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COMMUTATIVE FIELDS NORMED BY A G-VALUATIVE ORDER

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A G-valuative order is a kind of valuation defined on a field and ranging over an ordered group The valuaton of a field ranging over $\mathbf{R}^+$, the positive cone of $\mathbf{R}$, induces a topological structure on this field. So archimedian and nonarchimedian valuations of Q induce a metric or hypermetric structure on it and lead to the construction of the fields $\mathbf{R}$ and Q_p, of real and p-adic numbers, respectively, Here, we are making an attempt at an analogous study referring to a commutative field and using the G-valuative orders.

1. Generalities. 1.1. The purpose of this paper is twofold: given a commutative field, valuated by a G-valuative order v, to make it a topological field and simultaneously to complete it by using the uniform structure established for this field.

More precisely, in theorem 2.1 it is proved that a G-valuated field K becomes a topological field. Hence it has the structure of a uniform space.

Moreover a geometrical representation of the field is described in sections 3.1-3.3.

Finally, in 3.4 we construct a new field, completing the uniform structure of K.

1.2. It is known that every ordered space $(G, \leqq)$ can be embedded to a complete lattice which is the well-known "Mac Neille's complement" of G.

It is also known (see [2]) that every ordered space $(G, \leqq)$ can be embedded to another ordered space, called "Kurepa — Dokas' complement" (symb. $(\widetilde{G}, \leqq)$).

We have the following:

(1) $(\widetilde{G}, \leqq)$ is a complete lattice (see [8]).

(2) Let (A, B) be a cut in G, in the meaning of Mac Neille's theory. If (A, B) is a gap (that is A has not a supremum and B has not an infimum), then in Mac Neille's complement there exists a new element lying between A and B.

On the other hand, in $\widetilde{G}$ the new elements have been put at the end of every class A (resp. B), which has not an end point, that is maximum (resp. minimum).

So, for the a, b non-comparable elements of G, the infimum of a and b in $\widetilde{G}$ will be an element of $\widetilde{G}\setminus G=\{x\in\widetilde{G}: x\notin G\}$. Denote this element by $\inf_{\widetilde{G}}\{a, b\}$.

1.3. Let us consider a commutative field $(K, +, .)$, a partially ordered Abelian group $(G, +, \leqq)$, and its Kurepa—Dokas complement $(\widetilde{G}, \leqq)$. The structures in the ordered group are compatiable in the usual meaning; that is, for every a, b, c of G, the relation $a\leqq b$ implies $a+c\leqq b+c$.

We also considered a new element ∞, such that for each $\gamma\in G$, $\gamma<\infty$ and $\gamma+\infty=\infty+\gamma=\infty+\infty=\infty$ and let $\widehat{G}=G\cup\{\infty\}$.

A G-valuative order v (see [5]) having the commutative field $(K, +, .)$ as a domain and $(\widehat{G}, +, \leqq)$ as a range, is a function which satisfies the following conditions:

for every x, y of K

(i) $v(x)=\infty \Leftrightarrow x=0$

(ii) $v(x.y)=v(x)+v(y)$

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(iii) $v(x)=v(-x)$

(iv) $v(x+y)\geqq\inf_{\tilde{G}}\{v(x), x(y)\}$, the triangle inequality.

It has been proved that the last statement is equivalent (and this equivalence is used for every ambiguous case) to the following one:

if $v(x)>c$, $v(y)>c$ then $v(x+y)>c$, for $c\in G$.

1.4. We end this paragraph with an arrangement of the structure $(G, +, \leqq)$, which further will be often made use of.

If we denote by G^* the (maximal) torsion subgroup of G, then the factor group G/G^* will be a torsion free group and considering the structure $(G/G^*, +, \leqq)$, where $\leqq$ is the natural order induced by the initial one, we can extend this structure to a totally ordered Abelian group by Lorenzen — Simbireva — Everett's theorem (see [4], p. 39). Denote this total order by $\tilde{\leqq}$. Thus the extension of the initial order $\leqq$ of G to another partial order $\leqq_1$,

$$a\leqq_1 b \Leftrightarrow \bar{a} \tilde{\leqq} \bar{b},$$

where $\bar{a}$, $\bar{b}$ are the corresponding classes (mod G^*) of a and b respectively will be the consequence and the following statements will hold

(A1) Every class of G (mod G^*), for $G^*\neq\{0\}$, contains parallel elements.

(A2) if $\bar{a}\tilde{<}\bar{a}'$, then for each $b\in\bar{a}$ and $b'\in\bar{a}'$, we have $b<_1 b'$.

2. G-valuated topological fields. Supposing that a G-valuative order is defined on a field we induce a topology on it (cf. [7], p. 65 for the case of a linear ordered value group). In this paragraph the structure $(K, +, .)$ is a commutative field and v is a G-valuative order with domain the field K and ranging over an Abelian group; the image of $K^+\setminus=K\{0\}$ by v is a group $(G, +, \leqq)$ too (cf. [5], p. 66). In this case K is called the G-valuated by v and G the value group of v. The set $G^+=\{x\in G\colon 0\leqq x\}$ is called the positive cone of G.

Theorem 2.1. *If K is a G-valuated by a G-valuative order v with value group G and if for every subset of G there exists an upper bound in G^+, then a T_1-topology is introduced in K, which makes K a topological field, hence a Tychonoff uniform space.*

Proof. Let us consider the class Γ of the subsets $V_\gamma(x_0)$, $V_\gamma(x_0)=\{x\in K: v(x-x_0)>\gamma\}$, for every x in K, where γ goes through G^+. This class Γ forms a fundamental system of neighbourhoods of x_0 and the demonstration will be realized in four steps. (We denote by $-V_\gamma(x_0)$ (resp. by $[V_\gamma(x_0)]^{-1}$, $x_0\neq 0$) the subset:

$$\{x\in K\colon -x\in V_\gamma(x_0)\} \quad (\text{resp. } \{x\in K: x^{-1}\in V_\gamma(x_0)\}).$$

1rst step. $(K, +)$ is a topological group. First we remark that for every $\gamma\in G^+$ the subset $V_\gamma(0)$ is a subgroup of $(K, +)$ and that $V_\gamma(0)=-V_\gamma(0)$. Indeed; for x, y in $V_\gamma(0)$, $v(x-y)\geqq\inf_{\tilde{G}}\{v(x), v(y)\}>\gamma$, hence $x-y\in V_\gamma(0)$, while the second assertion is obvious.

Now it is sufficient to show that

(i) the class $\Gamma_0=(V_\gamma(0))_{\gamma\in G^+}$ is a filter-base in K.

(ii) $(\forall_\gamma\in G^+)\ (\exists\gamma_1\in G^+)\ [V_{\gamma_1}(0)+V_{\gamma_1}(0)\subset V_\gamma(0)]$.

The (i)-statement is a simple consequence of the fact that G is up directed and that: if $\gamma<\gamma'$, then $V_{\gamma'}(0)\subset V_\gamma(0)$. On the other hand, $V_\gamma(0)$ is a group, so $V_\gamma(0)+V_\gamma(0)\subset V_\gamma(0)$ and (ii) is proved

2nd step. $(K, +, .)$ is a topological field. We should prove

(iii) If x, y are elements of K and $\gamma \in G^+$, then there is a $\delta \in G^+$, such that $V_\delta(x) . V_\delta(y) \subset V_\gamma(xy)$ and

(iv) if $x \in K^* = K \setminus \{0\}$, and $\gamma \in G^+$, then there exists a $\delta \in G^+$ such that $[V_\delta(x)]^{-1} \subset V_\gamma(x^{-1})$.

D e m o n s t r a t i o n of (iii). Firstly,

(1) $$V_\gamma(x_0) = V_\gamma(0) + \{x_0\}.$$

Indeed, if $y \in V_\gamma(0) + \{x_0\}$, $y = x + x_0$ with $v(x) > \gamma$, then $v(x_0 - y) = v(x) > \gamma \Rightarrow y \in V_\gamma(x_0)$. On the other hand, if $y \in V_\gamma(x_0)$, then $v(x_0 - y) > \gamma$ and putting $x = y - x_0$, we have $v(x) > \gamma$, $y = x + x_0$ with $x \in V_\gamma(0)$, and (1) has been proved.

Now from (1) we have successively

$$V_\delta(x) . V_\delta(y) = [V_\delta(0) + \{x\}] . [V_\delta(0) + \{y\}] = V_\delta(0) . V_\delta(0) + V_\delta(0) [\{x\} + \{y\}] + \{x\} . \{y\}$$

and it is sufficient to find a $\delta \in G^+$ such that

(2) $$V_\delta(0) \cdot V_\delta(0) + V_\delta(0) \ [\{x\} + \{y\}] \subset V_\gamma(0)$$

if Z is an element of the left side of (2), then $Z = Z_1 . Z_2 + Z_3(x+y)$, where Z_1, Z_2, Z_3 in $V_\delta(0)$.

$$v(Z) = v(Z_1 . Z_2 + Z_3(x+y)) \geqq \inf_{\tilde{G}} \{v(Z_1 . Z_2),\ v(Z_3(x+y))\} = \inf_{\tilde{G}} \{v(Z_1) + v(Z_2),$$

(3) $$v(Z_3) + v(x+y)\} \leqq \inf_{\tilde{G}} \{\delta + \delta,\ \delta + v(x+y)\} = \delta + \inf_{\tilde{G}} \{\delta,\ v(x+y)\}.$$

It is evident from (2) and (3) that it is sufficient for δ to fulfil the relation

(4) $$\gamma \leqq \delta + \inf_{\tilde{G}} \{\delta,\ v(x+y)\}.$$

Suppose that $\delta > v(x+y)$ and consider a $\varkappa \in G^+$ such that, $\varkappa > -v(x+y)$. Hence for $\delta \geqq \gamma + \varkappa$, there holds $\delta + \inf_{\tilde{G}} \{\delta,\ v(x+y)\} > \delta - \varkappa \geqq \gamma$ and the statement (iii) has been proved.

D e m o n s t r a t i o n of (iv). Let $y \in V_\delta(x)$; then $v(y - x) > \delta$. The required statement "$y \in [V_\gamma(x^{-1})]^{-1}$" is equivalent to "$y^{-1} \in V_\gamma(x^{-1})$", that is to say,

(5) $$v\left(\frac{1}{y} - \frac{1}{x}\right) > \gamma,\ v\left(\frac{y-x}{xy}\right) = v(y-x) - v(x) - v(y) > \gamma.$$

But $v(x-y) - v(x) - v(y) > \delta - v(x) - v(y)$ and (5) is changed into

(6) $$\delta - v(x) - v(y) > \gamma.$$

If $\delta > v(x)$, we have:

$$v(y) \geqq \inf_{\tilde{G}} \{v(x-y),\ v(x)\} \geqq \inf_{\tilde{G}} \{\delta,\ v(x)\}$$

as well as $v(x) \geqq \inf_{\tilde{G}} \{\delta,\ v(y)\}$. The last two relations mean that $v(x)$ and $v(y)$ are equal or parallel.

On the other hand, there exists a $\varkappa \in G^+$, such that $\varkappa > v(x)$. So if $\delta > 2\varkappa + \gamma$, then for every y, such that $v(x-y) > \delta$ hold (because $v(x)$ and $v(y)$ are equal or parallel) $-v(y) > -\varkappa$ and $\delta - v(x) - v(y) > \delta - 2\varkappa > \gamma$.

Hence (5) is satisfied and (iv) is proved.

3rd s t e p. $(K, +, .)$ is a uniform space and certainly it is a proximity space because $(K, +)$ is a topological group.

4rth s t e p. $\{0\} = \overline{\{0\}}$, 0 is the neutral element of $(K, +)$.

Indeed, for $x \neq 0$ and $\delta > v(x)$, $0 \notin V_\delta(x)$.

Remark 2.1. If the partially ordered group is not up directed, the induced topology in K will be a discrete one.

Remark 2.2. Constructing the surroundings of uniformity, we consider for every $\gamma \in G^+$, the subset $D_\gamma = \{(x, y) \in K \times K : x - y \in V_\gamma(0)\}$ and that will mean that $D_\gamma = \{(x, y) \in K \times K : v(x-y) > \gamma\}$.

Remark 2.3. Constructing in the usual way the proximity induced by the above uniformity, we are not able to express this proximity in an explicit form. However there exists a relation — denoted below by δ-between the subsets of K, which is a proximity; the topologies induced by the preceding uniformity and by this proximity coincide.

So, if A and B are subsets of K define:

$A\delta B$ iff the subset $\{V(x-y); x \in A, y \in B\}$ is cofinal to G^+ or contains the element ∞.

We prove that δ is a proximity on K. It is sufficient to show the following statements (the remaining are obvious):

(B1) $A\delta(B \cup C)$ if "$A\delta B$ or $A\delta C$", for A, B, C subsets of K.

(B2) If $A\bar{\delta}B$, then there exist non void subsets C and D of K, such that $C \cap D = \emptyset$ and $A\bar{\delta}(K \setminus C)$, $B\bar{\delta}(K \setminus D)$.

Demonstration of (B1): First it is not difficult to prove the next two statements:

(i) If in the expression $A-(B \cup C)$, the symbol "$-$" is an algebraic operation, then it holds that: $A-(B \cup C) = (A-B) \cup (A-C)$

(ii) For every couple of subsets A and B of K, such that the union $A \cup B$ is cofinal to G^+, one of A and B is cofinal to G^+ and conversely.

Now, by (i), it holds that: $\{v(x-y) : x \in A$ and $y \in B \cup C\} = \{v(x-y) : (x \in A$ and $y \in B\}$ or $(x \in A$ and $y \in C)\} = \{v(x-y) : x \in A$ and $y \in B\} \cup \{v(x-y) : x \in A$ and $y \in C\}$.

These relations and (ii) imply that $A\delta(B \cup C) \Leftrightarrow (A\delta B)$ or $(A\delta C)$.

Demonstration of (B2): We suppose that $A\bar{\delta}B$. It means that there exists an element $\gamma \in G^+$, such that each element of $\{v(x-y) : x \in A$ and $y \in B\}$ is smaller than or parallel to γ. Consider $\gamma_1 \in G^+$, $\gamma_1 \geqq \gamma$ and the set M of all x in K, such that each element of the set $v(x-B)$ is smaller than or parallel to γ_1.

Next we consider $\gamma_2 \in G^+$, $\gamma_2 \geqq \gamma_1$, and the set $C = \{x \in M : (\exists a \in A)\ [v(x-a) > \gamma_2]\}$. Evidently: $A \subset M$, $M \neq \emptyset$ and $A \subset C$.

Suppose that $A\delta(K-C)$. Then the set: $\{v(x-y) : x \in A$ and $y \in K-C\}$ is cofinal to G^+, that is, there exist $a \in A$ and $x \in K-C$, such that $v(x-a) > \gamma_2$, hence $x \in C$, which is absurd. So,

$$\text{(a)} \qquad A\bar{\delta}(K \setminus C).$$

Symmetrically, choosing γ_2', $\gamma_1' \in G^+$, $\gamma_2' \geqq \gamma_1' \geqq \gamma$, we define the subsets: $N = \{x \in K$: each element of the set $v(x-A)$ is smaller than or parallel to $\gamma_1' \in G^+\}$ and $D = \{x \in N$: $(\exists b \in B)\ [v(x-b) > \gamma_2']\}$.

Similarly:

$$\text{(b)} \qquad B\bar{\delta}(K \setminus D).$$

We have to prove that

$$\text{(c)} \qquad C \cap D = \emptyset$$

Suppose that $x \in C \cap D$. Then $(\exists a \in A)\ [v(x-a) > \gamma_2]$, because $x \in C$. Besides, "x belongs to D" means that for every $a \in A$, each element $v(x-a)$ is smaller than or parallel to γ_1'. But we can suppose that $\gamma_2 > \gamma_1'$ and so the element x cannot satisfy the last two statements at one and the same time. Thus (c) has been proved.

The assertion (B2) is derived from the above (a), (b) and (c).

We must point out that the proximity structure is a separated one. In fact, for a and b elements of K, $a\delta b$ implies $a=b$.

We intend to prove that the topology induced by the above proximity coincides with the one induced in theorem 2.1.

For $A\neq\emptyset$ a subset of K, put $\tilde{A}$ its closure for the proximity and $\tilde{A}$ its closure for the second topology, i. e. $\tilde{A}=\{x\in K$: the subset $v(x-A)$ is cofinal to G^+ or contains $\infty\}$ and

$$\bar{A}=\{x\in K(\forall\gamma\in G^+)\ [V_\gamma(x)\cap A\neq\emptyset]\}.$$

Let $x\in\tilde{A}$. Then, for every $\gamma\in G^+$, there exists an element $a\in A$, such that $v(x-a)>\gamma$, hence $a\in V_\gamma(x)$.

Now let $x\in\bar{A}$. Then, for every $\gamma\in G^+$, $V_\gamma(x)\cap A\neq\emptyset$ and that means that there exists an $a\in A$, with $v(x-a)>\gamma$, hence the subset $v(x-A)$ is cofinal to G^+. The proof is completed.

3. **G-Hypermetroid Spaces.** 3.1. In this paragraph we use the term "G-hypermetroid space" for a space K, on which has been defined a function p, ranging over a partially ordered abelian group $(G, +, \leq)$ and having the properties (for every x, y, z of K):

(i) $p(x, y)=\infty\Leftrightarrow x=y$

(ii) $p(x, y)=p(y, x)$

(iii) $p(x, z)\geq\inf_{\tilde{G}}\{p(x, y), p(y, z)\}$ (the triangle inequality).

The above statements are the dual ones of those which hold for a hypermetric space. Also it is evident that the G-valuative order v of K, makes the field K a G-hypermetroid space by the relation $p(x, y)=v(x-y)$, while for every $\gamma\in G^+$, the subset $\{(x, y)\in K\times K: v(x-y)<\gamma\}$ is a surrounding for the uniformity defined on K by v.

3.2. As usual we will say that an (unordered) triple (x, y, z) of elements of K is a triangle with sides xy, yz, xz and vertices x, y, z. $p(x, y)$ expresses the "proximity of the vertices x and y". If $p(x, y)>p(w, z)$, we say that the side xy is larger than the side wz, while the term "isoscele" preserves the usual meaning. Finally the subset $B_r(a)=\{x\in K: p(a, x)\geq r\}$ is called "a sphere with a centre x and a radius r", $r\in G^+$.

We give two results related to these notions; some other results are established in an analogous way as in the case of hypermetric spaces.

Proposition 3.1. *If the sides of a triangle are comparable one to another, then the triangle is isoscele with a base larger than or equal to the equal sides.*

Proof. Let (x, y, z) be a triangle. Put $p(x, y)=a$, $p(x, z)=b$, $p(y, z)=\gamma$.

Suppose that $\gamma\geq a$. If $a=\gamma$, the proof is completed. Now let $\gamma>a$. Then $b\geq\inf_{\tilde{G}}\{a, \gamma\}=\inf_{\tilde{G}}\{a\}$ and because of the comparability of a and b, $b\geq a$.

On the other hand, $a\geq\inf_{\tilde{G}}\{b, \gamma\}$ and because $a<\gamma$, we have $a\geq b$.

Proposition 3.2. *In a G-hypermetroid space, every point of a sphere can be considered as its centre.*

Proof. For a sphere $B_r(a)$ and x, y being two elements of it, we have $p(x, y)\geq\inf_{\tilde{G}}\{p(x, a), p(y, a)\}\geq r$. Hence $B_r(a)=B_r(x)$.

3.3. We consider again the structures $(K, +, \cdot)$, $(G, +, \leq)$, the G-valuative order v of K onto $\hat{G}=G\cup\{\infty\}$, as well as the fully ordered Abelian group $(G/G^*, +, \tilde{\leq})$ and the extended structure of G, $(G, +, \leqq_1)$, as they have been described at the end of 1. Put $\bar{e}$ for the image-by v-of any element $e \in K$, $C_{\bar{e}}$ for the class of $\bar{e}$ (mod G^*) and S_e for the subset $\{x \in K: v(x) \in C_{\bar{e}}\}$. For simplicity we suppose that the index e of any S_e, is a preimage of an element belonging to a system Γ of representatives of the factor group G/G^*.

We also note, for each $e^* \in S_e$, $I_{e^*}=\{x \in \bar{S}_e : \bar{x}=\overline{e^*} \in C_{\bar{e}}\}$ and we always consider that $\bar{1}=0$, (0 is the neutral element of G and represents G^+ in Γ).

For this notation, S_1 covers G^* by v.

P r o p o s i t i o n 3.3. *$(I_1, .)$ is a subgroup of $(S_1, .)$, while $(S_1, .)$ is a subgroup of $(K^*, .)$, $K^*=K-\{0\}$.*
The proof is obvious.

P r o p o s i t i o n 3.4. *$S_1=I_1 . \chi$ where χ is a system of representatives of the factor group S_1/I_1.*

P r o o f. Let x be an arbitrary element of S_1 and $x^* \in \chi$, such that $\bar{x}=\overline{x^*}$. Then, if $e^*=x . x^{*-1}$, $\overline{e^*}=0$. It means that $e^* \in I_1$ and $x=x^* . e^*$, where $x^* \in \chi$ and $e^* \in I_1$.

On the other hand, for each $x^* \in \chi$, $\overline{x^*} \in G^*$ and $I_1 . \chi \subset S_1$.

P r o p o s i t i o n 3.5. *If $a \in S_1 \setminus I_1$, then $I_a=a . I_1$.*

P r o o f. Let $x \in I_a$. Then $\overline{x.a^{-1}}=\bar{x}-\bar{a}=0$, that means that $a^*=x . a^{-1} \in I_1$. Conversely; let $y \in (a . I_1)$, that is $y=a . x$, where $x \in I_1$. Then $\bar{y}=\overline{a . x}=\bar{a}+\bar{x}=\bar{a}$, hence $y \in I_a$.

P r o p o s i t i o n 3.6. *For each $e^* \in S_e$ there exists an element $x^* \in \chi$, such that $I_{e^*}=I_e . I_{x^*}$, where χ is defined above* (fig. 1).

P r o o f. Let $x^*=e^* . e^{-1}$. If $x' . x'' \in I_e . I_{x^*}$, then $\overline{x' . x''}=\overline{x'}+\overline{x''}=\bar{e}+\overline{x^*}=\overline{e . x^*}=\overline{e^*}$, hence $I_e . I_{x^*} \subset I_{e^*}$.

On the other hand, if $y \in I_{e^*}$, then $\overline{y . e^{*-1}}=\bar{y}-\overline{e^*}=0$, hence $y . e^{*-1} \in I_1$ and $y=e^*.i$, where $i \in I_1$. So, $y=(e . x^*) . i=e . (x^* . i)=e . k$, where $k \in J_{x^*}$; hence $y \in I_e . I_{x^*}$ and the proof is completed.

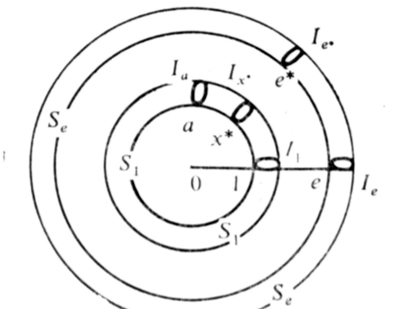

Fig. 1

The following proposition holds too. Its proof is obvious.

P r o p o s i t i o n 3.7. *Each class S_e (mod S_1) is equal to $I_e . \chi$, where χ is as above.*

Now it is easy to define a natural order $\underset{\sim}{\alpha}$ on K^* by the statements:

(1) If e, e^* are preimages of two representatives $\bar{e}$ and $\overline{e^*}$ resp. of G/G^*, then

$$e \underset{\sim}{\alpha} e^* \Leftrightarrow \overline{e^*} \mathrel{\tilde{\leqq}} \bar{e}.$$

(2) If x, y are elements of K^*, $x \in S_e$, $y \in S_{e^*}$, then "$x \alpha y$ if $e \alpha e^*$" and "$x = y$ or $x // y$ if $e = e^*$".

3.4. As it has been explained in **2**, K would be considered a uniform space and so we can define on K Cauchy filters and by them we can complete K to another uniform space and K will be a dense subset of it.

Evidently this procedure will be analogous to the construction of the fields R and Q_p of real and p-adic numbers respectively, from the set of rational numbers Q. This construction is realized, completing Q, by the natural or p-adic distance on Q.

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