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CURVATURE IDENTITIES FOR ALMOST CONTACT METRIC MANIFOLDS OF HERMITIAN TYPE

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ABSTRACT. We introduce the class of almost contact metric manifolds of Hermitian type as a subclass of the normal almost contact metric manifolds. Identities for the Riemanian curvature tensor on such a class are established.

0. Introduction. The problem about the identities for the Riemanian curvature tensor on Hermitian and almost Hermitian manifolds has been solved by A. Gray [9] and F. Tricerry and L.Vanhecke [4]. The complete decomposition of the space of tensors, having the same symmetries, as the Riemanian curvature tensor, into mutually orthogonal, $U(n)$-invariant and irreducible factors over an almost Hermitian real vector space in [4] is given. The same problem, but for the Hermitian connection and the corresponding Hermitian curvature tensor has been solved by Ganchev, Ivanov, Michova [2]. In the case of almost contact metric manifolds the analogous problems are not solved completely. Some identities for the Riemanian curvature tensor in the case of quasi-Sasakian manifolds, are given in [5]. In [1] the problem for the existence of Hermitian connection on an almost contact metric manifold is treated.

In the present paper we apply the scheme of [4], [9], using the method of local complex coordinate systems, as in [2], on the naturally arising class of almost contact metric manifolds of Hermitian type. On such a manifold, the identities for the Riemanian curvature tensor are given.

1. Preliminaries. Let M be a $2n+1$-dimensional differentiable manifold with *almost contact metric structure* (φ, ξ, η, g), where φ is a tensor field of type (1,1), ξ is a vector field, η is a 1-form, g is a definite metric tensor field , such that

$$\varphi^2 = -\mathrm{id} + \eta \otimes \xi, \quad \varphi(\xi) = 0, \quad \eta(\xi) = 1, \quad g = g \circ \varphi + \eta \otimes \eta.$$

On such a manifold arise the operators $h = -\varphi^2, \ \ v = \eta \otimes \xi$, such that

$$h(T_pM) = \mathrm{Ker}\,\eta_p, \quad v(T_pM) = \mathrm{Im}\,\eta_p \quad \text{and} \quad T_pM = h(T_pM) \oplus v(T_pM), \quad p \in M.$$

The *contact distribution* $D_p = \operatorname{Ker}\eta_p$, $p \in M$ contains $2n$-dimensional real vector spaces with almost complex structure φ_p and hence D_p carries an orthonormal φ-basis $\{e_\alpha; \varphi e_\alpha\}_{\alpha\in I}$, $I = \{1,\dots,n\}$. Let D_p^C be the complexification of D_p. Then the following decomposition $D_p^C = D_p^{10} \oplus D_p^{01}$ is valid, where the subspaces $D_p^{10}(D_p^{01})$ of vectors of type $(1,0)((0,1))$ are spanned by the vectors $Z_\alpha = e_\alpha - i\varphi e_\alpha$, $\alpha \in I$ $(Z_{\bar\alpha} = \overline{Z_\alpha}$, $\bar\alpha \in \bar I = \{\bar 1,\dots,\bar n\})$. Then D_p^C has basis $\{Z_a\}_{a\in I\cup\bar I}$ and putting $Z_o = \xi_p$ we have that the vectors $\{Z_A\}_{A\in I\cup\bar I\cup I_o}$ form a basis for the space $T_p^C M = D_p^C \oplus \xi_p$, where $I_o = \{o = \bar o\}$. Unless otherwise stated, Greek small letters will be run through the index-set I, Latin small – through $I\cup\bar I$ and Latin capital - through $I\cup\bar I\cup I_o$.

The structure (φ,ξ,η,g) is said to be *normal* if the tensor field $N = [\varphi,\varphi] + 2d\eta\otimes\xi$ of type (1,2) vanishes identically, where $[\varphi,\varphi]$ is the Nijenhuis tensor field, formed with $\varphi : [\varphi,\varphi](x,y)= =[\varphi x,\varphi y] - \varphi[\varphi x,y] - \varphi[x,\varphi y] - \varphi^2[x,y]$, $x,y\in T_pM$. In that case $D_p^{10}(D_p^{01})$ form an involutive distribution [3]. The tensor field of type $(0,3)$, corresponding to N, will be denoted by the same letter:

$$N(x,y,z) = g[N(x,y),z], \quad x,y,z\in T_pM.$$

Let $\Phi : \Phi(x,y) = g(x,\varphi y)$, $x,y\in T_pM$, be the fundamental 2-form on M, ∇ be the Levi-Civita connection of the metric g and $F = -\nabla\Phi$. The properties formulated in the following Lemma are well known [1],[6], [7]:

Lemma 1 . *Let $M^{2n+1}(\varphi,\xi,\eta,g)$ be an almost contact metric manifold. Then for all $x,y,z\in T_pM$ the next equalities are true:*

a) $F(x,y,z) = g[(\nabla_x\varphi)y,z] = -F(x,z,y) = -F(x,\varphi y,\varphi z) + [\eta\wedge(\nabla_x\eta)\circ\varphi](y,z);$

b) $3d\Phi(x,y,z) = -\underset{(xyz)}{\mathcal{G}} F(x,y,z)$, *($\mathcal{G}$ – cyclic summation);*

c) $3d\Phi(\xi,y,z) = -F(\xi,y,z) + (\nabla_y\eta)\varphi z - (\nabla_z\eta)\varphi y;$

d) $N(x,y,z) = -3d\Phi(\varphi x,y,z) + 3d\Phi(\varphi y,x,z) + 2F(z,y,\varphi x) + 2[\eta\otimes(\nabla\eta - 2d\eta)](x,y,z).$

2. Almost contact metric manifolds of Hermitian type. Analogously to the canonical complex structure on $\mathbf{R}^{2n}\equiv\mathbf{C}^n$ [8] we are going to consider the canonical normal structure on $\mathbf{R}^{2n+1}$.

We consider $\mathbf{R}^{2n+1}$ as $\mathbf{C}^n\times\mathbf{R}$ by the natural isomorphism

$$\mathbf{R}^{2n+1} \overset{1-1}{\longleftrightarrow} \mathbf{C}^n\times\mathbf{R},$$

$$\begin{aligned}(x,y,t)=(x^1,\dots,x^n,y^1,\dots y^n,t) &\longleftrightarrow (z,t) = (z^1,z^2,\dots,z^n,t)\\ &=(x^1+iy^1,\dots,x^n+iy^n,t).\end{aligned}$$

Let (φ,ξ,η) be the structure on $\mathbf{R}^{2n+1}$ defined by

$$\varphi = \delta_{\alpha\beta}(dx^\alpha\otimes\frac{\partial}{\partial y^\beta} - dy^\alpha\otimes\frac{\partial}{\partial x^\beta}), \quad \xi = \frac{\partial}{\partial t}, \quad \eta = dt,$$

where $(U; \{x^\alpha, y^\alpha, t\})$ is a local coordinate system, $\mathfrak{X}U = \mathrm{span}\{\frac{\partial}{\partial x^\alpha}, \frac{\partial}{\partial y^\alpha}, \frac{\partial}{\partial t}\}$, $\mathfrak{X}^*U = \mathrm{span}\{dx^\alpha, dy^\alpha, dt\}$ and $\delta_{\alpha\beta}$ is Kroneker's symbol.

It is clear that this structure is an almost contact structure and it is an extension of the canonical complex structure on $\mathbf{R}^{2n}$:

$$\varphi(\frac{\partial}{\partial x^\alpha}) = \frac{\partial}{\partial y^\alpha}, \quad \varphi(\frac{\partial}{\partial y^\alpha}) = -\frac{\partial}{\partial x^\alpha}, \quad \varphi(\frac{\partial}{\partial t}) = \varphi(\xi) = 0,$$

$$\eta \circ \varphi = 0, \quad \eta(\xi) = 1, \quad \varphi^2 = -\mathrm{id} + \eta \otimes \xi.$$

We call this structure *canonical normal structure on* $\mathbf{R}^{2n+1}$.

Proposition 1. *Let (φ, ξ, η) and $(\bar\varphi, \bar\xi, \bar\eta)$ be the canonical normal structures on $\mathbf{R}^{2n+1}$ and $\mathbf{R}^{2m+1}$ respectively and let f be a transformation, such that*

$$f : \mathbf{R}^{2n+1} \longrightarrow \mathbf{R}^{2m+1} \doteq \{\overline{x^1}, \dots, \overline{x^m}, \overline{y^1}, \dots, \overline{y^m}, \bar{t}\}$$

$$(x, y, t) \longrightarrow f(x, y, t) = (u^\alpha(x, y, t), v^\alpha(x, y, t), q(x, y, t)), \quad \alpha \in J = \{1, \dots, m\}.$$

Then:

a) $\bar\xi$ corresponds to ξ (and hence η to $\bar\eta$) by f, i.e. $f_*\xi = \tau\bar\xi$ ($\tau = \tau(x, y, t)$ nonvanishing function) iff u^α, v^α do not depend on t, i.e.

$$u^\alpha = u^\alpha(x, y), \quad v^\alpha = v^\alpha(x, y) \;\; and \;\; q = q(x, y, t), \quad \frac{\partial q}{\partial t} \neq 0, \quad \alpha \in J;$$

b) the diagram

$$\begin{array}{ccc} \mathfrak{X}\mathbf{R}^{2n+1} & \xrightarrow{f_*} & \mathfrak{X}\mathbf{R}^{2m+1} \\ \varphi \downarrow & & \downarrow \bar\varphi \\ \mathfrak{X}\mathbf{R}^{2n+1} & \xrightarrow{f_*} & \mathfrak{X}\mathbf{R}^{2m+1} \end{array}$$

is commutative, i.e. $f_* \circ \varphi = \bar\varphi \circ f_*$ iff q depends only on t, i.e.

$$u^\alpha = u^\alpha(x, y, t), \quad v^\alpha = v^\alpha(x, y, t), \quad q = q(t) \;\; and$$

$$(1) \qquad \left| \begin{array}{l} \dfrac{\partial u^\alpha}{\partial x^\beta} = \dfrac{\partial v^\alpha}{\partial y^\beta} \\ \dfrac{\partial u^\alpha}{\partial y^\beta} = -\dfrac{\partial v^\alpha}{\partial x^\beta}, \quad \alpha \in J, \quad \beta \in I; \end{array} \right.$$

c) the structure $(\bar\varphi, \bar\xi, \bar\eta)$ corresponds to (φ, ξ, η) by f i.e.

$$f_*\xi = \tau\bar\xi \;\; and \;\; f_* \circ \varphi = \bar\varphi \circ f_*$$

iff $u^\alpha = u^\alpha(x,y)$, $v^\alpha = v^\alpha(x,y)$, $q = q(t)$, $q'(t) \neq 0$ *and the Caushy – Riemann conditions (1) are valid.*

Proof. a) $f_*\xi - \tau\bar{\xi} = 0$ is equivalent to

$$(f_*\frac{\partial}{\partial t} - \tau\frac{\partial}{\partial \bar{t}})(\overline{x^\alpha})(\partial/\partial\overline{x^\alpha}) + (f_*\frac{\partial}{\partial t} - \tau\frac{\partial}{\partial \bar{t}})(\overline{y^\alpha})(\partial/\partial\overline{y^\alpha}) + (f_*\frac{\partial}{\partial t} - \tau\frac{\partial}{\partial \bar{t}})(\bar{t})\frac{\partial}{\partial \bar{t}} = 0,$$

i.e. $\frac{\partial u^\alpha}{\partial t}(\partial/\partial\overline{x^\alpha} + \frac{\partial v^\alpha}{\partial t}(\partial/\partial\overline{y^\alpha} + (\frac{\partial q}{\partial t} - \tau)\frac{\partial}{\partial \bar{t}} = 0$, which proves a).

b)

$$f_*\circ\varphi - \bar{\varphi}\circ f_* = 0 \iff \left| \begin{array}{l} 1)\ (f_*\circ\varphi - \bar{\varphi}\circ f_*)(\frac{\partial}{\partial x^\beta}) = 0, \\ 2)\ (f_*\circ\varphi - \bar{\varphi}\circ f_*)(\frac{\partial}{\partial y^\beta}) = 0, \\ 3)\ (f_*\circ\varphi - \bar{\varphi}\circ f_*)(\frac{\partial}{\partial t}) = 0 \ \text{ trivial.} \end{array} \right.$$

1) implies

$$(\frac{\partial u^\alpha}{\partial y^\beta} + \frac{\partial v^\alpha}{\partial x^\beta})(\partial/\partial\overline{x}^\alpha) + (\frac{\partial v^\alpha}{\partial y^\beta} - \frac{\partial u^\alpha}{\partial x^\beta})(\partial/\partial\overline{y}^\alpha) + \frac{\partial q}{\partial y^\beta}\frac{\partial}{\partial \bar{t}} = 0,$$

i.e.(1) are true and $\frac{\partial q}{\partial y^\beta} = 0$. Analogously 2) implies (1) and $\frac{\partial q}{\partial x^\beta} = 0$ and hence (1) and $q = q(t)$ are valid.

c) The proof follows from a) and b). □

Let $M^{2n+1}(\varphi,\xi.\eta)$ and $M^{2m+1}(\bar{\varphi},\bar{\xi},\bar{\eta})$ be almost contact manifolds.

Definition. The transformation $f : M^{2n+1}(\varphi,\xi,\eta) \longrightarrow M^{2m+1}(\bar{\varphi},\bar{\xi},\bar{\eta})$ is said to be *strict contact holomorphic* if f is *strict contact*, i.e. $f_*\xi = \bar{\xi}$ (and hence $f^*\bar{\eta} = \eta$) [3] and *contact holomorphic*, i.e. $(\bar{\varphi},\bar{\xi},\bar{\eta})$ corresponds to (φ,ξ,η) by f at once.

The set $H\Gamma^o_{2n+1}$ of all strict contact holomorphic transformations in $\mathbf{R}^{2n+1}$ is a pseudogroup and we consider $(2n+1)$-dimensional manifold with differentiable structure, compatible with $H\Gamma^o_{2n+1}$. Then M carries an almost contact structure (φ,ξ,η), induced by the canonical normal structure on $\mathbf{R}^{2n+1}$. Such a structure (φ,ξ,η) we shall call *strict normal*. If g is a metric, compatible with a strict normal structure (φ,ξ,η), i.e. $g = g\circ\varphi + \eta\otimes\eta$, then the structure (φ,ξ,η,g) (respectively the manifold $M(\varphi,\xi,\eta,g)$) will be called *almost contact metric of Hermitian type*.

Theorem 1. *Let* $M^{2n+1}(\varphi,\xi,\eta,g)$ *be an almost contact metric manifold of Hermitian type. Then:*

i) $d\eta = 0$, $[\varphi,\varphi] = 0$, (*i.e.* $N = 0$, $d\eta = 0$);

ii) at every point $p \in M$ *there exists a local complex coordinate system* $(U; \{z^1,\dots,z^n,t\})$ *such that* $z^\alpha = x^\alpha + iy^\alpha$ (x^α, y^α *are real coordinates),* $Z_0 = \xi = \frac{\partial}{\partial t} = \partial_0$, $D^{10}_pU(D^{01}_pU)$ *is spanned by the vectors* $\{Z_\alpha = \partial/\partial z^\alpha = \partial_\alpha\}_{\alpha\in I}$ $(\{Z_{\bar{\alpha}} = \partial/\partial\overline{z}^\alpha =$

$\partial_{\bar{\alpha}}\}_{\bar{\alpha}\in I})$, $D_p^C U = \text{span}\{Z_a = \partial_a\}_{a\in I\cup\bar{I}}$, *and* $T_p^C U = \text{span}\{Z_A = \partial_A\}_{A\in I\cup\bar{I}\cup I_0}$, $p \in U$.

Proof. i) Let $(U; \psi = \{x^1, \ldots, x^n, y^1, \ldots, y^n, t\})$ be a local real coordinate system, compatible with $H\Gamma^o_{2n+1}$. Then $\eta = \psi^*(dt)$, i.e. $dt = (\psi^{-1})^*\eta$. On the other hand $d\eta$ is $(\psi^{-1})^*$-invariant [3]: $d((\psi^{-1})^*\eta) = (\psi^{-1})^* d\eta = 0$, i.e. $d\eta = 0$. Further, for the Nijenhuis tensor $[\varphi, \varphi]$, we have

$$\psi_*[\varphi,\varphi] = [\psi_*\varphi, \psi_*\varphi] = [\delta_{\alpha\beta}(dx^\alpha \otimes \frac{\partial}{\partial y^\beta} - dy^\beta \otimes \frac{\partial}{\partial x^\alpha});$$

$$\delta_{\lambda\mu}(dx^\lambda \otimes \frac{\partial}{\partial y^\mu} - dy^\mu \otimes \frac{\partial}{\partial x^\lambda})] = 0,$$

and hence $[\varphi, \varphi] = 0$.

ii) We recall the following

Proposition [3]. *Let* $i : M^r \longrightarrow M^{2n+1}(\varphi, \xi, \eta, g)$ *be an immersed* r*-dimensional submanifold. Then* M^r *is an integral submanifold of the contact distribution* D *iff* η *and* $d\eta$*, restricted to* M^r *vanish.*

Now, using i) we assert that D is an involutive distribution and let M^{2n} be the maximal integral submanifold of D. Since $[\varphi, \varphi] = 0$, from Newlander-Nirenberg's theorem it follows that $M^{2n}(\varphi, g)$ is a Hermitian manifold and thus ii) is proved. □

3. Curvature identities for almost contact metric manifolds of Hermitian type. We refer to some well known algebraic facts. Let $V(\varphi, \xi, \eta, g)$ be a $2n+1$-dimensional almost contact metric real vector space. We resume the decomposition of the space of Riemannian curvature-like tensors on V:

$$\mathcal{R}(V) = \{R \in \otimes^4 V^* : R(x,y,z,u) = -R(y,x,z,u) = -R(x,y,u,z),$$

$$\underset{(xyz)}{\mathcal{G}} R(x,y,z,u) = 0\}$$

in the following

Lemma 2 . *The decomposition* $\mathcal{R}(V) = h\mathcal{R}(V) \oplus v\mathcal{R}(V) \oplus w\mathcal{R}(V)$ *is orthogonal with respect to the induced from* g *inner product in* $\otimes^4 V^*$ *and invariant under the action of the standard representation of the Lie-group* $U(n) \times 1$ *in* $\otimes^4 V^*$. *The defining conditions for the factors are:*

$$h\mathcal{R}(V) = \{R \in \mathcal{R}(V) : R(x,y,z,u) = hR(x,y,z,u) = R(hx,hy,hz,hu), x,y,z,u \in V\},$$

$$\begin{aligned} v\mathcal{R}(V) =& \{R \in \mathcal{R}(V) : R(x,y,z,u) = vR(x,y,z,u) \\ &= [\eta \wedge R(\xi, *, z, u)](x,y) + [\eta \wedge R(\xi, *, x, y)](z,u), x,y,z,u \in V\}, \\ w\mathcal{R}(V) =& \{R \in \mathcal{R}(V) : R(x,y,z,u) = wR(x,y,z,u) \\ &= \{\eta \wedge [\eta(x)R(\xi, y, *, \xi) - \eta(y)R(\xi, x, *, \xi)]\}(z,u), x,y,z,u \in V\}. \end{aligned}$$

The partial decomposition of the space $h\mathcal{R}(V)$ coincides with that of an almost Hermitian real vector space [4],[9]:

Lemma 3. *$h\mathcal{R}(V) = h\mathcal{R}_1(V) \oplus h\mathcal{R}_1^{\perp}(V) \oplus h\mathcal{R}_2^{\perp}(V) \oplus h\mathcal{R}_3^{\perp}(V)$, where*

$$h\mathcal{R}_{i+1}(V) = h\mathcal{R}_i(V) \oplus h\mathcal{R}_i^{\perp}(V), \quad i = 1,2,3 \quad (h\mathcal{R}_4(V) = h\mathcal{R}(V)),$$
$$h\mathcal{R}_i^{\perp}(V) = \{R \in h\mathcal{R}_{i+1}(V) : R = \frac{1}{2}(I - H_i)R\},$$
$$h\mathcal{R}_i(V) = \{R \in h\mathcal{R}_{i+1}(V) : R = \frac{1}{2}(I + H_i)R\},$$

I is the identity on $\otimes^4 V^*$ and H_i are involutive isometries on $h\mathcal{R}_{i+1}(V)$, $i = 1,2,3$, defined by

$$(H_1R)(x,y,z,u) = \frac{1}{2}[R(\varphi x,\varphi y,z,u) + R(y,\varphi z,\varphi x,u) + R(\varphi z,x,\varphi y,u)],$$
$$(H_2R)(x,y,z,u) = \frac{1}{2}[R(x,y,z,u) + R(\varphi x,\varphi y,z,u) + R(\varphi x,y,\varphi z,u) + R(\varphi x,y,z,\varphi u)],$$
$$(H_3R)(x,y,z,u) = R(\varphi x,\varphi y,\varphi z,\varphi u), \quad x,y,z,u \in hV.$$

We give the partial decompositions of $v\mathcal{R}(V)$ and $w\mathcal{R}(V)$, which are easy to check, in the following Lemmas.

Lemma 4. *$v\mathcal{R}(V) = v\mathcal{R}_1(V) \oplus v\mathcal{R}_1^{\perp}(V) \oplus v\mathcal{R}_2^{\perp}(V)$, where*

$$v\mathcal{R}_{i+1}(V) = v\mathcal{R}_i(V) \oplus v\mathcal{R}_i^{\perp}(V), \quad i = 1,2 \quad (v\mathcal{R}_3(V) = v\mathcal{R}(V)),$$
$$v\mathcal{R}_i^{\perp}(V) = \{R \in v\mathcal{R}_{i+1}(V) : R = \frac{1}{2}(I - V_i)R\},$$
$$v\mathcal{R}_i(V) = \{R \in v\mathcal{R}_{i+1}(V) : R = \frac{1}{2}(I + V_i)R\},$$

and V_i are involutive isometries in $v\mathcal{R}_{i+1}(V)$, defined by

$$(V_1R)(\xi,x,y,z) = \frac{1}{2}[R(\xi,x,\varphi y,\varphi z) + R(\xi,\varphi z,\varphi x,y) + R(\xi,\varphi y,z,\varphi x)],$$
$$(V_2R)(\xi,x,y,z) = R(\xi,\varphi x,y,\varphi z), \quad x,y,z \in hV.$$

Lemma 5. *$w\mathcal{R}(V) = w\mathcal{R}_1(V) \oplus w\mathcal{R}_1^{\perp}(V)$, where*

$$w\mathcal{R}_1^{\perp}(V) = \{R \in w\mathcal{R}(V) : R = \frac{1}{2}(I - W)R\},$$

$$w\mathcal{R}_1(V) = \{R \in w\mathcal{R}(V) : R = \frac{1}{2}(I + W)R\}$$

and W is the involutive isometry , defined by

$$(WR)(\xi, x, y, \xi) = R(\xi, \varphi x, \varphi y, \xi), \quad x, y \in hV.$$

Corollary. *On an almost contact metric real vector space $V(\varphi, \xi, \eta, g)$ the following partial decomposition of the space $\mathcal{R}(V)$ into mutual orthogonal and $U(n) \times 1$ invariant factors is valid:*

$$\mathcal{R}(V) = h\mathcal{R}_1(V) \oplus h\mathcal{R}_1^{\perp}(V) \oplus h\mathcal{R}_2^{\perp}(V) \oplus h\mathcal{R}_3^{\perp}(V) \oplus v\mathcal{R}_1(V) \oplus$$

$$v\mathcal{R}_1^{\perp}(V) \oplus v\mathcal{R}_2^{\perp}(V) \oplus w\mathcal{R}_1(V) \oplus w\mathcal{R}_1^{\perp}(V).$$

We shall apply these algebraic conclusions to the space of almost contact metric manifolds of Hermitian type.

Definition. An almost contact metric manifold $M^{2n+1}(\varphi, \xi, \eta, g)$ is said to be in the class $h\mathcal{R}_i$, $h\mathcal{R}_i^{\perp}$, $v\mathcal{R}_i$, $v\mathcal{R}_i^{\perp}$, $w\mathcal{R}_1$, $w\mathcal{R}_1^{\perp}$ iff R_p is in the subspace $h\mathcal{R}_i(T_pM)$, $h\mathcal{R}_i^{\perp}(T_pM)$, $v\mathcal{R}_i(T_pM)$, $v\mathcal{R}_i^{\perp}(T_pM)$, $w\mathcal{R}_1(T_pM)$, $w\mathcal{R}_1^{\perp}(T_pM)$ for any $p \in M$, respectively, where R is the Riemannian curvature tensor of type (0,4) on M.

In terms of local complex coordinates the unique determined **C**-linear extentions of the tensors on an almost contact mertic manifold of Hermitian type one can characterize by their essential (i.e. which may not be zero) components. Using Lemma 1, we have the following essential components:

$$g : g_{\alpha\bar{\beta}} = g_{\bar{\beta}\alpha} = g(Z_\alpha, Z_{\bar{\beta}}), \quad g_{oo} = 1;$$

$$\Phi : \Phi_{\alpha\bar{\beta}} = -\Phi_{\bar{\beta}\alpha} = -ig_{\alpha\bar{\beta}}; \quad F : F_{\bar{\alpha}\beta\gamma}, F_{\bar{\alpha}\beta o}$$

$d\Phi : \Phi_{\bar{\alpha}\beta\gamma}, \Phi_{\bar{\alpha}\beta o}$ and also their conjugated ones. Let $(g^{AB}) = (g_{AB})^{-1}$.

Lemma 6. *Let $M(\varphi, \xi, \eta, g)$ be an almost contact metric manifold of Hermitian type. Then:*

i) the contraction-components $\Phi_{AB}^{C} = \Phi_{ABD}g^{DC}$ satisfy

a) $\Phi^{\sigma}_{AB}\Phi_{\sigma CD} = \Phi^{\bar{\sigma}}_{CD}\Phi_{\bar{\sigma}AB}$,

b) $(\Phi_{\alpha *} \wedge \Phi_{S\beta *})_{\bar{\gamma}\bar{\delta}} = -(\Phi^{S}_{\beta *} \wedge \Phi_{S\alpha *})_{\bar{\gamma}\bar{\delta}} = (\Phi^{S}_{\bar{\gamma} *} \wedge \Phi_{S\bar{\delta} *})_{\alpha\beta}$;

ii) the essential components of $d\Phi$ and $\nabla\Phi = -F$ satisfy

a) $\Phi_{\bar{\alpha}\beta\gamma} = -\frac{1}{3}F_{\bar{\alpha}\beta\gamma} = -\frac{i}{3}(\partial_\beta g_{\bar{\alpha}\gamma} - \partial_\gamma g_{\bar{\alpha}\beta})$,

b) $\Phi_{0\bar{\alpha}\beta} = -\frac{2}{3}F_{\bar{\alpha}\beta 0} = \frac{2}{3}F_{\beta\bar{\alpha}0} = \frac{i}{3}\partial_0 g_{\bar{\alpha}\beta}$;

iii) the essential Kristofel's symbols of ∇ – Γ^C_{AB} *satisfy* $\Gamma^C_{AB} = \Gamma^C_{BA}$ *and are*

a) $\Gamma^{\gamma}_{\alpha\beta} = -\frac{3}{2}i\Phi^{\gamma}_{\alpha\beta} + g^{\gamma\bar{\sigma}}\partial_\alpha g_{\beta\bar{\sigma}}$,

b) $\Gamma^{s}_{\alpha\bar{\beta}} = -\frac{3}{2}i\Phi^{s}_{\alpha\bar{\beta}}$, $s \in I \cup I \cup I_0$,

c) $\Gamma^{\sigma}_{\alpha 0} = -\frac{3}{2}i\Phi^{\sigma}_{\alpha 0}$ *– consequence from b);*

iv) the partial derivatives of the essential Φ_{ABC} *are*

a) $\partial_\alpha \Phi_{\beta\gamma\bar{\delta}} = (\nabla_\alpha d\Phi)_{\beta\gamma\bar{\delta}} + \frac{3}{2}i[\Phi^{\sigma}_{\alpha *} \wedge \Phi_{\sigma\bar{\delta}*}]_{\beta\gamma} - [\partial_\alpha g_{\bar{\sigma}*} \wedge \Phi^{\bar{\delta}}_{\bar{\sigma}*}]_{\beta\gamma} - \frac{3}{2}i\Phi^{\bar{\sigma}}_{\alpha\bar{\delta}}\Phi_{\beta\gamma\bar{\sigma}}$,

b) $\partial_{\bar{\alpha}} \Phi_{\beta\gamma\bar{\delta}} = (\nabla_{\bar{\alpha}} d\Phi)_{\beta\gamma\bar{\delta}} - \frac{3}{2}i[\Phi^{S}_{\bar{\alpha} *} \wedge \Phi_{S\bar{\delta}*}]_{\beta\gamma}$,

c) $\partial_0 \Phi_{\beta\gamma\bar{\delta}} = (\nabla_0 d\Phi)_{\beta\gamma\bar{\delta}} - \frac{3}{2}i[\Phi^{\sigma}_{0 *} \wedge \Phi_{\sigma\bar{\delta}*}]_{\beta\gamma} - \frac{3}{2}i\Phi^{\bar{\sigma}}_{0\bar{\delta}}\Phi_{\beta\gamma\bar{\sigma}}$,

d) $\partial_\alpha \Phi_{\beta\bar{\gamma}0} = (\nabla_\alpha d\Phi)_{\beta\bar{\gamma}0} + \Phi^{\bar{\sigma}}_{\bar{\gamma}0}\partial_\alpha g_{\beta\bar{\sigma}} - \frac{3}{2}i\Phi^{\bar{\sigma}}_{\alpha\bar{\gamma}}\Phi_{\beta\bar{\sigma}0} - \frac{3}{2}i[\Phi^{\sigma}_{\alpha *} \wedge \Phi_{\sigma\bar{\gamma}*}]_{\beta 0}$,

e) $\partial_0 \Phi_{\beta\bar{\gamma}0} = (\nabla_0 d\Phi)_{\beta\bar{\gamma}0} + 3i\Phi^{\sigma}_{0\beta}\Phi_{\sigma\bar{\gamma}0}$.

Proof. (i) The condition a) follows directly. b) is a simple verification, using a). For instance we shall show the first equality:

$$(\Phi^{S}_{\alpha *} \wedge \Phi_{S\beta *})_{\bar{\gamma}\bar{\delta}} = (\Phi^{\sigma}_{\alpha\bar{\gamma}}\Phi_{\sigma\beta\bar{\delta}} + \Phi^{\bar{\sigma}}_{\alpha\bar{\gamma}}\Phi_{\bar{\sigma}\beta\bar{\delta}} + \Phi^{o}_{\alpha\bar{\gamma}}\Phi_{o\beta\bar{\delta}} - (\bar{\gamma} \longleftrightarrow \bar{\delta}) =$$

$$= (\Phi^{\bar{\sigma}}_{\beta\bar{\delta}}\Phi_{\bar{\sigma}\alpha\bar{\gamma}} + \Phi^{\sigma}_{\beta\bar{\delta}}\Phi_{\sigma\alpha\bar{\gamma}} + \Phi^{o}_{\beta\bar{\delta}}\Phi_{o\alpha\bar{\gamma}}) - (\bar{\gamma} \longleftrightarrow \bar{\delta}) = -(\Phi^{S}_{\beta *} \wedge \Phi_{S\alpha *})_{\bar{\gamma}\bar{\delta}};$$

ii) The equalities follow from Lemma 1b),c),d) and the definition of $d\Phi$;

iii) Applying ii) to the well known expression

$$\Gamma^C_{AB} = \frac{1}{2}g^{SC}(\partial_A g_{BC} + \partial_B g_{CA} - \partial_C g_{AB}),$$

we get a), b). To prove c) one can see that Lemma 1a) implies

$$F_{\bar{\alpha}\beta o} = i\Gamma^{o}_{\bar{\alpha}\beta} = -i\Gamma^{\bar{\sigma}}_{\bar{\alpha}o}g_{\bar{\sigma}\beta} \quad \text{and hence} \quad \Gamma^{\sigma}_{\alpha o} = -\Gamma^{o}_{\alpha\bar{\beta}}g^{\sigma\bar{\beta}} = -\frac{3}{2}i\Phi^{\sigma}_{\alpha 0}, \quad \text{using b);}$$

iv) It follows from the well known expression

$$(\nabla_A d\Phi)_{BCD} = \partial_A \Phi_{BCD} - \Gamma^S_{AB}\Phi_{SCD} - \Gamma^S_{AC}\Phi_{BSD} - \Gamma^S_{AD}\Phi_{BCS}$$

and i), ii), iii).

As a consequence of Lemma 6 iii) we formulate

Lemma 7. *Let* $M(\varphi, \xi, \eta, g)$ *be an almost contact metric manifold of Hermitian type. Then* $\nabla\varphi$ *vanishes on* $D^{10} \times D^{10}$ *and on* $\xi \times D^{10}$.

Let $R = [\nabla, \nabla] - \nabla_{[\ ;\ ]}$ be the Riemannian curvature tensor of type (1,3) on an almost contact metric manifold of Hermitian type $M(\varphi, \xi, \eta, g)$. Denoting by the same letter the corresponding curvature tensor of type (0.4): $R(x,y,z,u) = g[R(x,y)z,u]$, $x,y,z,u \in \mathfrak{X}_M^C$ and using Lemma 7, it is clear, that the next properties are true:

$$
(2)\quad
\begin{array}{|l}
i)\ R(x,y,z,u) = -R(y,x,z,u) = -R(x,y,u,z) = \overline{R(\bar{x},\bar{y},\bar{z},\bar{u})},\\
ii)\ \underset{(xyz)}{\mathcal{G}} R(x,y,z,u) = 0,\\
iii)\ R(x,y)\circ\varphi = \varphi\circ R(x,y) \ \text{ on } \ D^{10}(D^{01}) \ \text{ if } \ x,y \in D^{10}(D^{01}),\\
iv)\ R(\xi,x)\circ\varphi = \varphi\circ R(\xi,x) \ \text{ on } \ D^{10}(D^{01}), \ \text{ if } \ x \in D^{10}(D^{01}),\\
\quad \text{and as a consequense}\\
v)\ R(x,y,z,u) = R(z,u,x,y),\\
vi)\ R(x,y,z,\xi) = 0, \quad x,y,z \in D^{10}(D^{01}).
\end{array}
$$

Next we shall formulate the basic results of this paper.

Theorem 2. *The space $\mathcal{R}$ of the Riemannian curvature tensors on almost contact metric manifolds of Hermitian type, in terms of local complex coordinates, has the structure, given in the table:*

classes		essential components	characterization conditions
$h\mathcal{R}$	$h\mathcal{R}_3^\perp$	$R_{\bar\alpha\beta\gamma\delta}$	$R_{\bar\alpha\beta\gamma\delta} = -\frac{3}{2}i(\nabla_\beta d\Phi)_{\bar\alpha\gamma\delta} + \frac{9}{4}\Phi^{\bar\sigma}_{\bar\alpha\beta}\Phi_{\bar\sigma\gamma\delta}$
	$h\mathcal{R}_1^\perp$	$R_{\alpha\beta\bar\gamma\bar\delta}$	$R_{\alpha\beta\bar\gamma\bar\delta} = -\frac{3}{2}i[(\nabla_\alpha d\Phi)_{\beta\bar\gamma\bar\delta} - (\nabla_\beta d\Phi)_{\alpha\bar\gamma\bar\delta}] + \frac{9}{4}[\Phi^S_{\alpha *}\wedge\Phi_{S\beta *}]_{\bar\gamma\bar\delta}$ $\iff R_{\alpha\bar\beta\gamma\bar\delta} = -R_{\alpha\bar\delta\gamma\bar\beta}(=\frac{1}{2}R_{\alpha\gamma\bar\beta\bar\delta})$
	$h\mathcal{R}_1$	$R_{\alpha\bar\beta\gamma\bar\delta}$	$R_{\alpha\bar\beta\gamma\bar\delta} = R_{\alpha\bar\delta\gamma\bar\beta}(\iff R_{\alpha\beta\bar\gamma\bar\delta} = 0)$
$v\mathcal{R}$	$v\mathcal{R}_1^\perp$	$R_{o\bar\alpha\beta\gamma}$	$R_{o\bar\alpha\beta\gamma} = \frac{3}{2}i(\nabla_o d\Phi)_{\bar\alpha\beta\gamma} + \frac{9}{4}\Phi^{\bar\sigma}_{o\bar\alpha}\Phi_{\bar\sigma\beta\gamma} \iff R_{o\alpha\beta\bar\gamma} = -R_{o\beta\alpha\bar\gamma}$
	$v\mathcal{R}_1$	$R_{o\alpha\beta\bar\gamma}$	$R_{o\alpha\beta\bar\gamma} = R_{o\beta\alpha\bar\gamma}(\iff R_{o\bar\alpha\beta\gamma} = 0)$
$w\mathcal{R}$	$w\mathcal{R}_1$	$R_{o\bar\alpha\beta o}$	$R_{o\bar\alpha\beta o} = \frac{3}{2}i(\nabla_o d\Phi)_{\bar\alpha\beta o} + \frac{9}{4}\Phi^{\bar\sigma}_{o\bar\alpha}\Phi_{\bar\sigma\beta o}$

Proof. In terms of local complex coordinates the Lemmas 2,3,4,5 imply the following equalities in sense of isomorphism:

Lemma 2:

$h\mathcal{R}=\{R\in\mathcal{R}(T_p^C M),\ \ p\in M : R_{\alpha\beta\gamma\delta}, R_{\bar\alpha\beta\gamma\delta}, R_{\alpha\bar\beta\gamma\bar\delta}, R_{\alpha\beta\bar\gamma\bar\delta}$ – essential components$\}$,

$v\mathcal{R}=\{R\in\mathcal{R}(T_p^C M),\ \ p\in M : R_{0\alpha\beta\gamma}, R_{0\bar\alpha\beta\gamma}, R_{0\alpha\beta\bar\gamma}$ essential components$\}$,

$w\mathcal{R}=\{R\in\mathcal{R}(T_p^C M),\ \ p\in M : R_{o\alpha\beta\gamma o}, R_{o\alpha\bar\beta o}$ essential components$\}$.

Lemma 3:

$$h\mathcal{R}_3^{\perp}=\{R\in h\mathcal{R}: R_{\bar\alpha\beta\gamma\delta} - \text{ essential components}\},\quad h\mathcal{R}_3=\{R\in h\mathcal{R}: R_{\bar\alpha\beta\gamma\delta}=0\},$$
$$h\mathcal{R}_2^{\perp}=\{R\in h\mathcal{R}_3: R_{\alpha\beta\gamma\delta} - \text{ essential components}\},\quad h\mathcal{R}_2=\{R\in h\mathcal{R}_3: R_{\alpha\beta\gamma\delta}=0\},$$
$$h\mathcal{R}_1^{\perp}=\{R\in h\mathcal{R}_2: R_{\alpha\beta\bar\gamma\bar\delta} - \text{ essential components} \iff R_{\alpha\bar\beta\gamma\bar\delta}=-R_{\alpha\bar\delta\gamma\bar\beta}\},$$
$$h\mathcal{R}_1=\{R\in h\mathcal{R}_2: R_{\alpha\beta\bar\gamma\bar\delta}=0 \iff R_{\alpha\bar\beta\gamma\bar\delta}=R_{\alpha\bar\delta\gamma\bar\beta}\}.$$

Lemma 4:

$$v\mathcal{R}_2^{\perp}=\{R\in v\mathcal{R}: R_{0\alpha\beta\gamma} \text{ essential components}\},\quad v\mathcal{R}_2=\{R\in v\mathcal{R}: R_{0\alpha\beta\gamma}=0\},$$
$$v\mathcal{R}_1^{\perp}=\{R\in v\mathcal{R}_2: R_{0\bar\alpha\beta\gamma}=R_{0\gamma\beta\bar\alpha} \iff R_{0\alpha\beta\bar\gamma}=-R_{0\beta\alpha\bar\gamma}\},$$
$$v\mathcal{R}_1=\{R\in v\mathcal{R}_2: R_{0\bar\alpha\beta\gamma}=0 \iff R_{0\bar\alpha\beta\gamma}=R_{0\beta\alpha\bar\gamma}\}.$$

Lemma 5:

$$w\mathcal{R}_1^{\perp}=\{R\in w\mathcal{R}: R_{0\alpha\beta 0} - \text{ essential components}\},$$

$$w\mathcal{R}_1=\{R\in w\mathcal{R}: R_{0\alpha\bar\beta 0} - \text{ essential components}\}.$$

Further (2) imply $R_{\alpha\beta\gamma\delta}=R_{o\alpha\beta\gamma}=R_{o\alpha\beta o}=0$ and so the structure of $\mathcal{R}$ is obtained. The characterization conditions we get from the well known formula

$$R_{ABCD}=g_{DS}(\partial_A\Gamma^S_{BC}-\partial_B\Gamma^S_{AC}-\Gamma^D_{AC}\Gamma^S_{BD}+\Gamma^D_{BC}\Gamma^S_{AD}-C^D_{AB}\Gamma^S_{DC}),$$

by using Lemma 6 iii), iv).

Further by $[*\longleftrightarrow\varphi^*]$ we shall denote the expression, obtained from that in the previous brackets [,], replacing every argument * by $\varphi*$ and conversely, and by $<,>$-the scalar product in T^*_pM, $p\in M$, induced by the metric g.

Theorem 3. *Let $M(\varphi,\xi,\eta,g)$ be an almost contact metric manifold of Hermitian type. Then the following curvature identities are valid:*

$$(3)\ [R(x,y,z,u)-R(\varphi x,y,z,\varphi u)-R(\varphi x,\varphi y,z,u)-R(\varphi x,y,\varphi z,u)]+[*\leftrightarrow\varphi^*]=0;$$

$$R(\xi,y,z,u)-R(\xi,\varphi y,z,\varphi u)=R(\xi,\varphi y,\varphi z,u)+R(\xi,y,\varphi z,\varphi u); \tag{4}$$

$$R(\xi,x,y,\xi)=R(\xi,\varphi x,\varphi y,\xi),\quad x,y,z,u\in D_p. \tag{5}$$

Proof. As was noted above $R_{\alpha\beta\gamma\delta}=R_{o\beta\gamma\delta}=R_{o\alpha\beta o}=0$. Linearizing every of these equalities, replacing $\alpha,\beta,\gamma,\delta$ by $x-i\varphi x, y-i\varphi y, z-i\varphi z, u-i\varphi u$, respectively and taking the real parts, we get (3), (4), (5).

Remark. Restricting the identity (3) to the contact distribution (D, φ), we obtain the well known identity for a Hermitian manifold - [4], [9].

Exactly in the same way, taking the real and imaginary parts of the vanishing components $\Phi_{\alpha\beta\gamma}$ and $\Phi_{\alpha\beta o}$ (Lemma 6 ii)), we state

Lemma 8. *Let $M(\varphi,\xi,\eta,g)$ be an almost contact metric manifold of Hermitian type. Then for all $x,y,z \in D_p$*

i) the following identities are equivalent:

a) $d\Phi(x,y,z) - d\Phi(\varphi x,\varphi y,z) = d\Phi(\varphi x,y,\varphi z) + d\Phi(x,\varphi y,\varphi z)$,

b) $d\Phi(\varphi x,y,z) + d\Phi(x,\varphi y,z) = d\Phi(\varphi x,\varphi y,\varphi z) - d\Phi(x,y,\varphi z)$;

ii) the following identities are equivalent:

a) $d\Phi(x,y,\xi) - d\Phi(\varphi x,\varphi y,\xi) = 0$,

b) $d\Phi(\varphi x,y,\xi) + d\Phi(x,\varphi y,\xi) = 0$.

Theorem 4. *Let $M(\varphi,\xi,\eta,g)$ be an almost contact metric manifold of Hermitian type in class $h\mathcal{R}_3^{\perp}$. Then the following curvature identity is valid for all $x,y,z,u \in D_p$:*

$$
\begin{aligned}
(6)\quad & [R(x,y,z,u) + R(\varphi x,y,z,\varphi u) + R(\varphi x,\varphi y,z,u) + R(\varphi x,y,\varphi z,u)] - [* \longleftrightarrow \varphi^*] \\
& -\frac{3}{2}\{(\nabla_y d\Phi)(\varphi x,z,u) - (\nabla_y d\Phi)(x,\varphi z,u) - (\nabla_y d\Phi)(x,z,\varphi u) - (\nabla_{\varphi y} d\Phi)(x,z,u)\} \\
& -\frac{3}{2}\{* \longleftrightarrow \varphi *\} = \langle d\Phi(x,y,*) + d\Phi(\varphi x,\varphi y,*),\ d\Phi(z,u,*) - d\Phi(\varphi z,\varphi u,*)\rangle + \langle {x \to \varphi x \atop z \to \varphi z} \rangle \\
& +\langle d\Phi(\varphi x,y,*) - d\Phi(x,\varphi y,*),\ d\Phi(\varphi z,u,*) + d\Phi(z,\varphi u,*)\rangle.
\end{aligned}
$$

P r o o f. The left hand side of (6) is the real part of the linearizing as above expression $R_{\bar\alpha\beta\gamma\delta} + \frac{3}{2} i(\nabla_\beta d\Phi)_{\bar\alpha\gamma\delta}$ in Theorem 2 as above. We shall prove that $Re\{\frac{9}{4}[\Phi_{\bar\alpha\beta}\Phi_{\bar\sigma\gamma\delta}]\}$ is the right hand side of (6). Putting $\bar\alpha,\beta,\gamma,\delta$ as in Theorem 3 and $Z_\lambda = e_\lambda - i\varphi e_\lambda$, $Z\bar\sigma = e_\sigma + i\varphi e_\sigma$, where $e_\lambda = \partial/\partial x^\lambda$, $e^\lambda = dx^\lambda$, we have

$$
Re\{\frac{9}{4}[\Phi^{\bar\sigma}_{\bar\alpha\beta}\Phi_{\bar\sigma\gamma\delta}]\} = \frac{9}{4} Re\{g^{\lambda\bar\sigma}[\Phi_{\bar\alpha\beta\lambda}\Phi_{\bar\sigma\gamma\delta}]\}
$$

$$
\begin{aligned}
= \frac{9}{2} g^{\lambda\sigma}\{ & [d\Phi(x,y,e_\lambda) + d\Phi(\varphi x,\varphi y,e_\lambda)][d\Phi(z,u,e_\sigma) - d\Phi(\varphi z,\varphi u,e_\sigma) \\
& + d\Phi(\varphi z,u,\varphi e_\sigma) + d\Phi(z,\varphi u,\varphi e_\sigma)] \\
& + [d\Phi(x,y,\varphi e_\lambda) + d\Phi(\varphi x,\varphi y,\varphi e_\lambda)][d\Phi(z,u,\varphi e_\sigma) - d\Phi(\varphi z,\varphi u,\varphi e_\sigma) \\
& - d\Phi(\varphi z,u,e_\sigma) - d\Phi(z,\varphi u,e_\sigma)]\}\frac{9}{2} g^{\lambda\sigma}\{ {x \to \varphi x \atop z \to \varphi z} \},
\end{aligned}
$$

and applying Lemma 8, we get the result above.

By the same way, by using Theorem 2, (3), (6), one can prove

Theorem 5. *Let $M(\varphi,\xi,\eta,g)$ be an almost contact metric manifold of Hermitian type in class $h\mathcal{R}_3 = h\mathcal{R}_1 \oplus h\mathcal{R}_1^{\perp}$. Then the following curvature identity is valid for all $x,y,z,u \in D_p$:*

$$R(x,y,z,u) = R(x,y,\varphi z,\varphi u) + R(x,\varphi y,z,\varphi u) + R(x,\varphi y,\varphi z,u). \tag{7}$$

Theorem 2, Lemma 8 i) and (7) imply

Theorem 6. *Let $M(\varphi,\xi,\eta,g)$ be an almost contact metric manifold of Hermitian type in class $h\mathcal{R}_1^{\perp}$. Then the following curvature identity is valid for all $x,y,z,u \in D_p$:*

$$(8) \qquad 2[R(x,y,z,u) - R(\varphi x,\varphi y,z,u) + R(\varphi x,\varphi y,\varphi z,\varphi u) - R(x,y,\varphi z,\varphi u)]$$

$$= \frac{3}{2}\{[(\nabla_x d\Phi)(y,z,\varphi u) + (\nabla_x d\Phi)(y,\varphi z,u) - (\nabla_x d\Phi)(\varphi y,z,u) - (\nabla_{\varphi x} d\Phi)(y,z,u)]$$

$$-[x \longleftrightarrow y]\} - \frac{3}{2}\{* \longleftrightarrow \varphi *\} + \{\langle d\Phi(x,z,*) + d\Phi(\varphi x,\varphi z,*); d\Phi(y,u,*) + d\Phi(\varphi y,\varphi u,*)\rangle$$

$$-3[d\Phi(x,\xi,*) \wedge d\Phi(y,\xi,*)](z,u)\} - \{\begin{matrix} x \to \varphi x \\ y \to \varphi y \end{matrix}\}.$$

Theorem 2, and (8) imply

Theorem 7. *Let $M(\varphi,\xi,\eta,g)$ be an almost contact metric manifold of Hermitian type in class $h\mathcal{R}_1$. Then the following curvature identity is valid for all $x,y,z,u \in D_p$:*

$$R(x,y,z,u) + R(\varphi x,\varphi y,\varphi z,\varphi u) = R(x,y,\varphi z,\varphi u) + R(\varphi x,\varphi y,z,u).$$

Theorem 2, Lemma 8 ii) and (4) imply

Theorem 8. *Let $M(\varphi,\xi,\eta,g)$ be an almost contact metric manifold of Hermitian type in class $v\mathcal{R}_1^{\perp}$. Then the following curvature identity is valid for all $x,y,z \in D_p$:*

$$(9) \qquad 2[R(\xi,x,y,z) - R(\xi,x,\varphi y,\varphi z)]$$

$$= \frac{3}{2}[(\nabla_\xi d\Phi)(x,\varphi y,z) + (\nabla_\xi d\Phi)(x,y,\varphi z) - (\nabla_\xi d\Phi)(\varphi x,y,z) + (\nabla_\xi d\Phi)(\varphi x,\varphi y,\varphi z)]$$

$$-\langle d\Phi(x,\xi,*),\ d\Phi(y,z,*) - d\Phi(\varphi y,\varphi z,*)\rangle - \langle\begin{matrix} x \to \varphi x \\ y \to \varphi y \end{matrix}\rangle.$$

Theorem 2, and (9) imply

Theorem 9. *Let $M(\varphi,\xi,\eta,g)$ be an almost contact metric manifold of Hermitian type in class $v\mathcal{R}_1$. Then the following curvature identity is valid for all $x,y,z\in D_p$:*

$$R(\xi,x,y,z)=R(\xi,x,\varphi y,\varphi z).$$

Finally, Theorem 2, (5) and Lemma 8ii) imply

Theorem 10. *Let $M(\varphi,\xi,\eta,g)$ be an almost contact metric manifold of Hermitian type in class $w\mathcal{R}_1$. Then the following curvature identity is valid for all $x,y\in D_p$:*

$$2R(\xi,x,y,\xi)=\frac{3}{2}[(\nabla_\xi d\Phi)(\xi,\varphi x,y)+(\nabla_\xi d\Phi)(\xi,x,\varphi y)]-$$

$$-\langle d\Phi(x,\xi,*),d\Phi(y,\xi,*)\rangle-\langle d\Phi(\varphi x,\xi,*),d\Phi(\varphi y,\xi,*)\rangle.$$

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