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ON THE CAUCHY PROBLEM FOR SYSTEM OF FIRST ORDER PATRIAL DIFFERENTIAL-FUNCTIONAL EQUATIONS

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Consider the Cauchy problem

$$\text{a)}\quad \begin{aligned} &z_x^{(i)}(x, Y)=f^{(i)}(x, Y, Z(x, Y), z(\,.\,), z_Y^{(i)}(x, Y)),\\ &z^{(i)}(x, Y)=a^{(i)}(x, Y) \quad \text{for} \quad (x, Y)\in E_0, \end{aligned} \qquad i=1, \dots, m,$$

where $a^{(i)}$, $i=1, \dots, m$, are given functions defined on the initial set E_0, $Z(x, Y)=(z^{(1)}(x, Y)$ $\dots, z^{(m)}(x, Y))$. $Z(\cdot)=(z^{(1)}(\cdot), \dots, z^{(m)}(\cdot))$ and $z_Y^{(i)}(x, Y)=(z_{y_1}^{(i)}(x, Y), \dots, z_{y_n}^{(i)}(x, Y))$. We prove theorems concerning the estimation of solutions of partial differential-functional equations (a) and formulate a criterion of uniqueness of solutions of (a). This will generalize the results of Zima (1969) as well as of classical results concerning first order partial differential equations. If we assume in the theorems of this paper that the right hand sides of the systems of equations and the right hand sides of the suitable comparative systems do not include a functional argument, then we obtain more general theorems than those of Szarski (1967) and more exact than those of Laksmikantham (1969).

Equations with partial derivatives of the first order have the following property: the problem of existence of their solutions is strictly connected with the problem of solving of systems of ordinary differential equations. The investigations of properties of solutions of partial equations of the first order or their systems is also strictly connected with properties of characteristic systems of ordinary differential equations [1; 5; 8—10; 15; 17; 20; 21; 24]. Ordinary differential inequalities find numerous applications in the theory of first order partial differential equations. Such problems as estimations of solutions of partial equations, estimations of the domain of the solution, estimation of the difference between two solutions, criterions of uniqueness, are classical examples however not only ones [16; 23; 25].

A similar role in the theory of differential-functional equations with partial first order derivatives is played by differential-functional inequalities with ordinary derivatives. Some results in this field for partial equations with a retarded argument can be found in papers [29; 30].

The problems of existence of solutions of differential-functional equations were investigated by many authors. Among others, in papers [6; 7] the author tried to apply the method of characteristics to equations with a retarded argument. Certain types of differential-integral equations were considered in [27]. The paper [26] contains sufficient conditions for the existence of solutions of generalized Cauchy problem for differential-functional equations. In papers [11—13] the method of successive approximations is considered for partial equations with a retarded argument (see also [3; 4]). The paper [1] (see

SERDICA Bulgaricae mathematicae publicationes. Vol. 5, 1979, p. 327—339.

also [19]) contains sufficient conditions for the existence and uniqueness of continuous solutions and solutions of class C^1 of an initial-boundary problem for almost linear partial differential-functional systems in two independent variables.

In this paper we shall demonstrate theorems concerning the estimation of solutions of partial differential-functional equations with the help of solutions of systems of ordinary differential-functional equations. The theory of differential-functional inequalities will be applied for the estimation of the difference between solutions of two systems of partial differential-functional equations and to the formulation of a criterion of uniqueness of solutions of such systems. This will be a generalization of the results published in [30] as well as of classical results concerning first order equations with partial derivatives. If we assume in the theorems of this paper that the right hand sides of the systems with partial derivatives and the right hand sides of the suitable comparative systems do not include a functional argument, then we obtain more general theorems than those results in [25, Chapter VII] and more exact than those in [16, Chapter IX].

Let $C(E_0 \cup E, R)$ be a class of continuous functions from $E_0 \cup E$ into R where $R=(-\infty, +\infty)$ and

$$E_0=\{(x, Y): x_0-\tau_0 \leq x \leq x_0,\ \tau_0 \geq 0,\ Y=(y_1, \ldots, y_n),$$
$$\bar{r}_i(x) \leq y_i \leq \bar{s}_i(x),\quad i=1, \ldots, n\},$$
$$E=\{(x, Y):\ x_0 \leq x < x_0+a,\ a>0,\ r_i(\tilde{x}) \leq y_i \leq s_i(\tilde{x}),\quad i=1, \ldots, n\}.$$

Assume that

(i) $\bar{r}_i$ and $\bar{s}_i$, $i=1, \ldots, n$, are continuous functions on $[x_0-\tau_0, x_0]$ and $\bar{r}_i(x) \leq \bar{s}_i(x)$, $x \in [x_0-\tau_0, x_0]$,

(ii) $\tilde{r}_i$ and $\tilde{s}_i$, $i=1, \ldots, n$, are of class C^1 on $[x_0, x_0+a)$ and $\tilde{r}_i(x) < \tilde{s}_i(x)$. for $x \in [x_0, x_0+a)$, $\tilde{r}_i(x_0)=\bar{r}_i(x_0)$, $\tilde{s}_i(x_0)=\bar{s}_i(x_0)$.

Elements of $C(E_0 \cup E, R)$ will be denoted by $z(\cdot)$, $u(\cdot)$, $v(\cdot)$ and the like. Let $C^m(E_0 \cup E, R)$ be a set of all vector functions $U(\cdot)=(u^{(1)}(\cdot), \ldots, u^{(m)}(\cdot))$ where $u^{(i)}(\cdot) \in C(E_0 \cup E, R)$.

Let $A=(a_1, \ldots, a_p)$, $\bar{A}=(\bar{a}_1, \ldots, \bar{a}_p)$ be two points of the p-dimensional Euclidean space. We will write $A \leq \bar{A}$ if $a_i \leq \bar{a}_i$ for $i=1, \ldots, p$, and $A \underset{i}{<} \bar{A}$ if $a_i < \bar{a}_i$ for $i=1, \ldots, p$. The index i being fixed we write $A \underset{i}{\leq} \bar{A}$ if $a_j \leq \bar{a}_j$ for $j=1, \ldots, p$, and $a_i=\bar{a}_i$.

Suppose that functions $f^{(i)}$, $i=1, \ldots, m$, of the variables $(x, Y, Z, U(\cdot), Q)$, where $Z=(z^{(1)}, \ldots, z^{(m)})$, $Q=(q_1, \ldots, q_n)$, are defined on $E \times R^m \times C^m(E_0 \cup E, R) \times B$, where B is a domain which is contained in R^n.

In this paper we discuss a number of questions referring to the Cauchy problem for systems of first order partial differential-functional equations

$$(1)\qquad \begin{aligned} &z_x^{(i)}(x, Y)=f^{(i)}(x, Y, Z(x, Y), Z(\cdot), z_Y^{(i)}(x, Y)), \quad i=1, \ldots, m, \\ &z^{(i)}(x, Y)=\alpha^{(i)}(x, Y) \quad \text{for} \quad (x, Y) \in E_0, \quad i=1, \ldots, m, \end{aligned}$$

where $\alpha^{(i)}$, $i=1, \ldots, m$, are given functions defined on the initial set E_0 and $Z(x, Y)=(z^{(1)}(x, Y), \ldots, z^{(m)}(x, Y))$, $Z(\cdot)=(z^{(1)}(\cdot), \ldots, z^{(m)}(\cdot))$, $z_Y^{(i)}(x, Y)$

$=(z^{(i)}_{y_1}(x, Y), \ldots, z^{(i)}_{y_n}(x, Y))$. In particular, we will give applications of the theory of ordinary differential-functional inequalities to questions like: estimates of the solution of (1), estimates of the difference between two solutions, uniqueness criteria and continuous dependence of the solution on initial data and on the right hand sides of the system.

1. Assumptions and Lemmas. Assumption H_1. *Suppose that*

1. $\sigma=(\sigma_1, \ldots, \sigma_m)$ *is a non-negative and continuous function of the variables* $(t, Z, W(\cdot))$ *defined on* $R_+\times R^m_+\times C^m([-\tau_0, a), R_+)$ *and* $R_+=[0, +\infty)$, $C^m([-\tau_0, a), R_+)$ *is a set of all vector functions* $W(\cdot)=(w_1(\cdot), \ldots, w_m(\cdot))$, *where* $w_i(\cdot)$ *are continuous and non-negative on* $[-\tau_0, a)$,

2. σ_i *satisfy the Volterra condition, i. e. if* $W(\cdot)$, $\overline{W}(\cdot)\in C^m([-\tau_0, a), R_+)$ *and* $W(\tau)=\overline{W}(\tau)$ *for* $\tau\in[-\tau_0, t]$ *then* $\sigma_i(t, Z, W(\cdot))=\sigma_i(t, Z, \overline{W}(\cdot))$,

3. *for every fixed j, if* $Z\overset{j}{\leqq}\overline{Z}$ *then* $\sigma_j(t, Z, W(\cdot))\leqq\sigma_j(t, \overline{Z}, W(\cdot))$,

4. σ_i, $i=1, \ldots, m$, *are non decreasing with respect to the functional argument* $W(\cdot)$.

Assumption H_2. *Suppose that*

1. *the non-negative function* $g=(g^{(1)}, \ldots, g^{(m)})$ *of the variables* $(x, Y, Z, W(\cdot), Q)$ *is defined on* $E\times R^m_+\times C^m([-\tau_0, a), R_+)\times R^n_+$,

2. $g^{(i)}$ *satisfy the following Volterra condition:*
if $W(\cdot)$, $\widetilde{W}(\cdot)\in C^m([-\tau_0, a), R_+)$ *and* $W(\tau)=\widetilde{W}(\tau)$ *for* $\tau\in[-\tau_0, t]$ *then* $g^{(i)}(x_0+t, Y, Z, W(\cdot), Q)=g^{(i)}(x_0+t, Y, Z, \widetilde{W}(\cdot), Q)$,

3. *for each point* $(x, Y)\in E$ *there exist sets of integers* I_1, I_2, I_3 *such that* $I_1\cup I_2\cup I_3=\{1, \ldots, n\}$ *and* $y_j=\tilde{r}_j(x)$ *for* $j\in I_1$, $y_j=\tilde{s}_j(x)$ *for* $j\in I_2$, $\tilde{r}_j(x)<y_j<\tilde{s}_j(x)$ *for* $j\in I_3$; *we assume that*

$$g^{(i)}(x, Y, Z, W(\cdot), Q)-\sum_{j\in I_1}\tilde{r}'_j(x)q_j+\sum_{j\in I_2}\tilde{s}'_j(x)q_j\leqq\sigma_i(x-x_0, Z, W(\cdot)),\quad i=1, \ldots, m,$$

where $Q=(q_1, \ldots, q_n)$ *and* $q_j=0$ *for* $j\in I_3$,

4. *the function* $\sigma=(\sigma_1, \ldots, \sigma_m)$ *satisfies Assumption* H_1.

Remark 1. If E is the Haar pyramid

$$(2)\qquad \{(x, Y):\ x_0\leqq x<x_0+a,\ |y_i-\overset{0}{y}_i|\leqq a_i-L_i(x-x_0),\quad i=1, \ldots, n\},$$

where $0<L_i<+\infty$, $aL_i<a_i$, and

$$g^{(i)}(x, Y, Z, W(\cdot), Q)=\sigma_i(x-x_0, Z, W(\cdot))+\sum_{k=1}^{n}L_kq_k$$

then condition 3 of Assumption H_2 is satisfied.

Lemma 1. *Suppose that*

1. *Assumption* H_1 *is satisfied*

2. $\Phi=(\varphi_1, \ldots, \varphi_m)$ *is a continuous and non-negative function on* $[-\tau_0, a)$ *and* $\Phi(t)<H(t)$ *for* $t\in[-\tau_0, 0]$, *where* $H=(\eta_1, \ldots, \eta_m)$ *and* η_i *are continuous on* $[-\tau_0, 0]$.

3. $\Omega(t; H)=(\omega_1(t; H), \ldots, \omega_m(t; H))$ *is the maximum solution of the initial problem*

$$(3) \qquad \frac{dw_i(t)}{dt} = \sigma_i(t, W(t), W(\cdot)), \quad t \in [0, a),$$

$$w_i(t) = \eta_i(t) \quad \text{for} \quad t \in [-\tau_0, 0],$$

this solution is defined on $[-\tau_0, a)$,

4. *let* $B^{(i)} = \{t \in (0, a) : \Phi(t) \underset{i}{\leq} \Omega(t; H)\}$ *and for* $t \in B^{(i)}$ $D_- \varphi_i(t) < \sigma_i(t, \Phi(t), \Phi(\cdot))$ ($D_-\varphi(t)$ *is the left-hand lower Dini's derivative of* φ *at the point* t).

Under these assumptions we have $\Phi(t) < \Omega(t; H)$ *for* $t \in [0, a)$.

We omit the simple proof of this lemma (see [16, 28]).

Remark 2. In Lemma 1 we can assume that $\Omega(t; H)$ is an arbitrary solution (not necessarily maximum) of the initial problem (3) defined on $[-\tau_0, a)$.

In the sequel we will use the following

Lemma 2 (see [29]). *Suppose that* w *is a continuous function on* $[-\tau_0, a)$. *Then for each* $t \in (0, a)$ *there exists a* $\bar{t} \in [-\tau_0, t]$ *such that* $\max_{\tau \in [-\tau_0, t]} w(\tau) = w(\bar{t})$ *and the function* β *defined by*

$$\beta(t) = \inf \{\bar{t} : \max_{\tau \in [-\tau_0, t]} w(\tau) = w(\bar{t})\}, \quad t \in (0, a),$$

is non-decreasing on $(0, a)$ *and for* each $\tilde{t} \in (0, a)$ $\lim_{t \to \tilde{t}_-} \beta(t) = \beta(\tilde{t})$.

We define

$$S_t = \{Y : (x_0 + t, Y) \in E_0 \cup E\}, \quad -\tau_0 \leq t < a,$$

$$S = \{(x, Y) \in E : \text{there exists } j,\ 1 \leq j \leq n, \text{ such that } y_j = \tilde{s}_j(x) \text{ or } y_j = \tilde{r}_j(x)\},$$

$$H_x = \{(\xi, \Theta) : (\xi, \Theta) \in E_0 \cup E, \quad \xi \leq x\},$$

$$\bar{K} = \{(x, Y) : x = x_0,\ r_i(x_0) \leq y_i \leq s_i(x_0), \quad i = 1, \ldots, n\}.$$

We shall denote a function w of the variable t for t belonging to some interval (α, β) by the symbol $w(\cdot)$ or $(w(\tau))_{(\alpha, \beta)}$.

For a function $U(\cdot) \in C^m(E_0 \cup E, R)$, $U(\cdot) = (u^{(1)}(\cdot), \ldots, u^{(m)}(\cdot))$ we define

$$|U(x, Y)| = (|u^{(1)}(x, Y)|, \ldots, |u^{(m)}(x, Y)|),$$

$$|u_Y^{(i)}(x, Y)| = (|u_{y_1}^{(i)}(x, Y)|, \ldots, |u_{y_n}^{(i)}(x, Y)|),$$

and

$$(\max_{Y \ni S_\tau} |U(x_0 + \tau, Y)|)_{[-\tau_0, a)}$$

$$= ((\max_{Y \in S_\tau} |u^{(1)}(x_0 + \tau, Y)|)_{[-\tau_0, a)}, \ldots, (\max_{Y \in S_\tau} |u^{(m)}(x_0 + \tau, Y)|)_{[-\tau_0, a)}).$$

Definition 1. *A function* u *of the variables* (x, Y) *will be called the function of class D in* $E_0 \cup E$ *if* u *is continuous on* $E_0 \cup E$, *possesses Stolz's differential on* S *and has first derivatives in an interior of* E.

2. Comparison theorems for systems of partial differential-functional inequalities.

Theorem 1. *Suppose that*

1. *Assumptions* H_1 *and* H_2 *are satisfied,*

2. *the function* $\bar{U}=(\bar{u}^{(1)},\ldots,\bar{u}^{(m)})$ *of the variables* (x, Y) *is defined on* $E_0\cup E$ *and* $\bar{u}^{(i)}$ *are of class* D *in* $E_0\cup E$,

3. *the differential-functional inequalities*

$$u_x^{(i)}(x, Y) \tag{4}$$

$$\leq g^{(i)}(x, Y, |\bar{U}(x, Y)|, (\max_{Y\in S_\tau}|\bar{U}(x_0+\tau, Y)|)_{[-\tau_0, a)}, \; u_Y^{(i)}(x, Y)|),$$

$$(x, Y\in S_\tau^{\blacksquare})\in E, \; i=1,\ldots,m$$

and the initial inequalities

$$|\bar{u}^{(i)}(x, Y)|\leq\eta_i(x-x_0), \quad (x, Y)\in E_0, \quad i=1,\ldots, m, \tag{5}$$

are satisfied,

4. *the maximum solution* $\Omega(t; H)$, $H=(\eta_1,\ldots,\eta_m)$, *of the initial problem* (3) *is defined on* $[-\tau_0, b)$, *where* $b>a$.

Under these assumptions

$$|\bar{U}(x, Y)\leq\Omega(x-x_0; H) \tag{6}$$

for $(x, Y)\in E$.

P r o o f. Let us define $\bar{w}_i(t)=\max_{Y\in S_t}|\bar{u}^{(i)}(x_0+t, Y)|$, $t\in[-\tau_0, a)$, $i=1,\ldots, m$, and $\bar{W}(\cdot)=(\bar{w}_1(\cdot),\ldots,\bar{w}_m(\cdot))$. It is obvious that the estimation (6) on E is equivalent with

$$\bar{W}(t)\leq\Omega(t; H), \quad t\in[0, a). \tag{7}$$

For $\varepsilon>0$, denote by $\Omega(t; H, \varepsilon)=(\omega_1(t; H, \varepsilon),\ldots,\omega_m(t; H, \varepsilon))$ the maximum solution of the problem

$$\frac{dw_i(t)}{dt}=\sigma_i(t, W(t), W(\cdot))+\varepsilon, \quad w_i(t)=\eta_i(t)+\varepsilon, \quad t\in[-\tau_0, 0], \tag{8}$$

$$i=1,\ldots, m.$$

For $\varepsilon>0$ sufficiently small, $\Omega(t; H, \varepsilon)$ is defined on $[-\tau_0, a)$ and

$$\lim_{\varepsilon\to 0}\Omega(t; H, \varepsilon)=\Omega(t; H) \quad \text{on} \quad [-\tau_0, a).$$

To prove (7), it is sufficient to show that

$$\bar{W}(t)<\Omega(t; H, \varepsilon) \quad \text{for} \quad t\in[0, a). \tag{9}$$

Now, we will prove (9) using Lemma 1. Suppose that for some $\tilde{t}\in(0, a)$ we have $\bar{W}(\tilde{t})\overset{j}{\leq}\Omega(\tilde{t}; H, \varepsilon)$. It follows from the definition of $\Omega(t; H, \varepsilon)$ that $\bar{w}_j(\tilde{t})>0$ and that there is an $\tilde{Y}=(\tilde{y}_1,\ldots,\tilde{y}_n)$ such that

$$\bar{w}_j(\tilde{t})=|\bar{u}^{(j)}(\tilde{x}, \tilde{Y})|, \quad \tilde{x}=x_0+\tilde{t}. \tag{10}$$

Suppose that $(\tilde{x}, \tilde{Y})$ is an interior point of E. Then $\bar{u}_Y^{(j)}(\tilde{x}, \tilde{Y})=0$. From this equality, from (10) and by Assumption H_2 it follows that

$$D_{-}w_j(\tilde{t}) \leq u_x^{(j)}(\tilde{x}, \tilde{Y})$$

$$\leq g^{(j)}(\tilde{x}, \tilde{Y}, |U(\tilde{x}, \tilde{Y})|, (\max_{Y \in S_\tau} |U(x_0+\tau, Y)|)_{[-\tau_0, a)}, |u_Y^{(j)}(\tilde{x}, \tilde{Y})|)$$

$$\leq g^{(j)}(\tilde{x}, \tilde{Y}, W(\tilde{t}), W(\cdot), 0) \leq \sigma_j(\tilde{t}, W(\tilde{t}), W(\cdot)) + \varepsilon.$$

Suppose that $(\tilde{x}, \tilde{Y}) \in S$. We can assume (rearranging the indices if necessary) that we have $\tilde{y}_i = \tilde{r}_i(\tilde{x})$ for $i=1, \ldots, s$, $\tilde{y}_i = \tilde{s}_i(\tilde{x})$ for $i=s+1, \ldots, p$, $\tilde{r}_i(\tilde{x}) < \tilde{y}_i < \tilde{s}_i(\tilde{x})$ for $i=p+1, \ldots, n$.

From (10) it follows that there are two possibilities

(i) $$\bar{w}_j(\tilde{t}) = \bar{u}^{(j)}(\tilde{x}, \tilde{Y}),$$

(ii) $$\bar{w}_j(\tilde{t}) = -\bar{u}^{(j)}(\tilde{x}, \tilde{Y}).$$

Let us consider the case (i). We have

(11) $$\bar{u}_{y_i}^{(j)}(\tilde{x}, \tilde{Y}) \leq 0 \quad \text{for} \quad i=1, \ldots, s, \ \bar{u}_{y_i}^{(j)}(\tilde{x}, \tilde{Y}) \geq 0 \quad \text{for} \quad i=s+1, \ldots, p.$$

$$\bar{u}_{y_i}^{(j)}(\tilde{x}, \tilde{Y}) = 0 \quad \text{for} \quad i=p+1, \ldots, n.$$

Let $\bar{Y}(x) = (\tilde{r}_1(x), \ldots, \tilde{r}_s(x), \tilde{s}_{s+1}(x), \ldots, \tilde{s}_p(x), \tilde{y}_{p+1}, \ldots, \tilde{y}_n)$ and consider the composite function $\bar{u}^{(j)}(x, \bar{Y}(x))$. We have $\bar{u}^{(j)}(x_0+\tilde{t}, \bar{Y}(x_0+\tilde{t})) = \bar{w}_j(\tilde{t})$, $\bar{u}^{(j)}(x_0+\tau, \bar{Y}(x_0+\tau)) \leq \bar{w}_j(\tau)$, $\tau \in [-\tau_0, \tilde{t}]$, and therefore

$$D_{-}\bar{w}_j(\tilde{t}) \leq D_{-}[\bar{u}^{(j)}(x_0+t, \bar{Y}(x_0+t))]_{t=\tilde{t}}.$$

From Assumption H_2 and from (11) it follows that

$$D_{-}w_j(\tilde{t}) \leq u_x^{(j)}(x_0+\tilde{t}, Y(\tilde{x})) + \sum_{i=1}^{s} u_{y_i}^{(j)}(\tilde{x}, \tilde{Y})\tilde{r}_i'(\tilde{x}) + \sum_{i=s+1}^{p} u_{y_i}^{(j)}(\tilde{x}, \tilde{Y})\tilde{s}_i'(\tilde{x})$$

$$\leq g^{(j)}(\tilde{x}, \tilde{Y}, |U(\tilde{x}, \tilde{Y})|, (\max_{Y \in S_\tau} |U(x_0+\tau, Y)|)_{[-\tau_0, a)}, |u_Y^{(j)}(\tilde{x}, \tilde{Y})|)$$

$$-\sum_{i=1}^{s} (-u_{y_i}^{(j)}(\tilde{x}, \tilde{Y}))\tilde{r}_i'(\tilde{x}) + \sum_{i=s+1}^{p} u_{y_i}^{(j)}(\tilde{x}, \tilde{Y})\tilde{s}_i'(\tilde{x}) < \sigma_j(\tilde{t}, W(\tilde{t}), W(\cdot)) + \varepsilon.$$

Suppose that the possibility (ii) holds. Then we have

(12) $$u_{y_i}^{(j)}(\tilde{x}, \tilde{Y}) \geq 0 \quad \text{for} \quad i=1, \ldots, s, \ u_{y_i}^{(j)}(\tilde{x}, \tilde{Y}) \leq 0 \quad \text{for} \quad i=s+1, \ldots, p,$$

$$\bar{u}_{y_i}^{(j)}(\tilde{x}, \tilde{Y}) = 0 \quad \text{for} \quad i=p+1, \ldots, n,$$

and

(13) $$-\bar{u}^{(j)}(x_0+\tilde{t}, \bar{Y}(x_0+\tilde{t})) = \bar{w}_j(\tilde{t}), \quad -\bar{u}^{(j)}(x_0+\tau, \bar{Y}(x_0+\tau)) \leq \bar{w}_j(\tau), \quad \tau \in [-\tau_0, \tilde{t}].$$

We have

(14) $$D_{-}\bar{w}_j(\tilde{t}) \leq D_{-}[-\bar{u}^{(j)}(x_0+t, \bar{Y}(x_0+t))]_{t=\tilde{t}}.$$

From Assumption H_2 and from (12)—(14) it follows that

$$D_{-}w_j(\tilde{t})$$
$$\leq g^{(j)}(\tilde{x}, \tilde{Y}, |U(\tilde{x}, \tilde{Y})|, (\max_{Y\in S_\tau} |\bar{U}(x_0+\tau, Y)|)_{[-\tau_0, a)}, |\bar{u}_Y^{(j)}(\tilde{x}, \tilde{Y})|)$$
$$-\sum_{i=1}^{s} \bar{u}_{y_i}^{(j)}(\tilde{x}, \tilde{Y})\tilde{r}_i'(\tilde{x})+\sum_{i=s+1}^{p}(-\bar{u}_{y_i}^{(j)}(\tilde{x}, \tilde{Y}))\tilde{s}_i'(\tilde{x})<\sigma_j(\tilde{t}, W(\tilde{t}), W(\cdot))+\varepsilon.$$

It therefore follows that, in any case, the assumption (10) implies the differential inequality $D_{-}w_j(\tilde{t})<\sigma_j(\tilde{t}, W(\tilde{t}), W(\cdot))+\varepsilon$. Since $w_j(t) < \eta_j(t)+\varepsilon$ for $t\in[-\tau_0, 0], i=1, \ldots, m$ it follows that all the assumptions of Lemma 1 are satisfied. From this lemma we obtain (9). From (9) we obtain in the limit (letting ε tend to 0) inequality (7). This completes the proof.

In the case when E is the Haar pyramid defined by (2) and

(15) $$E_0=\{(x, Y):\ x_0-\tau_0\leq x\leq x_0,\ |y_i-\overset{0}{y_i}|\leq a_i,\ i=1, \ldots, n\},$$

we have the following

Theorem 2. *Suppose that*

1. *the function* $\bar{U}=(\bar{u}^{(1)}, \ldots, \bar{u}^{(m)})$ *of the variables* (x, Y) *is defined on* $E_0\cup E$, *where* E *and* E_0 *are defined by* (2) *and* (15) *respectively,*
2. $\bar{u}^{(i)}$, $i=1, \ldots, m$, *are of class* D *in* $E_0\cup E$,
3. *the function* $\sigma=(\sigma_1, \ldots, \sigma_m)$ *satisfies Assumption* H_1,
4. *the differential-functional inequalities*

(16) $$|\bar{u}_x^{(i)}(x, Y)|\leq\sigma_i(x-x_0, |\bar{U}(x, Y)|, (\max_{Y\in S_\tau}|U(x_0+\tau, Y)|)_{[-\tau_0, a)})$$
$$+\sum_{k=1}^{n} L_k|u_{y_k}^{(i)}(x, Y)|,\quad (x, Y)\in E,\quad i=1, \ldots, m,$$

and the initial inequalities

(17) $$|u^{(i)}(x, Y)|\leq\eta_i(x-x_0),\quad (x, Y)\in E_0,\quad i=1, \ldots, m,$$

are satisfied,

5. *the maximum solution* $\Omega(t; H)$, $H=(\eta_1, \ldots, \eta_m)$, *of the initial problem* (3) *is defined on* $[-\tau_0, b)$, *where* $b>a$.

Under these assumptions

(18) $$|\bar{U}(x, Y)|\leq\Omega(x-x_0;\ H)\quad\text{for}\quad (x, Y)\in E.$$

Remark 3. Suppose that for $(t, Z, W(\cdot))\in R_+\times R_+^m\times C^m([-\tau_0, a), R_+)$

(19) $$\sigma_i(t, Z, W(\cdot))=\sigma_i(t, Z, \max_{\tau\in[-\tau_0, t]} w_1(\tau), \ldots, \max_{\tau\in[-\tau_0, t]} w_m(\tau))$$

and for $(x, Y, Z, W(\cdot), Q)\in E\times R_+^m\times C^m([-\tau_0, a), R_+)\times R_+^n$

$$g^{(i)}(x, Y, Z, W(\cdot), Q)$$
$$=g^{(i)}(x, Y, Z, \max_{\tau\in[-\tau_0, x-x_0]} w_1(\tau), \ldots, \max_{\tau\in[-\tau_0, x-x_0]} w_m(\tau), Q).$$

Let

$$\max_{(\xi,\Theta)\in H_x} |U(\xi,\Theta)| = (\max_{(\xi,\Theta)\in H_x} |u^{(1)}(\xi,\Theta)|, \ldots, \max_{(\xi,\Theta)\in H_x} |u^{(m)}(\xi,\Theta)|).$$

Then assumption (4) in Theorem 1 has the following form:

$$(21)\qquad |\bar{u}_x^{(i)}(x,Y)| \le g^{(i)}(x, Y, |\bar{U}(x,Y)|, \max_{(\xi,\theta)\in Hx} |\bar{U}(\xi,\Theta)|, |\bar{u}_Y^{(i)}(x,Y)|)$$

$$(x, Y)\in E, \quad i=1,\ldots, m,$$

and condition (16) from Theorem 2 is

$$(22)\qquad |\bar{u}_x^{(i)}(x,Y)| \le \sigma_i(x-x_0, |\bar{U}(x,Y)|, \max_{(\xi,\Theta)\in H_x} |\bar{U}(\xi,\Theta)|)$$

$$+\sum_{k=1}^{n} L_k |u_{y_k}^{(i)}(x,y)|, \quad (x, Y)\in E, \quad i=1,\ldots, m.$$

It follows from Lemma 2 that there exist functions $\beta_1, \ldots, \beta_m$ such that $-\tau_0 \le \beta_i(t) \le t$, $t\in[0, a)$, $\max_{\tau\in[-\tau_0, t]} w_i(\tau) = w_i(\beta_i(t))$, $\lim_{t\to\tilde{t}_-} \beta_i(t) = \beta_i(\tilde{t})$, $\tilde{t}\in[0, a)$.

In this case we can assume that the comparison system (3) is the system of differential equations with the left-hand derivatives and with a deviated argument

$$(23)\qquad \begin{aligned} &(w_i(t))'_- = \sigma_i(t, W(t), W(\beta(t))), \quad t\in(0, a),\\ &w_i(t) = \eta_i(t) \quad \text{for} \quad t\in[-\tau_0, 0], \end{aligned}$$

where $W(\beta(t)) = (w_1(\beta_1(t)), \ldots, w_m(\beta_m(t)))$. Lemma 1 is true if we introduce instead of (3) the above system with the left-hand derivatives.

3. Estimation of solutions of differential-functional systems. In this section we give an estimation of solutions of the system (1) by means of solutions of a system of ordinary differential-functional equations.

Theorem 3. *Suppose that*

1. *the functions $f^{(i)}$, $i=1,\ldots, m$, of the variables $(x, Y, Z, U(\cdot), Q)$ are defined on $E\times R^m\times C^m(E_0\cup E, R)\times B$, where B is a domain which is contained in R^n,*

2. *$f^{(i)}$, $i=1,\ldots, m$, satisfy the Volterra condition, i. e. if $U(\cdot)$, $V(\cdot)\in C^m(E_0\cup E, R)$ and $U(\xi,\Theta)=V(\xi,\Theta)$ for $(\xi,\Theta)\in H_x$ then $f^{(i)}(x, Y, Z, U(\cdot), Q) = f^{(i)}(x, Y, Z, V(\cdot), Q)$, $i=1,\ldots, m$,*

3. *the estimations*

$$(24)\qquad |f^{(i)}(x, Y, Z, U(\cdot), Q)| \le g^{(i)}(x, Y, |Z|, (\max_{Y\in S_\tau} |U(x_0+\tau, Y)|)_{[-\tau_0, a)}, |Q|),$$

are satisfied on $E\times R^m\times C^m(E_0\cup E, R)\times B$ for $i=1,\ldots, m$,

4. *Asmptions H_1 and H_2 are satisfied,*

5. *$U=(\bar{u}^{(1)}\ldots, \bar{u}^{(m)})$ is a solution of (1) in $E_0\cup E$ and $u_{(\bar{i})}$ are of class D in $E_0\cup E$, $\bar{u}_Y^{(i)}(x, Y)\in B$ for $(x, Y)\in E$,*

6. *for $(x, Y)\in E_0$*

$$(25)\qquad |\bar{U}(x, Y)| \le H(x-x_0),$$

where $H=(\eta_1, \ldots, \eta_m)$ and η_i are continuous on $[-\tau_0, 0]$,

7. *the maximum solution $\Omega(t; H)$ of the initial problem* (3) *is defined on* $[-\tau_0, b)$, *where* $b>a$.

Under these assumptions

$$|\bar{U}(x, Y)| \leq \Omega(x-x_0; H), \quad (x, Y)\in E. \tag{26}$$

Proof. By (24), (25), the solution $\bar{U}$ satisfies all the assumptions of Theorem 1 and, hence, inequalities (26) hold true.

Remark 4. Suppose that

1. the sets E and E_0 are defined by (2) and (15) respectively,
2. conditions 1, 2, 5—7 of Theorem 3 are satisfied,
3. the estimations

$$|f^{(i)}(x, Y, Z, U(\cdot), Q)| \leq \sigma_i(x-x_0, |Z|, (\max_{Y\in S_\tau} |U(x_0+\tau, Y)|)_{[-\tau_0, a]})$$

$$+ \sum_{k=1}^{n} L_k |q_k|, \quad i=1,\ldots, m,$$

are satisfied on $E\times R^m\times C^m(E_0\cup E, R)\times B$,

4. Assumption H_1 is satisfied.

Under these assumptions $|\bar{U}(x, Y)| \leq \Omega(x-x_0; H)$, $(x, Y)\in E$.

Remark 5. Suppose that

1. the conditions 1, 2, 5, 6 of Theorem 1 are satisfied,
2. the estimations

$$|f^{(i)}(x, Y, Z, U(.), Q)| \leq g^{(i)}(x, Y, |Z|, \max_{(\xi, \Theta)\in H_x} |U(\xi, \Theta)|, |Q|),$$

$$(x, Y)\in E, \quad i=1,\ldots, m,$$

are satisfied on $E\times R^m\times C^m(E_0\cup E, R)\times B$,

3. the functions σ_i and $g^{(i)}$, $i=1,\ldots, m$, defined by (19), (20) satisfy Assumptions H_1, H_2,
4. the maximum solution $\Omega(t; H)$ of the initial problem (23) is defined on $[-\tau_0, b)$, $b>a$.

Under these assumptions $|\bar{U}(x, Y)| \leq \Omega(x-x_0; H)$ on E.

Remark 6. It is easy to formulate a theorem analogous to Theorem 3 in the case when E and E_0 are defined by (2), (15) and the functions σ_i, $i=1,\ldots, m$, are defined by (19).

4. Estimation of the difference between solutions, uniqueness criteria. Let us consider two systems, (1) and the system

$$\begin{aligned} &z_x^{(i)}(x, Y)=F^{(i)}(x, Y, Z(x, Y), Z(\cdot), z_Y^{(i)}(x, Y)), \\ &z^{(i)}(x, Y)=\beta^{(i)}(x, Y) \quad \text{for} \quad (x, Y)\in E_0. \end{aligned} \quad i=1,\ldots, m, \tag{27}$$

We have

Theorem 4. Suppose that

1. *the functions $f^{(i)}$, $F^{(i)}$, $t=1,\ldots, m$, are defined on $E\times R^m\times C^m(E_0\cup E, R)\times B$ and satisfy the Volterra condition,*
2. *Assumptions H_1 and H_2 are satisfied*
3. *the conditions*

$$(28)\qquad |f^{(i)}(x, Y, Z, U(\cdot), Q)-F^{(i)}(x, Y, Z, V(\cdot), \bar{Q})|$$
$$\leq g^{(i)}(x, Y, |Z-\bar{Z}|, (\max_{Y\in S_t}|U(x_0+t, Y)-V(x_0+t, Y)|)_{[-\tau_0, a)}, |Q-\bar{Q}|)$$

are satisfied on $E\times R^m\times C^m(E_0\cup E, R)\times B$ *for* $i=1,\ldots, m$,

4. $\bar{U}=(u^{(1)},\ldots, u^{(m)})$ *and* $\bar{V}=(v^{(1)},\ldots, v^{(m)})$ *are solutions of* (1) *and* (27) *respectively and*

$$(29)\qquad |U(x, Y)-V(x, Y)|\leq H(x-x_0)\quad \text{for}\quad (x, Y)\in E_0,$$

5. $u^{(i)}, v^{(i)}$ *are of class D on* $E_0\cup E$ *and* $u_Y^{(i)}(x, Y)$, $v_Y^{(i)}(x, Y)\in B$ *for* $(x, Y)\in E$,

6. *the maximum solution* $\Omega(t; H)$ *of* (3) *is defined on* $[-\tau_0, b)$, $b>a$. *Under these assumptions*

$$(30)\qquad |\bar{U}(x, Y)-\bar{V}(x, Y)|\leq \Omega(x-x_0; H)\quad \text{for}\quad (x, Y)\in E.$$

P r o o f. If we put $\tilde{U}(x, Y)=\bar{U}(x, Y)-\bar{V}(x, Y)$, then $\tilde{U}$ satisfies all the conditions of Theorem 1 and, hence (30) holds true.

R e m a r k 7. (see Th. 2) If

1. E and E_0 are defined by (2) and (15) respectively,
2. Assumption H_1 is satisfied,
3. the conditions 1, 4—6 of Theorem 4 are satisfied,
4. the estimations

$$|f^{(i)}(x, Y, Z, U(\cdot), Q)-F^{(i)}(x, Y, \bar{Z}, V(\cdot), \bar{Q})|$$
$$\leq \sigma_i(x-x_0, |Z-\bar{Z}|, (\max_{Y\in S_\tau}|U(x_0+\tau, Y)-V(x_0+\tau, Y)|)_{[-\tau_0, a)})$$
$$+\sum_{k=1}^{n} L_k|q_k-\bar{q}_k|,\quad i=1,\ldots, m,$$

are satisfied on $E\times R^m\times C^m(E_0\cup E, R)\times B$, then $|\bar{U}(x, Y)-\bar{V}(x, Y)|\leq \Omega(x-x_0; H)$ for $(x, Y)\in E$.

Theorem 4 implies

T h e o r e m 5. *Suppose that*

1. *Assumptions* H_1 *and* H_2 *are satisfied,*
2. *the functions* $f^{(i)}$, $i=1,\ldots, m$, *satisfy the Volterra condition on* $E\times R^m\times C^m(E_0\cup E, R)\times B$ *and*

$$(31)\qquad \begin{aligned}&|f^{(i)}(x, Y, Z, U(\cdot), Q)-f^{(i)}(x, Y, \bar{Z}, V(\cdot), \bar{Q})|\\ &\leq g^{(i)}(x, Y, |Z-\bar{Z}|, (\max_{Y\in S_\tau}|U(x_0+\tau, Y)-V(x_0+\tau, Y)|)_{[-\tau_0, a)}, |Q-\bar{Q}|).\end{aligned}$$

3. the function $(\omega_1(t),\ldots, \omega_m(t))=\Omega(t)\equiv 0$ for $t\in[-\tau_0, +\infty)$ is a maximum solution of the initial problem

$$(32)\qquad \frac{dw_i(t)}{dt}=\sigma_i(t, W(t), W(\cdot)),\ w_i(t)=0\ \text{for}\ t\in[-\tau_0, 0],\ i=1,\ldots, m.$$

Under these assumptions the Cauchy problem (1) *admits at most one solution $\bar{U}$ on E with $\bar{u}^{(i)}$ of class D on $E_0 \cup E$ (see analogous theorem for ordinary differential-functional equations* [14]).

Remark 8. If E and E_0 are sets defined by (2) and (15) respectively then in Theorem 5 we can assume that

$$(33)\qquad \begin{aligned} &|f^{(i)}(x, Y, Z, U(\cdot), Q)-f^{(i)}(x, Y, \bar{Z}, V(\cdot), \bar{Q})| \\ &\leq \sigma_i(x-x_0, |Z-\bar{Z}|, (\max_{Y\in S_\tau}|U(x_0+\tau, Y)-V(x_0+\tau, Y)|)_{[-\tau_0, a)})+\sum_{k=1}^{n} L_k |q_k-\bar{q}_k|, \\ &i=1, \ldots, m, \end{aligned}$$

where the function $\sigma=(\sigma_1, \ldots, \sigma_m)$ satisfies Assumption H_1 and condition 3 of Theorem 5 (see Th. 2).

Remark 9. If we assume instead of (31) in Theorem 5 that

$$|f^{(i)}(x, Y, Z, U(\cdot), Q)-f^{(i)}(x, Y, \bar{Z}, V(\cdot), \bar{Q})|$$

$$\leq \sigma_i(x-x_0, |Z-\bar{Z}|, \max_{(\xi, \Theta)\in H_x}|U(\xi, \Theta)-V(\xi, \Theta)|)+\sum_{k=1}^{n} L_k |q_k-\bar{q}_k|$$

then there exist functions $\beta_1, \ldots, \beta_m$ such that the comparison system (32) is of the form $(w_i(t))'=\sigma_i(t, W(t), w_1(\beta_1(t)), \ldots, w_m(\beta_m(t)))$, $i=1, \ldots, m$, where $-\tau_0 \leq \beta_i(t) \leq t$ for $t\in[0, a)$.

5. Continuous dependence of the solution on initial data and on the right-hand sides of system.

Theorem 6. *Suppose that*

1. *Assumptions H_1 and H_2 are satisfied,*

2. *$f^{(i)}$ and $F^{(i)}$, $i=1, \ldots, m$, satisfy the Volterra condition and estimations* (31) *hold true,*

3. *$\bar{U}$ and $\bar{V}$ are solutions of* (1) *and* (27) *respectively, $\bar{u}^{(i)}$, $\bar{v}^{(i)}$ are of class D on $E_0 \cup E$ and $u_Y^{(i)}(x, Y)$, $v_Y^{(i)}(x, Y)\in B$ for $(x, Y)\in E$,*

4. *the function $\Omega(t)\equiv 0$ for $t\in[-\tau_0, +\infty)$ is a maximum solution of* (32).

Under these assumptions, to every $\varepsilon>0$ there is a $\delta>0$ such that if

$$(34)\qquad |f^{(i)}(x, Y, Z, U(\cdot), Q)-F^{(i)}(x, Y, Z, U(\cdot), Q)|<\delta, \; i=1, \ldots, m,$$

on $E\times R^m\times C^m(E_0\cup E, R)\times B$ and

$$(35)\qquad |\alpha^{(i)}(x, Y)-\beta^{(i)}(x, Y)|<\delta, \; i=1, \ldots, m,$$

on E_0 then

$$(36)\qquad |\bar{u}^{(i)}(x, Y)-\bar{v}^{(i)}(x, Y)|<\varepsilon, \; i=1, \ldots, m, \text{ for } (x, Y)\in E.$$

Proof. For $\varepsilon>0$ we can choose $\delta>0$ so that the right-hand maximum solution $\Omega(t; H, \delta)=(\omega_1(t; H, \delta), \ldots, \omega_m(t; H, \delta))$ of the problem

$$\frac{dw_i(t)}{dt}=\sigma_i(t, W(t), W(\cdot))+\delta, \; w_i(t)=\eta_i(t) \text{ for } t\in[-\tau_0, 0], \; i=1, \ldots, m,$$

where $H=(\eta_1, \ldots, \eta_m)$ and $0\leq\eta_i(t)\leq\delta$ on $[-\tau_0, 0]$, be defined on $[0, a)$ and

$$(37)\qquad \omega_i(t; H, \delta)<\varepsilon \text{ for } t\in[0, a), \; i=1, \ldots, m.$$

Suppose that (34) and (35) hold true with the above choosen δ. Then we define

$$\eta_i(t) = \max_{Y \in S_t} |\alpha^{(i)}(x_0+t, Y) - \beta^{(i)}(x_0+t, Y)|, \quad t \in [-\tau_0, 0], \quad i=1, \ldots, m.$$

From (31), (34) it follows that

$$|f^{(i)}(x, Y, Z, U(\cdot), Q) - F^{(i)}(x, Y, \bar{Z}, V(\cdot), \bar{Q})|$$

$$\leq g^{(i)}(x, Y, |Z-\bar{Z}|, (\max_{Y \in S_\tau} |U(x_0+\tau, Y) - V(x_0+\tau, Y)|)_{[-\tau_0, a)}, |Q-\bar{Q}|) + \delta.$$

Hence, by Theorem 4 inequalities

(38) $$|u^{(i)}(x, Y) - v^{(i)}(x, Y)| \leq \omega_i(x-x_0; H, \delta), \quad i=1, \ldots, m,$$

hold true in E. From (37), (38) follows (36).

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