Serdica
Bulgariacae mathematicae publicationes

Сердика
Българско математическо списание

A GENERALIZATION OF TIMAN'S THEOREM FOR APPROXIMATION OF FUNCTIONS BY ALGEBRAIC POLYNOMIALS

DIMITAR G. DIMITROV, VASIL A. POPOV

Estimates are given for the local Hausdorff approximation with a parameter of a given continous function in the interval $[-1, 1]$ by means of algebraic polynomials. These estimations in the boundary case lead to the well-known Timan estimation for approximation of continuous functions with algebraic polynomials. On the other hand from this estimations follows the universal Sendov's estimation for the approximation of bounded function in Hansdorff metric.

We shall denote by $r(f, g; \alpha)$ the Hausdorff distance with a parameter α between the functions f and g bounded in the interval $[-1, 1]$. Concerning the definition and the fundamental properties of the Hausdorff distance between functions see [2]; [3]. Let us only recall that if f and g are continuous functions in the interval $\Delta=[-1, 1]$, then the Hausdorff distance with parameter α, $\alpha>0$, between them is defined by

$$r(f, g; \alpha)=\max\{\max_{x\in\Delta}\min_{y\in\Delta}\max_{\in\Delta}\{\alpha^{-1}|x-y|, |f(x)-g(y)|\},$$
$$\max_{x\in\Delta}\min_{y\in\Delta}\max\{\alpha^{-1}|x-y|, |f(y)-g(x)|\}\}.$$

A basic result of the theory of approximation of functions with respect to the Hausdorff distance is the universal estimate, obtained by Bl. Sendov [4]. Let H_n be the set of all algebraic polynomials of degree n. The best Hausdorff approximation with parameter $\alpha>0$ of a bounded in the interval Δ function f is defined by $E_n(f; \alpha)=\inf\{r(f, p; \alpha): p\in H_n\}$.

The above mentioned result of Sendov asserts that

$$E_n(f; 1)=O(n^{-1}\ln n). \tag{1}$$

This theorem will be improved if one takes into account the Nikolski's effect: the approximation by algebraic polynomials is better at the ends of the interval than in its middle.

If we define the Hausdorff deviation at the point $x\in\Delta$ with parameter α by

$$|f(x)-g(x)|_\alpha=\max\{\max_{y\in\Delta}\max\{\alpha^{-1}|x-y|, |f(x)-g(y)|\},$$
$$\min_{y\in\Delta}\max\{\alpha^{-1}|x-y|, |f(y)-g(x)|\}\},$$

then the following result holds [5].

Theorem A. *Let f be a continous function in the interval Δ. An absolute constant $c(M)$ exists, depending only of $M=\max_{x\in\Delta}|f(x)|$, such that* *for every natural number n there exists such an algebraical polynomial* $p_n \in H_n$, *that for every* $x \in \Delta$ *we have*

$$(2) \qquad |f(x)-p_n(x)|_1 \leq c(M)\{n^{-1}\sqrt{1-x^2}\ln n+(n^{-1}\ln n)^2\}.$$

Timan's theorem states [1]: *if f is a continuous function in* Δ, *then for every natural number n there exists an algebraic polynomial* $p_n \in H_n$, *such that for every* $x \in \Delta$ *we have*

$$(3) \qquad |f(x)-p_n(x)| \leq c'\omega(f;\ n^{-1}\sqrt{1-x^2}+n^{-2}),$$

where $\omega(f;\ \delta)=\max_{|x-y|\leq\delta}|f(x)-f(y)|$, $x, y \in \Delta$, *is the modulus of continuity of the function f in the interval* Δ, *while* c' *is an absolute constant.*

It is easy to see that for $\alpha \to 0$ the Hausdorff deviation with parameter α at the point x tends to the absolute value of the difference between the continuous functions f and g

$$(4) \qquad |f(x)-g(x)|_\alpha \xrightarrow[\alpha\to 0]{} |f(x)-g(x)|.$$

Timan's theorem does not follow directly from estimation (2). The problem arises to obtain such an estimation for the Hausdorff approximation with parameter α by means of algebraic polynomials, so that Timan's estimation be a consequence of it when $\alpha \to 0$.

Theorem 1. *Let f be a continuous function in the interval* $[-1, 1]$. *For each natural number n and each* α, $0<\alpha\leq 1$, *there exists an algebraic polynomial* $P_{n,\alpha} \in H_n$ *such that for every* $x \in [-1, 1]$ *the following inequality holds*

$$(5) \qquad |f(x)-P_{n,\alpha}(x)|_\alpha \leq c\,\omega(f;\ \Delta_m(x))/\{1+\alpha\Delta_m^{-1}(x)\,\omega(f;\ \Delta_m(x))\},$$

where $\Delta_m(x)=m^{-1}\sqrt{1-x^2}+m^{-2}$, $m=n/\ln(e+\alpha n^2)$, *while c is an absolute constant.*

As becomes immediately obvious, for every $\alpha>0$ we have

$$|f(x)-P_{n,\alpha}(x)|_\alpha \leq c\,\omega(f;\ \Delta_m(x))/\{1+\alpha\Delta_m^{-1}(x)\,\omega(f;\ \Delta_m(x))\}$$
$$\leq c\Delta_m(x)/\alpha \leq c'(\sqrt{1-x^2}\,n^{-1}\ln n+(n^{-1}\ln n)^2)/\alpha,$$

i. e. the result (2) (as well as (1)) follows from the result (5). Besides, if we let $\alpha \to 0$, owing to the unifrom boundedness of $P_{n,\alpha}$, we can choose a uniformly convergent subsequence P_{n,α_i}, P_{n,α_i} tends uniformly in $[-1, 1]$ to some $P_n \in H_n$, $\alpha_i \to 0$, and hence, from (5) and (4), when $\alpha_i \to 0$ we obtain Timan's estimation (3).

Let us note that a generalization for the Hausdorff distance with a parameter of Jackson's theorem in the case of approximation by trigonometrical polynomials and entire functions of exponential type (similar to theorem 1) was obtained in [3]. But theorem 1 cannot be obtained from this generalization.

The method we employ for proving theorem 1 is analogous to that in [3], applying, however, the corresponding modifications resulting from Nikolski's effect in the algebraical case.

Proof of theorem 1. Let f be a continuous function in the interval $[-1, 1]$ and $s>0$ be a natural number. Let us set $m=4s$. For such m we construct the function f_m in the following way:

We set: $\qquad x_i=-\cos i\pi/m,\ i=0,\dots,m,$

$M_i=\max\ \{f(x): x\in[x_{4i},\ x_{4i+4}]\},\ m_i=\min\ \{f(x): x\in[x_{4i},\ x_{4i+4}]\},\ i=0,\dots,s-1$

$$f_m(x)=\begin{cases} m_i & x\in[x_{4i},\ x_{4i+1}],\ i=0,\dots,s-1;\\ M_i, & x\in[x_{4i+2},\ x_{4i+3}],\ i=0,\dots,s-1;\\ \text{linear and continuous in the} & \\ \text{other subintervals } [x_j,\ x_{j+1}]. & \end{cases}$$

The following inequality holds

$$|f(x)-f_m(x)|_\alpha\leq 8\pi^2\min\ \{\alpha^{-1}\Delta_m(x),\ \omega(f;\Delta_m(x))\}. \tag{6}$$

Really, if the point $x\in[x_{4i},\ x_{4i+4}]$, $0\leq i\leq s-1$, then by construction

$$m_i\leq f_m(x)\leq M_i,\ f_m(x_{4i})=m_i,\ f_m(x_{4i+2})=M_i.$$

Whereby we get successively

$$|f(x)-f_m(x)|\leq\omega(f;|x_{4i}-x_{4(i+1)}|) \tag{7}$$

and for every x there exist points y_x and z_x, $y_x\ z_x\in[x_{4i},\ x_{4i+4}]$, such that

(8) $f(x)=f_m(y_x),\ f_m(x)=f(z_x),\ |x-y_x|\leq|x_{4i}-x_{4(i+1)}|,\ |x-z_x|\leq|x_{4i}-x_{4(i+1)}|.$

Let $x=\cos t$, then $t\in[4i\pi/m,\ 4(i+1)\pi/m]$. We have $(|t-4(2i+1)\pi/2m|\leq 2\pi/m)$

$$|x_{4i}-x_{4(i+1)}|=|\cos(4i\pi/m)-\cos(4(i+1)\pi/m)|$$

$$=2\sin\frac{2\pi}{m}\left|\sin\frac{4(2i+1)\pi}{2m}\right|\leq\frac{4\pi}{m}\left|\sin\left(\left(\frac{2(2i+1)\pi}{m}-t\right)+t\right)\right| \tag{9}$$

$$\leq\frac{4\pi}{m}\left|\sin t\cos\left(\frac{2(2i+1)\pi}{m}-t\right)+\cos t\sin\left(\frac{2(2i+1)\pi}{m}-t\right)\right|$$

$$\leq\frac{4\pi}{m}\left(|\sin t|+\frac{2\pi}{m}\right)\leq\frac{8\pi^2}{m}\left(\sqrt{1-x^2}+\frac{1}{m}\right)=8\pi^2\Delta_m(x).$$

From (7)–(9) and the definition of the Hausdorff difference with a parameter α at the point x we obtain (6).

For the function $f_m(\cos t)$ we construct the modified Jackson's operator

$$U_{m,r}(f_m;t)=\mu_{m,r}\int_{-\pi}^{\pi}f_m(\cos(t+u))\,\psi_{m,r}(u)\,du,$$

where $\psi_{m,r}(t)=((\sin 30\,mt/2)/\sin t/2)^{2r}$, $m, r\geq 2$ are positive integers and norming factor $\mu_{m,r}$ is defined by the condition

$$\mu_{m,r}\int_{-\pi}^{\pi}\psi_{m,r}(u)\,du=1.$$

$U_{m,r}$ is an even trigonometrical polynomial of the order $\leq 30\,mr$. We have:

$$0<\mu_{m,r}<\left(2\int_0^{\pi/30\,m}\left(\frac{\sin 15\,mt}{\sin t/2}\right)^{2r}dt\leq\left(\frac{\pi}{60m}\right)^{2r-1}/4. \tag{10}$$

We shall estimate now

$$|f_m(x)-U_{m,r}(f_m;\ \arccos x)|_\alpha.$$

For each fixed $x\in[-1,\ 1]$ we set $x=\cos t$, $t\in[0,\ 2\pi]$, $\Omega_m(x)=[-\cos(t-1/2m),\ -\cos(t+1/2m)]$, when $t-1/2m<0$ we set $\Omega_m(x)=[-1,\ -\cos(t+1/2m)]$, when $t+1/2m\geqq 2\pi$ we set $\Omega_m(x)=[-\cos(t-1/2m),\ 1]$. Let us mention that for the length $l(\Omega_m(x))$ of the interval $\Omega_m(x)$ we have

$$\begin{aligned} &l(\Omega_m(x))=|2\sin t\sin 1/2m|\leqq m^{-1}\sqrt{1-x^2}\ \text{ if } t-1/2m\geqq 0,\ t+1/2m\leqq 2\pi,\\ (11)\quad &l(\Omega_m(x))\leqq|1-\cos 1/m|\leqq 1/2m^2\ \text{ if } t-1/2m<0 \text{ or } t+1/2m>2\pi. \end{aligned}$$

For every $y\in\Omega_m(x)$ we have

$$\min_{z\in\Omega_m(x)}\{f_m(z)-f_m(x)\}\leqq f_m(y)-f_m(x)\leqq\max_{z\in\Omega_m(x)}\{f_m(z)-f_m(x)\}.$$

From here, using the definition of the operator $U_{m,r}$, we obtain

$$\min_{z\in\Omega_m(x)}\{f_m(z)-f_m(x)\}\leqq\mu_{m,r}\int_{-1/2m}^{1/2m}(f_m(\cos(u+t))-f_m(\cos t))\,\psi_{m,r}(u)\,du$$

$$\leqq\max_{z\in\Omega_m(x)}\{f_m(z)-f_m(x)\}.$$

Since f_m is a continuous function it follows from the above inequalities that there exists $y_x\in\Omega_m(x)$ such that

$$f_m(y_x)-f_m(x)=\mu_{m,r}\int_{-1/2m}^{1/2m}(f_m(\cos(u+t))-f_m(\cos t))\,\psi_{m,r}(u)\,du,$$

or, expressed in a different way

(12) $$|f_m(y_x)-U_{m,r}(\arccos x)|\leqq\varphi(x),$$

where

$$\varphi(x)=\mu_{m,r}\left\{\int_{-\pi}^{-1/2m}+\int_{1/2m}^{\pi}|f_m(\cos(u+t))-f_m(\cos t)|\,\psi_{m,r}(u)\,du\right\}.$$

We shall estimate now $\varphi(x)$. First of all we must estimate $|f_m(\cos(u+t))-f_m(\cos t)|$ by means of the modulus of continuity of f. By the construction of the finction f_m we have

$$f_m(\cos t)=f(\cos(t+\theta_1(t))),\quad f_m(\cos(t+u))=f(\cos(t+u+\theta_2(t,\ u))),$$

$$\text{where}\qquad|\theta_1(t)|\leqq 4\pi/m,\ |\theta_2(t,\ u)|\leqq 4\pi/m.$$

Consequently

$$|f_m(\cos(t+u))-f_m(\cos t)|=|f(\cos(t+\theta_1))-f(\cos(t+u+\theta_2)|$$

$$\leqq\omega(f;|\cos(t+\theta_1)-\cos(t+u+\theta_2)|)\leqq 2\omega\left(f;\left|\sin\left(t+\frac{u+\theta_1+\theta_2}{2}\right)\sin\frac{u-\theta_1+\theta_2}{2}\right|\right)$$

(13)

$$\leqq 8\pi\omega\left(f;\left(\sqrt{1-x^2}+\frac{|u|}{2}+\frac{1}{m}\right)\left(\frac{|u|}{2}+\frac{1}{m}\right)\right).$$

Therefore

$$\varphi_m(x) \leqq 16\,\pi\mu_{m,r}\int_{1/2m}^{\pi}\omega\left(f;\ \left(\sqrt{1-x^2}+\frac{|u|}{2}+\frac{1}{m}\right)\left(\frac{u}{2}+\frac{1}{m}\right)\psi_{m,r}(u)\,du.\right.$$

Since $u \geqq 1/2m,\ 2u \geqq 1/m$, we have

$$\varphi(x) \leqq 16\pi\mu_{m,r}\int_{1/2m}^{\pi}\omega\left(f;\ \left(\sqrt{1-x^2}+\frac{5}{2}\ u\right)\frac{5u}{2}\right)\psi_{m,r}(u)\,du$$

$$\leqq 16\pi\,\mu_{m,r}\int_{1/2m}^{\pi}\left\{\omega\left(f;\ \frac{5u}{2}\sqrt{1-x^2}\ \right)+\omega\left(f;\ \frac{25}{4}u^2\right)\right\}\psi_{m,r}(u)\,du \tag{14}$$

$$\leqq 16\,\pi\mu_{m,r}\int_{1/2m}^{\pi}\left\{\ 5um\,\omega\ \left(f;\ \frac{\sqrt{1-x^2}}{m}\right)+\frac{25}{2}\ u^2\,m^2\,\omega\left(f;\ \frac{1}{m^2}\right)\right\}\psi_{m,r}(u)\,du.$$

Using (10) and the definition of $\psi_{m,r}$ we obtain from (14) ($r \geqq 2$)

$$\varphi(x) \leqq 4\pi\left(\frac{\pi}{60\,m}\right)^{2r-1}\int_{1/2m}^{\pi}\left\{\ 5um\,\omega\ \left(f;\ \frac{\sqrt{1-x^2}}{m}\right)+\frac{25}{2}u^2\,m^2\,\omega\left(f;\ \frac{1}{m^2}\right)\right\}\frac{du}{(u/\pi)^{2r}}$$

$$\leqq -2\pi^2\left(\frac{\pi^2}{60m}\right)^{2r-1}\left\{10m\,\omega\ \left(f;\ \frac{\sqrt{1-x^2}}{m}\right)u^{-2r+2}+25m^2\,\omega\ \left(f;\ \frac{1}{m^2}\right)u^{-2r+3}\right\}\Bigg|_{1/2m}^{\pi}$$

$$\leqq c_1\left(\frac{\pi^2}{30}\right)^{2r}\omega\left(f;\ \frac{\sqrt{1-x^2}}{m}+\frac{1}{m^2}\right),$$

where c_1 is an absolute constant.

Since $\pi^2/30 \leqq 1/3 \leqq 1/e$, we obtain

$$\varphi(x) \leqq c_1 e^{-2r}\,\omega\ \left(f;\ \frac{\sqrt{1-x^2}}{m}+\frac{1}{m^2}\right)=c_1 e^{-2r}\,\omega(f;\ \Delta_m(x)). \tag{15}$$

From (12) and (15) we obtain that for every $x \in [-1,\ 1]$ there exists y_x such that

$$|f_m(y_x)-U_{m,r}(\arccos x)| \leqq c_1\,e^{-2r}\,\omega\left(f;\ \frac{\sqrt{1-x^2}}{m}+\frac{1}{m^2}\right),\ y_x \in \Omega_m(x), \tag{16}$$

and c_1 is an absolute constant.

From (11) it follows that

$$|y_x-x| \leqq \frac{\sqrt{1-x^2}}{m}+\frac{1}{2m^2}<\Delta_m(x). \tag{17}$$

On the other hand, since $y_x \in \Omega_m(x)$, we have

$$|f_m(x)-f_m(y_x)| = |f_m(\cos t)-f_m((\cos(t+u))|,$$

where $|u| \leqq 1/2m$. Using (13) we obtain that

$$|f_m(x)-f_m(y_x)| \leqq 8\pi\omega\ \left(f;\ \left(\sqrt{1-x^2}+\frac{2}{m}\right)\frac{2}{m}\right) \leqq 32\pi\omega(f;\ \Delta_m(x)). \tag{18}$$

From (16) and (18) it follows ($r \geqq 2$)

$$|f_m(x)-U_{m,r}(\arccos x)| \leqq 32\pi\omega(f;\ \Delta_m(x))+c_1\,e^{-2r}\,\omega(f;\ \Delta_m(x)) \tag{19}$$

$$\leq c_2 \omega(f;\ \Delta_m(x)).$$

The inequalities (16), (17) and (19) give us

(20)
$$\min_{y\in[-1,1]} \max \left\{\frac{1}{\alpha}|x-y|,\ |U_{m,r}(\arc\cos x)-f_m(y)|\right\}$$
$$\leq \min\{c_2\omega(f;\ \Delta_m(x)),\ \max\{\alpha^{-1}\Delta_m(x),\ c_1 e^{-2r}\omega(f;\Delta_m(x))\}\}=\Phi(x,\ m,\ r),$$

where c_1 and c_2 are absolute constants.

This is an estimation for the deviation of $U_{m,r}$ from f_m. Now we shall obtain an estimation for the deviation of f_m from $U_{m,r}$. For the purpose we shall estimate the difference between f_m and $U_{m,r}$ at the points $z_i=\arccos\frac{(2i-1)\pi}{2m}$, $i=1,\ldots,m-1$.

Since

$$f_m\left(-\cos\left(\frac{(2i-1)\pi}{2m}+u\right)\right)=f_m\left(-\cos\left(\frac{(2i-1)\pi}{2m}\right)\right) \text{ for } |u|\leq 1/2m,$$

we obtain immidiately

$$|f_m(z_i)-U_{m,r}(\arc\cos z_i)|\leq\varphi_m(z_i)$$

or, using (15)

(21) $$|f_m(z_i)-U_{m,r}(\arc\cos z_i)|\leq c_1 e^{-2r}\omega(f;\ \Delta_m(x)),\ i=1,\ldots,m-1.$$

Since f_m is monotone in the interval $[z_i,\ z_{i+1}]$, $i=1,\ldots,m-2$, and $U_{m,r}$ is a continuous function, we see that for every $x\in[z_i,\ z_{i+1}]$ there exists $\theta_x\in[z_i,\ z_{i+1}]$ such that $(x=\cos t,\ \theta_x=\cos(t+u),\ |u|\leq 1/m)$

$$|x-\theta_x|\leq\left|2\sin\left(t+\frac{u}{2}\right)\sin\frac{u}{2}\right|\leq\frac{\sqrt{1-x^2}}{m}+\frac{1}{m^2}=\Delta_m(x),$$

(22)
$$|f_m(x)-U_{m,r}(f_m;\ \arc\cos\theta_x)|\leq c_1 e^{-2r}\omega(f;\ \Delta_m(x)).$$

For x belonging to one of the two end intervals $[-1,\ z_1]$ and $[z_{m-1},\ 1]$ we have $f_m(x)=f_m(z_1)$ or $f_m(x)=f_m(z_{m-1})$ and (22) is also fulfilled with $\theta_x=z_{m-1}$.

From (22) and (19) we obtain, analogically of (20)

(23) $$\min_{y\in[-1,1]}\max\left\{\frac{1}{\alpha}|x-y|,\ |f_m(x)-U_{m,r}(\arc\cos y)|\right\}\leq\Phi(x,\ m,\ r).$$

The inequalities (20) and (23) give us

(24) $$|f_m(x)-U_{m,r}(\arc\cos x)|_\alpha\leq\Phi(x,\ m,\ r).$$

Let us set now $r=\left[\frac{\ln(e^4-\alpha n^2)}{2}\right]$, $m=4\left[\frac{n}{120r}\right]$ ($[a]$ denotes the integer part of a), and let n be so large that $\left[\frac{n}{120r}\right]\geq 1$ (remember that $\alpha\leq 1$). Then $r\geq 2$, $m=4s$, $30rm\leq n$, consequently $U_{m,r}(\arc\cos x)$ is an algebraic polynomial of n-th degree. On the other hand,

$$\Phi(x,\ m,\ r)=\min\{c_2\omega(f;\ \Delta_m(x)),\ \max\{\alpha^{-1}\Delta_m(x),\ c_1 e^{-2r}\omega(f;\ \Delta_m(x))\}\}$$

(25)
$$\leq\min\left\{c_2\omega(f;\ \Delta_m(x)),\ \max\left\{\alpha^{-1}\Delta_m(x),\ \frac{ec_1\omega(f;\ \Delta_m(x))}{e^4+\alpha n^2}\right\}\right\}$$
$$\leq c_3\min\{\alpha^{-1}\Delta_m(x),\ \omega(f;\ \Delta_m(x))\},$$

where c_3 is an absolute constant.

From (6), (24) and (25) we obtain

$$|f(x)-U_{m,r}(x)|_\alpha \leqq |f(x)-f_m(x)|_\alpha + |f_m(x)-U_{m,r}(\arccos x)|_\alpha \tag{26}$$
$$\leqq c_4 \min\left\{\frac{\Delta_m(x)}{\alpha},\ \omega(f;\Delta_m(x))\right\} \leqq 2c_4 \frac{\omega(f;\Delta_m(x))}{1+\alpha\Delta_m^{-1}(x)\,\omega(f,\Delta_m(x))}.$$

Here $m=4\left[n/120\left[\frac{\ln(e^4+\alpha n^2)}{2}\right]\right]$, n is so large that $\left[n/120\left[\frac{\ln(e^4+\alpha n^2)}{2}\right]\right] \geqq 1$, $U_{m,r}$ is algebraic polynomial of n-th degree, c_4 is an absolute constant.

Obviously if we increase the constant c_4 we can obtain from (26) that for every natural number n there exists $P_n \in H_n$ such that for every $x \in [-1, 1]$ we have

$$|f(x)-P_n(x)|_\alpha \leqq c \frac{\omega(f;\Delta_m(x))}{1+\alpha\Delta_m^{-1}(x)\,\omega(f;\Delta_m(x))},$$

where $\Delta_m(x)=\frac{\sqrt{1-x^2}}{m}+\frac{1}{m^2}$, $m=n/\ln(e+2n^2)$ and c is an absolute constant.

Theorem 1 is proved.

REFERENCES

1. А. Ф. Тиман, Усиление теоремы Джексона с наилучшим приближением непрерывных функций многочленами на конечном отрезке вещественной оси. *Доклады АН СССР*, 81, 1951, 509—511.
2. Бл. Сендов. Некоторые вопросы теории приближений функций и множеств в метрике Хаусдорфа. *Успехи мат. наук*, 27, 1969, № 5, 141—178.
3. Bl. Sendov, V. A. Popov. On a generalization of Jaskson's theorem, *J. Approxim. Theory*, 9, 1973, 102—111.
4. Бл. Сендов. Апроксимиране на функции с алгебрични полиноми по отношение на една метрика от хаусдорфовски тип. *Годишник Соф. Унив., Физ. -мат. фак.*, 55, 1962, 1—39.
5. Бл. Сендов, В. А. Попов. Аналог теоремы С. М. Никольского для приближения функции алгебраическими многочленами в хаусдорфовой метрике: *Конструктивная теория функций*. Труды Межд. конф. по Конструктивной теории функций, Варна, 1970 София, 1972, 95—105.

Centre for Mathematics and Mechanics
1090 Sofia P. O. Box 373

Received 8.2. 1978