$ *by* $F_\alpha(\varphi) = \alpha\varphi$.

Proposition 1.9. *The mapping defined on* $\mathbf{K}$ *by* $\alpha \to F_\alpha$ *is a field isomorphism of* $\mathbf{K}$ *into* $\mathfrak{N}$.

Proof. Left to the reader.

Definition 1.10. *For $\alpha \in \mathbf{K}$ identify α with F_α by writing $\alpha = F_\alpha$.*

Remark 1.11. After this embedding, with the convention that $\Phi * F = F * \Phi$ if $\Phi \in \mathfrak{N}$ and $F \in \mathfrak{M}$, $\mathfrak{M}$ becomes an admissible unitial $\mathfrak{N}$-vector module.

Definition 1.12. *For $f \in \mathcal{B}$ let F_f be the function defined on $\mathcal{A}$ by $F_f(\varphi) = f * \varphi$.*

Proposition 1.13. *The mapping defined on $\mathcal{B}$ by $f \to F_f$ is a vector space isomorphism of $\mathcal{B}$ into $\mathfrak{M}$ such that $F_{f*\varphi} = F_f * F_\varphi$ for all. $f \in B$ and $\varphi \in A$.*

Proof. Left to the reader.

Definition 1.14. *For $f \in \mathcal{B}$, identify f with F_f by writing $f = F_f$.*

Remark 1.15. After this embedding $\mathfrak{M}$ may also be considered as an admissible $\mathcal{A}$-vector module.

Proposition 1.16. *Let $F \in \mathfrak{M}$ and $\varphi \in \mathcal{A}$. Then $F * \varphi \in \mathcal{B}$ if and only if $\varphi \in D_F$. Moreover, if $\varphi \in D_F$, then $F * \varphi = F(\varphi)$.*

Proof. Suppose first that $\varphi \in D_F$. Then

$$(F * \varphi)(\psi) = (F * F_\varphi)(\psi) = F(F_\varphi(\psi)) = F(\varphi * \psi) = F(\varphi) * \psi = F_{F(\varphi)}(\psi)$$

for all $\psi \in \mathcal{A}$. This implies that $F * \varphi = F_{F(\varphi)} = F(\varphi) \in \mathcal{B}$.

Suppose now that $F * \varphi \in \mathcal{B}$. Then $(F * \varphi) * \sigma = \varphi * (F * \sigma) = \varphi * F(\sigma)$ for all $\sigma \in D_F$ Hence it follows that $(\varphi, F * \varphi) \in \bar{F} = F$ and so $\varphi \in D_F$.

Theorem 1.17. *Let $\Phi \in \mathfrak{N}$. Then the following conditions are equivalent*

(i) *Φ is invertible in $\mathfrak{N}$;*

(ii) *Φ is not a divisor of zero in $\mathfrak{M}$;*

(iii) *$\Phi(D_\Phi) \cap \mathcal{A}$ is not a divisor of zero in $\mathcal{B}$.*

Proof. It is clear that (i) implies (ii). Now suppose that (ii) holds and $f \in \mathcal{B}$ such that $f * \Phi(\sigma) = 0$ for all $\sigma \in \Phi^{-1}(\mathcal{A})$. Then we have

$$(F_f * \Phi)(\sigma) = F_f(\Phi(\sigma)) = f * \Phi(\sigma) = 0$$

for all $\sigma \in \Phi^{-1}(\mathcal{A})$. Since $\Phi^{-1}(\mathcal{A})$ is not a divisor of zero in $\mathcal{B}$, this implies tha $F_f * \Phi = 0$. Hence by (ii) it follows that $F_f = 0$, i. e., $f = 0$.

Finally suppose that (iii) holds. If $\Phi(\varphi) = \Phi(\psi)$, then a simple calculationt shows that $(\varphi - \psi) * \Phi(\sigma) = 0$ for all $\sigma \in D_\Phi$. Hence by (iii) it follows that $\varphi = \psi$. Consequently Φ is injective. Moreover, since

$$\Phi^{-1}(\varphi) * \psi = \Phi^{-1}(\varphi) * \Phi(\Phi^{-1}(\psi)) = \Phi(\Phi^{-1}(\varphi)) * \Phi^{-1}(\psi) = \varphi * \Phi^{-1}(\psi)$$

for all $\varphi, \psi \in \Phi(D_\Phi) \cap \mathcal{A}$, we have $\Phi^{-1} \mid \Phi(D_\Phi) \cap \mathcal{A} \in \mathfrak{M}$. Thus

$$(\Phi * \overline{\Phi^{-1} \mid \Phi(D_\Phi) \cap \mathcal{A}})(\varphi) = \Phi(\Phi^{-1}(\varphi)) = \varphi = 1(\varphi)$$

for all $\varphi \in \Phi(D_\Phi) \cap \mathcal{A}$, whence $\Phi * \overline{\Phi^{-1} \mid \Phi(D_\Phi) \cap \mathcal{A}} = 1$.

Remark 1.18. In several important special cases there are elements in $\mathcal{A}$ which are not divisors of zero in $\mathcal{B}$. In these cases we prefer to use the following notation.

Definition 1.19. *If $\varphi \in \mathcal{A}$ is not a divisor of zero in $\mathcal{B}$ and $f \in \mathcal{B}$ then let $f/\varphi = \overline{\{(\varphi, f)\}}$.*

Remark 1.20. Observe that, if $F \in \mathfrak{M}$ and $\varphi \in D_F$ such that φ is not a divisor of zero in $\mathcal{B}$, then $F = F(\varphi)/\varphi$.

Theorem 1.21. *Suppose that $\varphi, \psi \in \mathcal{A}$ are not divisors of zero in $\mathcal{B}$ and let $f, g \in \mathcal{B}$ and $\chi \in \mathcal{A}$. Then $f/\varphi = g/\psi$ iff $f * \psi = \varphi * g$ and moreover*

$$\frac{f}{\varphi} \oplus \frac{g}{\psi} = \frac{f*\psi + \varphi * g}{\varphi * \psi} \quad \textit{and} \quad \frac{f}{\varphi} * \frac{\chi}{\psi} = \frac{f*\chi}{\varphi * \psi}.$$

Proof. If $f/\varphi = g/\psi$, then $(\varphi, f) \in f/\varphi = g/\psi = \overline{\{(\psi, g)\}}$, and so $f*\psi = \varphi * g$. Conversely, if $f*\psi = \varphi * g$, then

$$\frac{f}{\varphi}(\varphi * \psi) = \frac{f}{\varphi}(\varphi) * \psi = f * \psi = \varphi * g = \varphi * \frac{g}{\psi}(\psi) = \frac{g}{\psi}(\varphi * \psi).$$

Hence, since $\varphi * \psi$ is not a divisor of zero in $\mathcal{B}$, it follows that $f/\varphi = g/\psi$.

Finally the equalities

$$\left(\frac{f}{\varphi} \oplus \frac{g}{\psi}\right)(\varphi * \psi) = \frac{f}{\varphi}(\varphi * \psi) + \frac{g}{\psi}(\varphi * \psi) = \frac{f}{\varphi}(\varphi) * \psi + \varphi * \frac{g}{\psi}(\psi)$$
$$= f * \psi + \varphi * g = \frac{f*\psi + \varphi * g}{\varphi * \psi}(\varphi * \psi)$$

and

$$\left(\frac{f}{\varphi} * \frac{\chi}{\psi}\right)(\varphi * \psi) = \frac{f}{\varphi}\left(\frac{\chi}{\psi}(\varphi * \psi)\right) = \frac{f}{\varphi}\left(\varphi * \frac{\chi}{\psi}(\psi)\right) = \frac{f}{\varphi}(\varphi * \chi) = \frac{f}{\varphi}(\varphi) * \chi = f * \chi = \frac{f*\chi}{\varphi * \psi}(\varphi * \psi)$$

imply the corresponding rules for $\oplus$ and $*$.

Remark 1.22. We shall call the $\mathfrak{N}$-module $\mathfrak{M}$ the multiplier extension of the admissible $\mathcal{A}$-vector module $\mathcal{B}$.

The elements of $\mathfrak{M}$ may be termed as quotient multipliers. If the elements of $\mathcal{B}$ are functions and $*$ is a certain kind of convolutions, then the elements of $\mathfrak{M}$ will also be called generalized functions.

2. The Mikusiński-type convergences.

Definition 2.1. *Suppose that*

(IV) $L_{\mathcal{A}}$-lim *is an L-convergence on $\mathcal{A}$ such that the algebra operations are sequentially continuous;*

(V) $L_{\mathcal{B}}$-lim *is an L-convergence on $\mathcal{B}$ such that the vector space and the module operations are sequentially continuous;*

(VI) $L_{\mathcal{A}}$-lim *is stronger than the L-convergence induced on $\mathcal{A}$ by* $L_{\mathcal{B}}$-lim.

Remark 2.2. A relation $L\text{-lim} \subset X^{\mathbf{N}} \times X$ is called an L-convergence on X [5] if

(1) $x \in L\text{-lim}_{n\to\infty} x$ for all $x \in X$,

(2) $x \in L\text{-lim}_{n\to\infty} x_n$ implies that $x \in L\text{-lim}_{n\to\infty} x_{k_n}$ for any subsequence $(x_{k_n})_{n=1}^{\infty}$ of $(x_n)_{n=1}^{\infty}$,

(3) $x, x' \in L\text{-lim}_{n\to\infty} x_n$ implies that $x = x'$. By (3), L-lim is a function thus we may write $x = L-\lim_{n\to\infty} x_n$ instead of $x \in L\text{-lim}_{n\to\infty} x_n$.

Definition 2.3. *Let* $L_{\mathfrak{N}}\text{-lim} \subset \mathfrak{N}^{\mathbf{N}} \times \mathfrak{N}$ *be such that for* $(\Phi_n)_{n=1}^{\infty} \in \mathfrak{N}^{\mathbf{N}}$ *and* $\Phi \in \mathfrak{N}$, $\Phi \in L_{\mathfrak{N}}\text{-lim}_{n\to\infty} \Phi_n$ *iff*

$$\{\varphi \in \bigcap_{n=1}^{\infty} \Phi_n^{-1}(\mathcal{A}) \cap \Phi^{-1}(\mathcal{A}) : L_{\mathcal{A}}\text{-}\lim_{n\to\infty} \Phi_n(\varphi) = \Phi(\varphi)\}$$

is not a divisor of zero in $\mathcal{B}$.

Let $L_{\mathfrak{M}}$-lim $\subset \mathfrak{M}^{\mathbf{N}} \times \mathfrak{M}$ *be such that for* $(F_n)_{n=1}^{\infty} \in \mathfrak{M}^{\mathbf{N}}$ *and* $F \in \mathfrak{M}$, $F \in L_{\mathfrak{M}}\text{-}\lim_{n\to\infty} F_n$ *iff*

$$\{\varphi \in \bigcap_{n=1}^{\infty} D_{F_n} \cap D_F : L_{\mathcal{B}}\text{-}\lim_{n\to\infty} F_n(\varphi) = F(\varphi)\}$$

is not a divisor of zero in $\mathcal{B}$.

Theorem 2.4. (i) $L_{\mathfrak{N}}$-lim *is an L-convergence on* $\mathfrak{N}$ *such that the ring operations are sequentially continuous.*

(ii) $L_{\mathfrak{M}}$-lim *is an L-convergence on* $\mathfrak{M}$ *such that the module operations are sequentially continuous.*

(iii) *The usual convergence on* **K** *and* $L_{\mathcal{A}}$-lim *are stronger than the L-convergences induced by* $L_{\mathfrak{N}}$-lim *on* **K** *and* $\mathcal{A}$, *respectively. Moreover* $L_{\mathcal{B}}$-lim *and* $L_{\mathfrak{N}}$-lim *are stronger than the L-convergences induced by* $L_{\mathfrak{M}}$-lim *on* $\mathcal{B}$ *and* $\mathfrak{N}$, *respectively.*

Proof. It is clear that $L_{\mathfrak{N}}$-lim and $L_{\mathfrak{M}}$-lim satisfy (1) and (2). Moreover, a simple calculation shows that (iii) holds.

If $F_{(i)} \in L_{\mathfrak{M}}\text{-}\lim_{n\to\infty} F_n$, $i=1, 2$, then

$$D_i = \{\varphi \in \bigcap_{n=1}^{\infty} D_{F_n} \cap D_F : L_{\mathcal{B}}\text{-}\lim_{n\to\infty} F_n(\varphi) = F_{(i)}(\varphi)\}$$

is not a divisor of zero in $\mathcal{B}$. Moreover, it is clear that D_i is an ideal in $\mathcal{A}$. Thus $D_1 \cap D_2$ is also not a divisor of zero in $\mathcal{B}$. Furthermore we have $F_{(1)}(\varphi) = F_{(2)}(\varphi)$ for all $\varphi \in D_1 \cap D_2$. This implies that $F_{(1)} = F_{(2)}$. Consequently $L_{\mathfrak{M}}$-lim is a function. Hence by (iii) it is clear that $L_{\mathfrak{N}}$-lim is also a function.

Finally we must show the sequential continuity of the corresponding operations. For example, we show that the multiplication $*: \mathfrak{M} \times \mathfrak{N} \to \mathfrak{M}$ is sequentially continuous. For this suppose that $L_{\mathfrak{M}}\text{-}\lim_{n\to\infty} F_n = F$ and $L_{\mathfrak{N}}\text{-}\lim_{n\to\infty} \Phi_n = \Phi$. Then

$$D = \{\varphi \in \bigcap_{n=1}^{\infty} D_{F_n} \cap D_F : L_{\mathcal{B}}\text{-}\lim_{n\to\infty} F_n(\varphi) = F(\varphi)\}$$

and

$$E = \{\varphi \in \bigcap_{n=1}^{\infty} \Phi_n^{-1}(\mathcal{A}) \cap \Phi^{-1}(\mathcal{A}) : L_{\mathcal{A}}\text{-}\lim_{n\to\infty} \Phi_n(\varphi) = \Phi(\varphi)\}$$

are not divisors of zero in $\mathcal{B}$. Moreover by (v) we have

$$L_{\mathcal{B}}\text{-}\lim_{n\to\infty} (F_n * \Phi_n)(\varphi * \psi) = L_{\mathcal{B}}\text{-}\lim_{n\to\infty} F_n(\varphi) * \Phi_n(\psi) = F(\varphi) * \Phi(\psi) = (F * \Phi)(\varphi * \psi)$$

for all $\varphi \in D$ and $\psi \in E$. Hence since $D * E$ is not a divisor of zero in $\mathcal{B}$ it follows that

$$L_{\mathfrak{M}}\text{-}\lim_{n\to\infty} F_n * \Phi_n = F * \Phi.$$

Remark 2.5. We shall call the L-convergences $L_{\mathfrak{N}}$-lim and $L_{\mathfrak{M}}$-lim the Mikusiński-type convergences on $\mathfrak{N}$ and $\mathfrak{M}$, respectively, since they are natural generalizations of the type I convergence of Mikusiński which is commonly used in the convolution calculus [10].

The type I convergence is not topological [1]. However, in case of periodic convolution the Mikusiński-type convergence is metrizable [3, 16, 17].

Despite of the fact that the Mikusiński-type convergences in general are not topological, it seems reasonable to consider $\mathfrak{N}$ and $\mathfrak{M}$ topologized by $L_{\mathfrak{N}}$-lim and $L_{\mathfrak{M}}$-lim, respectively [1, 2, 5]. (If L-lim is an L-convergence on X, then $G\subset X$ is called open if $x\in G$ and $x=L\text{-}\lim_{n\to\infty} x_n$ imply that $x_n\in G$ for all sufficiently large n.)

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