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RADICAL TRANSVERSAL SCREEN PSEUDO-SLANT LIGHTLIKE SUBMANIFOLDS OF INDEFINITE SASAKIAN MANIFOLDS*

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Abstract. In this paper, we introduce the notion of radical transversal screen pseudo-slant lightlike submanifolds of indefinite Sasakian manifolds giving characterization theorem with some non-trivial examples of such submanifolds. Integrability conditions of distributions D_1, D_2 and $RadTM$ on radical transversal screen pseudo-slant lightlike submanifolds of an indefinite Sasakian manifold, have been obtained. We also obtain necessary and sufficient conditions for foliations determined by above distributions to be totally geodesic.

1. Introduction. The theory of lightlike submanifolds of a semi-Riemannian manifold was introduced by Duggal and Bejancu [6]. A submanifold M of

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a semi-Riemannian manifold $\overline{M}$ is said to be lightlike submanifold if the induced metric g on M is degenerate, i.e. there exists a non-zero $X \in \Gamma(TM)$ such that $g(X,Y) = 0$, $\forall Y \in \Gamma(TM)$. In [5], B. Y. Chen defined slant immersions in complex geometry as a natural generalization of both holomorphic immersions and totally real immersions. In [10], A. Lotta introduced the concept of slant immersions of a Riemannian manifold into an almost contact metric manifold. A. Carriazo defined and studied bi-slant submanifolds of almost Hermitian and almost contact metric manifolds and further gave the notion of pseudo-slant submanifolds [2].

The geometry of radical transversal, transversal, generalized transversal lightlike submanifolds has been studied in [18, 19]. In [16], authors give the notion of screen pseudo-slant lightlike submanifolds of indefinite Sasakian manifolds. In this article, we introduce the notion of radical transversal screen pseudo-slant lightlike submanifolds of indefinite Sasakian manifolds. This new class of lightlike submanifolds of an indefinite Sasakian manifold includes radical transversal, transversal, generalized transversal lightlike submanifolds as its sub-cases. The paper is arranged as follows. There are some basic results in Section 2. In Section 3, we introduce radical transversal screen pseudo-slant lightlike submanifolds of an indefinite Sasakian manifold giving some examples. Section 4 is devoted to the study of foliations determined by distributions on radical transversal screen pseudo-slant lightlike submanifolds of indefinite Sasakian manifolds.

2. Preliminaries. A submanifold (M^m, g) immersed in a semi-Riemannian manifold $(\overline{M}^{m+n}, \overline{g})$ is called a lightlike submanifold [6] if the metric g induced from $\overline{g}$ is degenerate and the radical distribution $RadTM$ is of rank r, where $1 \leq r \leq m$. Let $S(TM)$ be a screen distribution which is a semi-Riemannian complementary distribution of $RadTM$ in TM, that is

$$TM = RadTM \oplus_{orth} S(TM). \tag{2.1}$$

Now consider a screen transversal vector bundle $S(TM^{\perp})$, which is a semi-Riemannian complementary vector bundle of $RadTM$ in $TM^{\perp}$. Since for any local basis $\{\xi_i\}$ of $RadTM$, there exists a local null frame $\{N_i\}$ of sections with values in the orthogonal complement of $S(TM^{\perp})$ in $[S(TM)]^{\perp}$ such that $\overline{g}(\xi_i, N_j) = \delta_{ij}$ and $\overline{g}(N_i, N_j) = 0$, it follows that there exists a lightlike transversal vector bundle $ltr(TM)$ locally spanned by $\{N_i\}$. Let $tr(TM)$ be complementary (but not orthogonal) vector bundle to TM in $T\overline{M}|_M$. Then

$$tr(TM) = ltr(TM) \oplus_{orth} S(TM^{\perp}), \tag{2.2}$$

(2.3) $$T\overline{M}|_M = TM \oplus tr(TM),$$

(2.4) $$T\overline{M}|_M = S(TM) \oplus_{orth} [RadTM \oplus ltr(TM)] \oplus_{orth} S(TM^{\perp}).$$

Following are four cases of a lightlike submanifold $\Big(M, g, S(TM), S(TM^{\perp})\Big)$:
Case 1. r-lightlike if $r < \min(m, n)$,
Case 2. co-isotropic if $r = n < m$, $S\left(TM^{\perp}\right) = \{0\}$,
Case 3. isotropic if $r = m < n$, $S(TM) = \{0\}$,
Case 4. totally lightlike if $r = m = n$, $S(TM) = S(TM^{\perp}) = \{0\}$.
The Gauss and Weingarten formulae are given as

(2.5) $$\overline{\nabla}_X Y = \nabla_X Y + h(X, Y),$$

(2.6) $$\overline{\nabla}_X V = -A_V X + \nabla^t_X V,$$

for all $X, Y \in \Gamma(TM)$ and $V \in \Gamma(tr(TM))$, where $\nabla_X Y, A_V X$ belong to $\Gamma(TM)$ and $h(X, Y), \nabla^t_X V$ belong to $\Gamma(tr(TM))$. ∇ and ∇^t are linear connections on M and on the vector bundle $tr(TM)$ respectively. The second fundamental form h is a symmetric $F(M)$-bilinear form on $\Gamma(TM)$ with values in $\Gamma(tr(TM))$ and the shape operator A_V is a linear endomorphism of $\Gamma(TM)$. From (2.5) and (2.6), for any $X, Y \in \Gamma(TM)$, $N \in \Gamma(ltr(TM))$ and $W \in \Gamma(S(TM^{\perp}))$, we have

(2.7) $$\overline{\nabla}_X Y = \nabla_X Y + h^l(X, Y) + h^s(X, Y),$$

(2.8) $$\overline{\nabla}_X N = -A_N X + \nabla^l_X N + D^s(X, N),$$

(2.9) $$\overline{\nabla}_X W = -A_W X + \nabla^s_X W + D^l(X, W),$$

where $h^l(X, Y) = L(h(X, Y))$, $h^s(X, Y) = S(h(X, Y))$, $D^l(X, W) = L(\nabla^t_X W)$, $D^s(X, N) = S(\nabla^t_X N)$. L and S are the projection morphisms of $tr(TM)$ on $ltr(TM)$ and $S(TM^{\perp})$ respectively. ∇^l and ∇^s are linear connections on $ltr(TM)$ and $S(TM^{\perp})$ called the lightlike connection and screen transversal connection on M respectively.

Now by using (2.7)–(2.9) and metric connection $\overline{\nabla}$, we obtain

(2.10) $$\overline{g}(h^s(X, Y), W) + \overline{g}(Y, D^l(X, W)) = g(A_W X, Y),$$

$$\overline{g}(D^s(X,N),W) = \overline{g}(N, A_W X). \tag{2.11}$$

Denote the projection of TM on $S(TM)$ by $\overline{P}$. Then from the decomposition of the tangent bundle of a lightlike submanifold, for any $X, Y \in \Gamma(TM)$ and $\xi \in \Gamma(RadTM)$, we have

$$\nabla_X \overline{P}Y = \nabla^*_X \overline{P}Y + h^*(X, \overline{P}Y), \tag{2.12}$$

$$\nabla_X \xi = -A^*_\xi X + \nabla^{*t}_X \xi. \tag{2.13}$$

By using above equations, we obtain

$$\overline{g}(h^l(X, \overline{P}Y), \xi) = g(A^*_\xi X, \overline{P}Y), \tag{2.14}$$

$$\overline{g}(h^*(X, \overline{P}Y), N) = g(A_N X, \overline{P}Y), \tag{2.15}$$

$$\overline{g}(h^l(X,\xi),\xi) = 0, \qquad A^*_\xi \xi = 0. \tag{2.16}$$

It is important to note that in general ∇ is not a metric connection. Since $\overline{\nabla}$ is metric connection, by using (2.7), we get

$$(\nabla_X g)(Y,Z) = \overline{g}(h^l(X,Y),Z) + \overline{g}(h^l(X,Z),Y). \tag{2.17}$$

A semi-Riemannian manifold $(\overline{M}, \overline{g})$ is called an ϵ-almost contact metric manifold [8] if there exists a $(1,1)$ tensor field ϕ, a vector field V called characteristic vector field and a 1-form η, satisfying

$$\phi^2 X = -X + \eta(X)V, \quad \eta(V) = \epsilon, \quad \eta \circ \phi = 0, \quad \phi V = 0, \tag{2.18}$$

$$\overline{g}(\phi X, \phi Y) = \overline{g}(X,Y) - \epsilon\eta(X)\eta(Y), \tag{2.19}$$

for all $X, Y \in \Gamma(T\overline{M})$, where $\epsilon = 1$ or -1. It follows that

$$\overline{g}(V,V) = \epsilon, \tag{2.20}$$

$$\overline{g}(X,V) = \eta(X), \tag{2.21}$$

$$\overline{g}(X, \phi Y) = -\overline{g}(\phi X, Y). \tag{2.22}$$

Then $(\phi, V, \eta, \overline{g})$ is called an ϵ-almost contact metric structure on $\overline{M}$.

An ϵ-almost contact metric structure $(\phi, V, \eta, \overline{g})$ is called an indefinite Sasakian structure if and only if

$$(\overline{\nabla}_X \phi)Y = \overline{g}(X, Y)V - \epsilon\eta(Y)X, \tag{2.23}$$

for all $X, Y \in \Gamma(T\overline{M})$, where $\overline{\nabla}$ is Levi-Civita connection with respect to $\overline{g}$.

A semi-Riemannian manifold endowed with an indefinite Sasakian structure is called an indefinite Sasakian manifold. From (2.23), for any $X \in \Gamma(T\overline{M})$, we get

$$\overline{\nabla}_X V = -\phi X. \tag{2.24}$$

Let $(\overline{M}, \overline{g}, \phi, V, \eta)$ be an ϵ-almost contact metric manifold. If $\epsilon = 1$, then $\overline{M}$ is said to be a spacelike almost contact metric manifold and if $\epsilon = -1$, then $\overline{M}$ is called a timelike almost contact metric manifold. In this paper, we consider indefinite Sasakian manifolds with spacelike characteristic vector field V.

3. Radical transversal screen pseudo-slant lightlike submanifolds. In this section, we introduce the notion of radical transversal screen pseudo-slant lightlike submanifolds of ϵ-almost contact metric manifolds. At first, we state the following Lemma for later use:

Lemma 3.1. *Let M be a $2q$-lightlike submanifold of an ϵ-almost contact metric manifold $\overline{M}$, of index $2q$ such that $2q < \dim(M)$ with structure vector field tangent to M. Then the screen distribution $S(TM)$ on lightlike submanifold M is Riemannian.*

The proof of above Lemma follows as in Lemma 4.1 of [13], so we omit it.

Definition 3.1. *Let M be a $2q$-lightlike submanifold of an ϵ-almost contact metric manifold $\overline{M}$ of index $2q$ such that $2q < \dim(M)$ with structure vector field tangent to M. Then we say that M is a radical transversal screen pseudo-slant lightlike submanifold of $\overline{M}$ if the following conditions are satisfied:*
(i) $\phi RadTM = ltr(TM)$,
(ii) there exists non-degenerate orthogonal distributions D_1 and D_2 on M such that

(a) $S(TM) = D_1 \oplus_{orth} D_2 \oplus_{orth} \{V\}$,

(b) *the distribution D_1 is anti-invariant, i.e.* $\phi D_1 \subset S(TM^{\perp})$,

(c) *the distribution D_2 is slant with angle $\theta(\neq \pi/2)$, i.e. for each $x \in M$ and each non-zero vector $X \in (D_2)_x$, the angle θ between ϕX and the vector* *subspace* $(D_2)_x$ *is a constant* $(\neq \pi/2)$, *which is independent of the choice of* $x \in M$ *and* $X \in (D_2)_x$.

This constant angle θ *is called the slant angle of distribution* D_2. *A radical transversal screen pseudo-slant lightlike submanifold is said to be proper if* $D_1 \neq \{0\}$, $D_2 \neq \{0\}$ *and* $\theta \neq 0$.

From the above definition, we have the following decomposition

$$TM = RadTM \oplus_{orth} D_1 \oplus_{orth} D_2 \oplus_{orth} \{V\}. \tag{3.1}$$

In particular, we have
(i) if $D_2 = 0$, then M is a transversal lightlike submanifold,
(ii) if $D_1 = 0$ and $\theta = 0$, then M is a radical transversal lightlike submanifold,
(iii) if $D_1 \neq 0$ and $\theta = 0$, then M is a generalized transversal lightlike submanifold.

Thus the above new class of lightlike submanifolds of an ϵ-almost contact metric manifold includes radical transversal, transversal, generalized transversal lightlike submanifolds as its sub-cases which have been studied in [18, 19].

Let $(\mathbb{R}^{2m+1}_{2q}, \overline{g}, \phi, \eta, V)$ denote the manifold $\mathbb{R}^{2m+1}_{2q}$ with its usual ϵ-almost contact metric structure given by

$$\eta = \frac{1}{2}\left(dz - \sum_{i=1}^{m} y^i dx^i\right), \quad V = 2\partial z,$$

$$\overline{g} = \eta \otimes \eta + \frac{1}{4}\left\{ -\sum_{i=1}^{q}(dx^i \otimes dx^i + dy^i \otimes dy^i) + \sum_{i=q+1}^{m}(dx^i \otimes dx^i + dy^i \otimes dy^i)\right\},$$

$$\phi\left(\sum_{i=1}^{m}(X_i \partial x_i + Y_i \partial y_i) + Z\partial z\right) = \sum_{i=1}^{m}(Y_i \partial x_i - X_i \partial y_i) + \left(\sum_{i=1}^{m} Y_i y^i\right)\partial z,$$

where (x^i, y^i, z) are the cartesian coordinates on $\mathbb{R}^{2m+1}_{2q}$. Now we construct some examples of radical transversal screen pseudo-slant lightlike submanifolds of a spacelike almost contact metric manifold.

Example 1. Let $(\mathbb{R}^{13}_2, \overline{g}, \phi, \eta, V)$ be a spacelike almost contact metric manifold, where $\overline{g}$ is of signature $(-,+,+,+,+,+,-,+,+,+,+,+,+)$ with respect to the canonical basis $\{\partial x_1, \partial x_2, \partial x_3, \partial x_4, \partial x_5, \partial x_6, \partial y_1, \partial y_2, \partial y_3, \partial y_4, \partial y_5, \partial y_6, \partial z\}$.

Suppose M is a submanifold of $\mathbb{R}^{13}_2$ given by $x^1 = u_1$, $y^1 = u_2$, $x^2 = -u_1 \cos\alpha + u_2 \sin\alpha$, $y^2 = u_1 \sin\alpha + u_2 \cos\alpha$, $x^3 = y^4 = u_3$, $x^4 = y^3 = u_4$,

$x^5 = u_5,\ y^5 = u_6,\ x^6 = k\cos u_6,\ y^6 = k\sin u_6,\ z = u_7$, where k is a non-zero constant.
The local frame of TM is given by $\{Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7\}$, where

$$
\begin{aligned}
Z_1 &= 2(\partial x_1 - \cos\alpha\partial x_2 + \sin\alpha\partial y_2 + y^1\partial z - \cos\alpha y^2\partial z),\\
Z_2 &= 2(\partial y_1 + \sin\alpha\partial x_2 + \cos\alpha\partial y_2 + \sin\alpha y^2\partial z),\\
Z_3 &= 2(\partial x_3 + \partial y_4 + y^3\partial z),\\
Z_4 &= 2(\partial x_4 + \partial y_3 + y^4\partial z),\\
Z_5 &= 2(\partial x_5 + y^5\partial z),\\
Z_6 &= 2(\partial y_5 - k\sin u_6\partial x_6 + k\cos u_6\partial y_6 - k\sin u_6 y^6\partial z),\\
Z_7 &= V = 2\partial z.
\end{aligned}
$$

Hence $RadTM = \text{span}\{Z_1, Z_2\}$ and $S(TM) = \text{span}\{Z_3, Z_4, Z_5, Z_6, V\}$.
Now $ltr(TM)$ is spanned by $N_1 = -\partial x_1 - \cos\alpha\partial x_2 + \sin\alpha\partial y_2 - y^1\partial z - \cos\alpha y^2\partial z$, $N_2 = -\partial y_1 + \sin\alpha\partial x_2 + \cos\alpha\partial y_2 + \sin\alpha y^2\partial z$ and $S(TM^\perp)$ is spanned by

$$
\begin{aligned}
W_1 &= 2(\partial x_3 - \partial y_4 + y^3\partial z),\\
W_2 &= 2(\partial x_4 - \partial y_3 + y^4\partial z),\\
W_3 &= 2(k\cos u_6\partial x_6 + k\sin u_6\partial y_6 + k\cos u_6 y^6\partial z),\\
W_4 &= 2(k^2\partial y_5 + k\sin u_6\partial x_6 - k\cos u_6\partial y_6 + k\sin u_6 y^6\partial z).
\end{aligned}
$$

It follows that $\phi Z_1 = 2N_2$, $\phi Z_2 = -2N_1$, which implies that $\phi RadTM = ltr(TM)$. On the other hand, we can see that $D_1 = span\{Z_3, Z_4\}$ such that $\phi Z_3 = W_2$, $\phi Z_4 = W_1$, which implies that D_1 is anti-invariant with respect to ϕ and $D_2 = \text{span}\{Z_5, Z_6\}$ is a slant distribution with slant angle $\theta = \arccos(1/\sqrt{1+k^2})$. Hence M is a radical transversal screen pseudo-slant 2-lightlike submanifold of $\mathbb{R}^{13}_2$.

Example 2. Let $(\mathbb{R}^{13}_2, \overline{g}, \phi, \eta, V)$ be a spacelike almost contact metric manifold, where $\overline{g}$ is of signature $(-,+,+,+,+,+,-,+,+,+,+,+,+)$ with respect to the canonical basis $\{\partial x_1, \partial x_2, \partial x_3, \partial x_4, \partial x_5, \partial x_6, \partial y_1, \partial y_2, \partial y_3, \partial y_4, \partial y_5, \partial y_6, \partial z\}$.

Suppose M is a submanifold of $\mathbb{R}^{13}_2$ given by $x^1 = y^2 = u_1$, $x^2 = y^1 = u_2$, $x^3 = u_3\cos\beta$, $y^3 = u_3\sin\beta$, $x^4 = u_4\sin\beta$, $y^4 = u_4\cos\beta$, $x^5 = u_5\cos u_6$, $y^5 = u_5\sin u_6$, $x^6 = \cos u_5$, $y^6 = \sin u_5$, $z = u_7$, $u_5 \neq 0$.
The local frame of TM is given by $\{Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7\}$, where

$$Z_1 = 2(\partial x_1 + \partial y_2 + y^1\partial z),$$

$$
\begin{aligned}
Z_2 &= 2(\partial x_2 + \partial y_1 + y^2 \partial z), \\
Z_3 &= 2(\cos\beta \partial x_3 + \sin\beta \partial y_3 + y^3 \cos\beta \partial z), \\
Z_4 &= 2(\sin\beta \partial x_4 + \cos\beta \partial y_4 + y^4 \sin\beta \partial z), \\
Z_5 &= 2(\cos u_6 \partial x_5 + \sin u_6 \partial y_5 - \sin u_5 \partial x_6 + \cos u_5 \partial y_6 + \cos u_6 y^5 \partial z \\
&\quad - \sin u_5 y^6 \partial z), \\
Z_6 &= 2(-u_5 \sin u_6 \partial x_5 + u_5 \cos u_6 \partial y_5 - u_5 \sin u_6 y^5 \partial z), \\
Z_7 &= V = 2\partial z.
\end{aligned}
$$

Hence $RadTM = \text{span}\{Z_1, Z_2\}$ and $S(TM) = \text{span}\{Z_3, Z_4, Z_5, Z_6, V\}$.
Now $ltr(TM)$ is spanned by $N_1 = -\partial x_1 + \partial y_2 - y^1 \partial z$, $N_2 = -\partial x_2 + \partial y_1 - y^2 \partial z$ and $S(TM^\perp)$ is spanned by

$$
\begin{aligned}
W_1 &= 2(\sin\beta \partial x_3 - \cos\beta \partial y_3 + y^3 \sin\beta \partial z), \\
W_2 &= 2(\cos\beta \partial x_4 - \sin\beta \partial y_4 + y^4 \cos\beta \partial z), \\
W_3 &= 2(\cos u_6 \partial x_5 + \sin u_6 \partial y_5 + \sin u_5 \partial x_6 - \cos u_5 \partial y_6 + \cos u_6 y^5 \partial z \\
&\quad + \sin u_5 y^6 \partial z), \\
W_4 &= 2(u_5 \cos u_5 \partial x_6 + u_5 \sin u_5 \partial y_6 + u_5 \cos u_5 y^6 \partial z).
\end{aligned}
$$

It follows that $\phi Z_1 = -2N_2$, $\phi Z_2 = -2N_1$, which implies that $\phi RadTM = ltr(TM)$. On the other hand, we can see that $D_1 = \text{span}\{Z_3, Z_4\}$ such that $\phi Z_3 = W_1$, $\phi Z_4 = W_2$, which implies that D_1 is anti-invariant with respect to ϕ and $D_2 = \text{span}\{Z_5, Z_6\}$ is a slant distribution with slant angle $\pi/4$. Hence M is a radical transversal screen pseudo-slant 2-lightlike submanifold of $\mathbb{R}^{13}_2$.

Now, for any vector field X tangent to M, we put $\phi X = PX + FX$, where PX and FX are tangential and transversal parts of ϕX respectively. We denote the projections on $RadTM$, D_1 and D_2 in TM by P_1, P_2 and P_3 respectively. Similarly, we denote the projections of $tr(TM)$ on $ltr(TM)$, $\phi(D_1)$ and D' by Q_1, Q_2 and Q_3 respectively, where D' is non-degenerate orthogonal complementary subbundle of $\phi(D_1)$ in $S(TM^\perp)$. Then, for any $X \in \Gamma(TM)$, we get

$$X = P_1 X + P_2 X + P_3 X + \eta(X) V. \tag{3.2}$$

Now applying ϕ to (3.2), we have

$$\phi X = \phi P_1 X + \phi P_2 X + \phi P_3 X, \tag{3.3}$$

which gives

$$\phi X = \phi P_1 X + \phi P_2 X + f P_3 X + F P_3 X, \tag{3.4}$$

where fP_3X (resp. FP_3X) denotes the tangential (resp. transversal) component of ϕP_3X. Thus we get $\phi P_1X \in \Gamma(ltr(TM))$, $\phi P_2X \in \Gamma(\phi D_1) \subset \Gamma(S(TM^\perp))$, $fP_3X \in \Gamma(D_2)$ and $FP_3X \in \Gamma(D')$. Also, for any $W \in \Gamma(tr(TM))$, we have

$$W = Q_1W + Q_2W + Q_3W. \tag{3.5}$$

Applying ϕ to (3.5), we obtain

$$\phi W = \phi Q_1W + \phi Q_2W + \phi Q_3W, \tag{3.6}$$

which gives

$$\phi W = \phi Q_1W + \phi Q_2W + BQ_3W + CQ_3W, \tag{3.7}$$

where BQ_3W (resp. CQ_3W) denotes the tangential (resp. transversal) component of ϕQ_3W. Thus we get $\phi Q_1W \in \Gamma(RadTM)$, $\phi Q_2W \in \Gamma(D_1)$, $BQ_3W \in \Gamma(D_2)$ and $CQ_3W \in \Gamma(D')$.

Now, by using (2.23), (3.4), (3.7) and (2.7)–(2.9) and identifying the components on $RadTM$, D_1, D_2, $ltr(TM)$, $\phi(D_1)$, D' and $\{V\}$, we obtain

$$\begin{aligned} P_1(A_{\phi P_2Y}X) + P_1(A_{\phi P_1Y}X) + P_1(A_{FP_3Y}X) &= P_1(\nabla_X fP_3Y) \\ &- \phi h^l(X,Y) + \eta(Y)P_1X, \end{aligned} \tag{3.8}$$

$$\begin{aligned} P_2(A_{\phi P_2Y}X) + P_2(A_{\phi P_1Y}X) + P_2(A_{FP_3Y}X) &= P_2(\nabla_X fP_3Y) \\ &- \phi Q_2h^s(X,Y) + \eta(Y)P_2X, \end{aligned} \tag{3.9}$$

$$\begin{aligned} P_3(A_{\phi P_2Y}X) + P_3(A_{\phi P_1Y}X) + P_3(A_{FP_3Y}X) &= P_3(\nabla_X fP_3Y) \\ &- BQ_3h^s(X,Y) - fP_3\nabla_XY + \eta(Y)P_3X, \end{aligned} \tag{3.10}$$

$$\nabla^l_X\phi P_1Y + D^l(X,\phi P_2Y) + h^l(X,fP_3Y) + D^l(X,FP_3Y) = \phi P_1\nabla_XY, \tag{3.11}$$

$$\begin{aligned} Q_2\nabla^s_X\phi P_2Y + Q_2\nabla^s_XFP_3Y =& \phi P_2\nabla_XY - Q_2D^s(X,\phi P_1Y) \\ &- Q_2h^s(X,fP_3Y), \end{aligned} \tag{3.12}$$

$$\begin{aligned} Q_3\nabla^s_X\phi P_2Y + Q_3\nabla^s_XFP_3Y - FP_3\nabla_XY &= CQ_3h^s(X,Y) \\ &- Q_3h^s(X,fP_3Y) - Q_3D^s(X,\phi P_1Y), \end{aligned} \tag{3.13}$$

$$\eta(\nabla_XfP_3Y) - \eta(A_{\phi P_1Y}X) - \eta(A_{\phi P_2Y}X) - \eta(A_{FP_3Y}X) = \overline{g}(\phi X,\phi Y). \tag{3.14}$$

Theorem 3.2. *Let M be a $2q$-lightlike submanifold of a spacelike almost contact metric manifold $\overline{M}$ with structure vector field tangent to M. Then M is a radical transversal screen pseudo-slant lightlike submanifold of $\overline{M}$ if and only if*

(i) $\phi ltr(TM)$ is a distribution on M such that $\phi ltr(TM) = RadTM$,

(ii) distribution D_1 is anti-invariant with respect to ϕ, i.e. $\phi D_1 \subset S(TM^{\perp})$,

(iii) there exists a constant $\lambda \in (0, 1]$ such that $P^2X = -\lambda X$.

Moreover, there also exists a constant $\mu \in [0, 1)$ such that $BFX = -\mu X$, for all $X \in \Gamma(D_2)$, where D_1 and D_2 are non-degenerate orthogonal distributions on M such that $S(TM) = D_1 \oplus_{orth} D_2 \oplus_{orth} \{V\}$ and $\lambda = \cos^2\theta$, θ is slant angle of D_2.

Proof. Let M be a radical transversal screen pseudo-slant lightlike submanifold of a spacelike almost contact metric manifold $\overline{M}$. Then distribution D_1 is anti-invariant with respect to ϕ and $\phi RadTM = ltr(TM)$. Thus $\phi X \in \Gamma(ltr(TM))$, for all $X \in \Gamma(RadTM)$. Hence $\phi(\phi X) \in \Gamma(\phi(ltr(TM)))$, which implies $-X \in \Gamma(\phi(ltr(TM)))$, for all $X \in \Gamma(RadTM)$, which proves (i) and (ii).

Now for any $X \in \Gamma(D_2)$, we have $|PX| = |\phi X| \cos\theta$, which implies

$$\cos\theta = \frac{|PX|}{|\phi X|}. \tag{3.15}$$

In view of (3.15), we get $\cos^2\theta = \dfrac{|PX|^2}{|\phi X|^2} = \dfrac{g(PX, PX)}{g(\phi X, \phi X)} = \dfrac{g(X, P^2X)}{g(X, \phi^2X)}$, which gives

$$g(X, P^2X) = \cos^2\theta\, g(X, \phi^2X). \tag{3.16}$$

Since M is radical transversal screen pseudo-slant lightlike submanifold, $\cos^2\theta = \lambda(\text{const}) \in (0, 1]$ and therefore from (3.16), we get $g(X, P^2X) = \lambda g(X, \phi^2X) = g(X, \lambda\phi^2X)$, which implies

$$g(X, (P^2 - \lambda\phi^2)X) = 0. \tag{3.17}$$

Since $(P^2 - \lambda\phi^2)X \in \Gamma(D_2)$ and the induced metric $g = g|_{D_2\times D_2}$ is non-degenerate (positive definite), from (3.17), we have $(P^2 - \lambda\phi^2)X = 0$, which implies

$$P^2X = \lambda\phi^2X = -\lambda X. \tag{3.18}$$

Now, for any vector field $X \in \Gamma(D_2)$, we have

$$\phi X = PX + FX, \tag{3.19}$$

where PX and FX are tangential and transversal parts of ϕX respectively.

Applying ϕ to (3.19) and taking tangential component, we get

$$-X = P^2X + BFX. \tag{3.20}$$

From (3.18) and (3.20), we get

$$BFX = -\mu X, \tag{3.21}$$

where $1 - \lambda = \mu(\text{const}) \in [0, 1)$. This proves (iii).

Conversely suppose that conditions (i), (ii) and (iii) are satisfied. From (i), we have $\phi N \in \Gamma(RadTM)$, for all $N \in \Gamma(ltr(TM))$. Hence $\phi(\phi N) \in \Gamma(\phi(RadTM))$, which implies $-N \in \Gamma(\phi(RadTM))$, for all $N \in \Gamma(ltr(TM))$. Thus $\phi RadTM = ltr(TM)$. From (3.20), for any $X \in \Gamma(D_2)$, we get

$$-X = P^2X - \mu X, \tag{3.22}$$

which implies

$$P^2X = -\lambda X, \tag{3.23}$$

where $1 - \mu = \lambda(\text{const}) \in (0, 1]$.

Now

$$\begin{aligned}\cos\theta = \frac{g(\phi X, PX)}{|\phi X||PX|} &= -\frac{g(X, \phi PX)}{|\phi X||PX|} = -\frac{g(X, P^2X)}{|\phi X||PX|}\\ = -\lambda\frac{g(X, \phi^2 X)}{|\phi X||PX|} &= \lambda\frac{g(\phi X, \phi X)}{|\phi X||PX|}.\end{aligned}$$

From above equation, we get

$$\cos\theta = \lambda\frac{|\phi X|}{|PX|}. \tag{3.24}$$

Therefore (3.15) and (3.24) give $\cos^2\theta = \lambda(\text{const})$.

Hence M is a radical transversal screen pseudo-slant lightlike submanifold. □

Corollary 3.1. *Let M be a radical transversal screen pseudo-slant lightlike submanifold of a spacelike almost contact metric manifold $\overline{M}$ with slant angle θ, then for any $X, Y \in \Gamma(D_2)$, we have*

(i) $g(PX, PY) = \cos^2\theta\, g(X, Y)$,

(ii) $\overline{g}(FX, FY) = \sin^2\theta\, g(X, Y)$.

The proof of above Corollary follows by using similar steps as in proof of Corollary 3.2 of [12].

Lemma 3.3. *Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$. Then we have*
(*i*) $g(\nabla_X Y, V) = \overline{g}(Y, \phi X)$,
(*ii*) $g([X, Y], V) = 2\overline{g}(Y, \phi X)$, *for all* $X, Y \in \Gamma(TM - \{V\})$.

Proof. Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$. Since $\overline{\nabla}$ is a metric connection, from (2.7) and (2.24), for any $X, Y \in \Gamma(TM - \{V\})$, we have

$$g(\nabla_X Y, V) = \overline{g}(Y, \phi X). \tag{3.25}$$

From (2.22) and (3.25), we have $g([X, Y], V) = 2\overline{g}(Y, \phi X)$. □

Theorem 3.4. *Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$ with structure vector field tangent to M. Then $RadTM \oplus \{V\}$ is integrable if and only if*
(*i*) $Q_2 D^s(Y, \phi X) = Q_2 D^s(X, \phi Y)$ *and* $Q_3 D^s(Y, \phi X) = Q_3 D^s(X, \phi Y)$,
(*ii*) $P_3 A_{\phi X} Y = P_3 A_{\phi Y} X$, *for all* $X, Y \in \Gamma(RadTM \oplus \{V\})$.

Proof. Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$. Let $X, Y \in \Gamma(RadTM \oplus \{V\})$. From (3.12), we have $Q_2 D^s(X, \phi Y) = \phi P_2 \nabla_X Y$, which gives

$$Q_2 D^s(X, \phi Y) - Q_2 D^s(Y, \phi X) = \phi P_2 [X, Y].$$

In view of (3.13), we get $Q_3 D^s(X, \phi Y) = C Q_3 h^s(X, Y) + F P_3 \nabla_X Y$, which implies $Q_3 D^s(X, \phi Y) - Q_3 D^s(Y, \phi X) = F P_3 [X, Y]$. Also from (3.10), we have

$$P_3 A_{\phi Y} X + B Q_3 h^s(X, Y) = -f P_3 \nabla_X Y,$$

which gives $P_3 A_{\phi X} Y - P_3 A_{\phi Y} X = f P_3 [X, Y]$. This proves the theorem. □

Theorem 3.5. *Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$ with structure vector field tangent to M. Then D_1 is integrable if and only if*
(*i*) $Q_3(\nabla^s_Y \phi X) = Q_3(\nabla^s_X \phi Y)$ *and* $P_3 A_{\phi X} Y = P_3 A_{\phi Y} X$,
(*ii*) $D^l(X, \phi Y) = D^l(Y, \phi X)$, *for all* $X, Y \in \Gamma(D_1)$.

Proof. Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$. Let $X, Y \in \Gamma(D_1)$. From (3.11), we have $D^l(X, \phi Y) = \phi P_1 \nabla_X Y$, we have $D^l(X, \phi Y) - D^l(Y, \phi X) = \phi P_1 [X, Y]$.

In view of (3.10), we get $P_3A_{\phi Y}X + BQ_3h^s(X,Y) = -fP_3\nabla_X Y$, which implies $P_3A_{\phi X}Y - P_3A_{\phi Y}X = fP_3[X,Y]$. Also from (3.13), we have

$$Q_3(\nabla^s_X \phi Y) - CQ_3h^s(X,Y) = FP_3\nabla_X Y,$$

which gives

$$Q_3(\nabla^s_X \phi Y) - Q_3(\nabla^s_Y \phi X) = FP_3[X,Y].$$

This concludes the theorem. □

Theorem 3.6. *Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$ with structure vector field tangent to M. Then $D_2 \oplus \{V\}$ is integrable if and only if*

(i) $D^l(X,FY) - h^l(Y,fX) = D^l(Y,FX) - h^l(X,fY)$,

(ii) $Q_2(\nabla^s_X FY - h^s(Y,fX)) = Q_2(\nabla^s_Y FX - h^s(X,fY))$,

for all $X,Y \in \Gamma(D_2 \oplus \{V\})$.

Proof. Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$. Let $X,Y \in \Gamma(D_2 \oplus \{V\})$. From (3.11), we have $h^l(X,fY) + D^l(X,FY) = \phi P_1\nabla_X Y$, which gives $h^l(X,fY) - h^l(Y,fX) + D^l(X,FY) - D^l(Y,FX) = \phi P_1[X,Y]$. In view of (3.12), we get $Q_2\nabla^s_X FY + Q_2h^s(X,fY) = \phi P_2\nabla_X Y$, which implies $Q_2\nabla^s_X FY - Q_2\nabla^s_Y FX + Q_2h^s(X,fY) - Q_2h^s(Y,fX) = \phi P_2[X,Y]$. Thus, we obtain the required results. □

Theorem 3.7. *Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$ with structure vector field tangent to M. Then the induced connection ∇ on M is not a metric connection.*

Proof. Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$. Suppose that the induced connection ∇ on M is a metric connection. Then $\nabla_X \phi N \in \Gamma(RadTM)$ for all $X \in \Gamma(TM)$ and $N \in \Gamma(ltr(TM))$. From (2.7), (2.8) and (2.23), for any $X \in \Gamma(TM)$ and $N \in \Gamma(ltr(TM))$, we have

$$\begin{aligned}\nabla_X \phi N + h^l(X,\phi N) + h^s(X,\phi N) = {}& -\phi A_N X + \phi\nabla^l_X N \\ & + \phi Q_2 D^s(X,N) + \phi Q_3 D^s(X,N) + \overline{g}(X,N)V.\end{aligned} \tag{3.26}$$

Now, on comparing tangential components of both sides of (3.26), we get

$$\begin{aligned}\nabla_X \phi N = {}& -fP_3A_N X + \phi\nabla^l_X N + \phi Q_2 D^s(X,N) \\ & + BQ_3 D^s(X,N) + \overline{g}(X,N)V.\end{aligned} \tag{3.27}$$

Since $TM = RadTM \oplus_{orth} D_1 \oplus_{orth} D_2 \oplus_{orth} \{V\}$, from (3.27), we obtain

$$\nabla_X \phi N - \phi \nabla^l_X N = 0,\ BQ_3 D^s(X,N) - fP_3 A_N X = 0, \tag{3.28}$$

$$\phi Q_2 D^s(X,N) = 0, \quad \overline{g}(X,N)V = 0. \tag{3.29}$$

Now taking $X = \xi \in \Gamma(Rad(TM))$ in (3.29), we get $\overline{g}(\xi, N)V = 0$. Thus $V = 0$, which is a contradiction. Hence M does not have a metric connection. □

4. Foliations determined by distributions.

In this section, we obtain necessary and sufficient conditions for foliations determined by distributions on a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Sasakian manifold to be totally geodesic.

Definition 4.1. *A radical transversal screen pseudo-slant lightlike submanifold M of an indefinite Sasakian manifold $\overline{M}$ is said to be mixed geodesic screen pseudo-slant lightlike submanifold if its second fundamental form h satisfies $h(X,Y) = 0$, for all $X \in \Gamma(D_1)$ and $Y \in \Gamma(D_2)$. Thus M is mixed geodesic radical transversal screen pseudo-slant lightlike submanifold if $h^l(X,Y) = 0$ and $h^s(X,Y) = 0$, for all $X \in \Gamma(D_1)$ and $Y \in \Gamma(D_2)$.*

Theorem 4.1. *Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$ with structure vector field tangent to M. Then $RadTM \oplus \{V\}$ defines a totally geodesic foliation if and only if $P_1 \nabla_X f P_3 Z = P_1 A_{\phi P_2 Z} X + P_1 A_{FP_3 Z} X$, for any $X, Y \in \Gamma(RadTM \oplus \{V\})$ and $Z \in \Gamma(D_1 \oplus D_2)$.*

P r o o f. Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$. It is easy to see that $RadTM \oplus \{V\}$ defines a totally geodesic foliation if and only if

$$\nabla_X Y \in \Gamma(RadTM \oplus \{V\}), \quad \text{for all} \quad X, Y \in \Gamma(RadTM \oplus \{V\}).$$

Since $\overline{\nabla}$ is metric connection, using (2.7), (2.19), (2.23) and (3.4), for any $X, Y \in \Gamma(RadTM \oplus \{V\})$ and $Z \in \Gamma(D_1 \oplus D_2)$, we get

$$\overline{g}(\nabla_X Y, Z) = -\overline{g}(\overline{\nabla}_X(\phi P_2 Z + fP_3 Z + FP_3 Z), \phi Y).$$

From (2.7), (2.9) and above equation, we get

$$\overline{g}(\nabla_X Y, Z) = -\overline{g}(\nabla_X fP_3 Z - A_{\phi P_2 Z} X - A_{FP_3 Z} X, \phi Y),$$

which implies

$$\overline{g}(\nabla_X Y, Z) = -\overline{g}(P_1\nabla_X f P_3 Z - P_1 A_{\phi P_2 Z}X - P_1 A_{FP_3 Z}X, \phi Y).$$

This concludes the theorem. □

Theorem 4.2. *Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$ with structure vector field tangent to M. Then D_1 defines a totally geodesic foliation if and only if*

(i) $\overline{g}(h^s(X, fZ), \phi Y) = -\overline{g}(\nabla^s_X FZ, \phi Y)$,

(ii) $h^s(X, \phi N)$ *has no component in* $\phi(D_1)$,

for all $X, Y \in \Gamma(D_1)$, $Z \in \Gamma(D_2)$ *and* $N \in \Gamma(ltr(TM))$.

P r o o f. Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$. The distribution D_1 defines a totally geodesic foliation if and only if $\nabla_X Y \in \Gamma(D_1)$, for all $X, Y \in \Gamma(D_1)$. Since $\overline{\nabla}$ is metric connection, from (2.7), (2.19) and (2.23), for any $X, Y \in \Gamma(D_1)$ and $Z \in \Gamma(D_2)$, we obtain $\overline{g}(\nabla_X Y, Z) = -\overline{g}(\overline{\nabla}_X \phi Z, \phi Y)$, which gives

$$\overline{g}(\nabla_X Y, Z) = -\overline{g}(h^s(X, fZ) + \nabla^s_X FZ, \phi Y).$$

Now from (2.7), (2.19) and (2.23), for any $X, Y \in \Gamma(D_1)$ and $N \in \Gamma(ltr(TM))$, we have $\overline{g}(\nabla_X Y, N) = -\overline{g}(\phi Y, \overline{\nabla}_X \phi N)$, which implies

$$\overline{g}(\nabla_X Y, N) = -\overline{g}(\phi Y, h^s(X, \phi N)).$$

Thus, we obtain the required results. □

Theorem 4.3. *Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$ with structure vector field tangent to M. Then $D_2 \oplus \{V\}$ defines a totally geodesic foliation if and only if*

(i) $\overline{g}(fY, A_{\phi Z}X) = \overline{g}(FY, \nabla^s_X \phi Z)$,

(ii) $\overline{g}(fY, A^*_{\phi N}X) = \overline{g}(FY, h^s(X, \phi N))$,

for all $X, Y \in \Gamma(D_2 \oplus \{V\})$, $Z \in \Gamma(D_1)$ *and* $N \in \Gamma(ltr(TM))$.

P r o o f. Let M be a radical transversal screen pseudo-slant lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$. The distribution $D_2 \oplus \{V\}$ defines a totally geodesic foliation if and only if $\nabla_X Y \in \Gamma(D_2 \oplus \{V\})$, for all $X, Y \in \Gamma(D_2 \oplus \{V\})$. Since $\overline{\nabla}$ is metric connection, from (2.7), (2.19) and (2.23), for any $X, Y \in \Gamma(D_2 \oplus \{V\})$ and $Z \in \Gamma(D_1)$, we obtain

$$\overline{g}(\nabla_X Y, Z) = -\overline{g}(\phi Y, \overline{\nabla}_X \phi Z),$$

which gives

$$\overline{g}(\nabla_X Y, Z) = \overline{g}(fY, A_{\phi Z}X) - \overline{g}(FY, \nabla^s_X \phi Z).$$

Now, from (2.7), (2.19) and (2.23), for any $X, Y \in \Gamma(D_2 \oplus \{V\})$ and $N \in \Gamma(ltr(TM))$, we have

$$\overline{g}(\nabla_X Y, N) = -\overline{g}(\phi Y, \overline{\nabla}_X \phi N),$$

which implies

$$\overline{g}(\nabla_X Y, N) = \overline{g}(fY, A^*_{\phi N}X) - \overline{g}(FY, h^s(X, \phi N)).$$

Thus, the theorem is completed. □

Theorem 4.4. *Let M be a mixed geodesic radical transversal screen pseudo-slant lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$ with structure vector field tangent to M. Then D_1 defines a totally geodesic foliation if and only if $\nabla^s_X FZ$ and $h^s(X, \phi N)$ have no components in $\phi(D_1)$, for all $X \in \Gamma(D_1)$, $Z \in \Gamma(D_2)$ and $N \in \Gamma(ltr(TM))$.*

Proof. Let M be a mixed geodesic radical transversal screen pseudo-slant lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$. The distribution D_1 defines a totally geodesic foliation if and only if $\nabla_X Y \in \Gamma(D_1)$, for all $X, Y \in \Gamma(D_1)$. Since $\overline{\nabla}$ is metric connection, from (2.7), (2.19) and (2.23), for any $X, Y \in \Gamma(D_1)$ and $Z \in \Gamma(D_2)$, we get $\overline{g}(\nabla_X Y, Z) = -\overline{g}(\phi Y, \overline{\nabla}_X \phi Z)$, which gives $\overline{g}(\nabla_X Y, Z) = -\overline{g}(\nabla^s_X FZ + h^s(X, fZ), \phi Y)$. Now, from (2.7), (2.19) and (2.23), for all $X, Y \in \Gamma(D_1)$ and $N \in \Gamma(ltr(TM))$, we get $\overline{g}(\nabla_X Y, N) = -\overline{g}(\phi Y, \overline{\nabla}_X \phi N)$, which implies $\overline{g}(\nabla_X Y, N) = -\overline{g}(\phi Y, h^s(X, \phi N))$. This proves the theorem. □

Theorem 4.5. *Let M be a mixed geodesic radical transversal screen pseudo-slant lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$ with structure vector field tangent to M. Then the induced connection ∇ on $D_1 \oplus D_2$ is a metric connection if and only if*

(i) $D^s(X, \phi\xi)$ has no component in $\phi(D_1)$,

(ii) $\overline{g}(fW, A_{\phi\xi}Z) = \overline{g}(FW, D^s(Z, \phi\xi))$,

for all $X, \in \Gamma(D_1)$, $Z, W \in \Gamma(D_2)$ and $\xi \in \Gamma(RadTM)$.

Proof. Let M be a mixed geodesic radical transversal screen pseudo-slant lightlike submanifold of an indefinite Sasakian manifold $\overline{M}$. Then $h^l(X, Z) = 0$, for all $X \in \Gamma(D_1)$ and $Z \in \Gamma(D_2)$. Since $\overline{\nabla}$ is metric connection, from (2.7), (2.19) and (2.23), for any $X, Y \in \Gamma(D_1)$ and $\xi \in \Gamma(RadTM)$, we have $\overline{g}(h^l(X, Y), \xi) = -g(\phi Y, \overline{\nabla}_X \phi\xi)$, which gives $\overline{g}(h^l(X, Y), \xi) = -g(\phi Y, D^s(X, \phi\xi))$.

In view of (2.7), (2.19) and (2.23), for any $Z, W \in \Gamma(D_2)$ and $\xi \in \Gamma(RadTM)$, we get $\overline{g}(h^l(Z, W), \xi) = -\overline{g}(\phi W, \overline{\nabla}_Z \phi\xi)$, which implies

$$\overline{g}(h^l(Z, W), \xi) = \overline{g}(fW, A_{\phi\xi} Z) - \overline{g}(FW, D^s(Z, \phi\xi)).$$

Thus, we obtain the required results. □

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