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ABSORPTION OF THE POISSON PROCESS BETWEEN TWO BOUNDARIES

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In this paper, we consider the absorption of the simple Poisson process between two curved boundaries. An explicit formula is derived for the probability that a sample function of a Poisson process will never be outside of these bounds (upper and lower) until a fixed moment.

1. Introduction. Let $\xi_1, \xi_2, \dots$ be a sequence of independent and identically exponentially distributed random variables with mean $\mathbf{E}\xi_1 = 1$. We can consider the random variables (r.v.)

$$t_1 = \xi_1,\ t_2 = \xi_1 + \xi_2, \dots,\ t_m = \xi_1 + \dots + \xi_m, \dots$$

as the successive points of a time-homogeneous Poisson process $\kappa(t)$ with intensity 1.

The r.v. t_m, $m = 1, 2, \dots$ can be interpreted as moments of successive arrivals of claims for an insurance company or a bank. If we take the size of any claim equal to 1 then $\kappa(t) = \sum_n I_{\{t_n \le t\}}$ represents the aggregate claim amount at time t of the company.

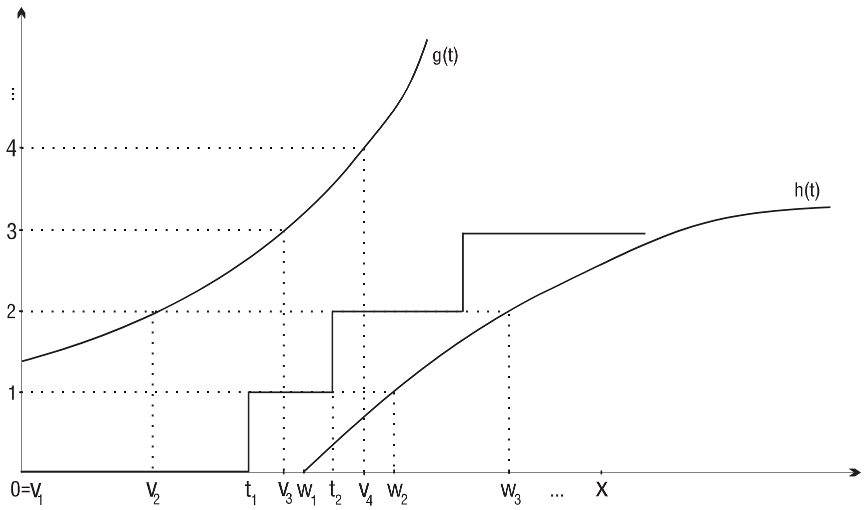

Figure 1

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Further we consider an upper boundary $g(t)$ which is nonnegative increasing real function defined on $\mathbf{R}^+$ and such as $\lim_{t\to\infty} g(t) = +\infty$ and a lower boundary $h(t)$ which is non-decreasing (figure 1). The interpretation of $g(t)$ is the accumulated bank incomes earned in the time interval $(0, t]$. The interpretation of $h(t)$ is that at any moment t, $g(t) - h(t)$ is the obligatory capital needed for the liquidity of the bank and $g(t) - \kappa(t)$ represents the current available cash.

Define the r.v. T as the moment of the first crossing of the trajectory of $\kappa(t)$ with either the upper or the lower boundary. If there is a crossing with the upper boundary it means that the bank has no money to pay the claim. The crossing with the lower boundary means that the bank retains too much non-invested money.

Our purpose is to determine the probability $P(T > x)$ for any real number x.

Let us note that there are many investigations on the absorption problem. The papers by Gallot (1966, 1993), Ph. Picard and C. Lefevre (1997) are only concerned with upper boundaries. In Daniels (1963), Zacks (1991) and Ph. Picard, C. Lefevre (1996), both kinds of boundaries are discussed.

2. Main results. Let us define

$$g^{-1}(y) = \inf\{t : g(t) \geq y\},\ h^{-1}(y) = \inf\{t : h(t) \geq y\};$$
$$v_i = \max\{0, g^{-1}(i)\},\ w_i = h^{-1}(i-1),\ i = 1, 2, \dots \text{ (Figure 1)}$$

For convenience we suppose $v_i < w_i$ for $i = 1, 2, \dots$.

In order that the trajectory of the process $\kappa(t)$ to be under the boundary $g(t)$, it is necessary and sufficient to accomplish the event

$$\bigcap_{i=1}^{\infty} \{\kappa(v_i) \leq i - 1\}.$$

The trajectory of the process $\kappa(t)$ until the moment x is under the boundary $g(t)$ if and only if the event $\bigcap_{i=1}^{\infty} \{\kappa(\min(v_i, x)) \leq i - 1\}$ takes place.

On the other hand the trajectory does not cross the lower boundary $h(t)$ till the moment x if and only if the event $\bigcap_{i=1}^{\infty} \{\kappa(w_i) \geq \min(i, h(x))\}$ occurs.

Introduce

$$h = \begin{cases} h(x), & h(x) \in \mathbf{N} \\ [h(x)] + 1, & h(x) \notin \mathbf{N} \end{cases}$$

where $[h(x)]$ is the integer part of x and $\mathbf{N}$ is the set of nonnegative integers. Clearly, for any integer i is true that $\min(i, h(x)) = \min(i, h)$.

For the probability $P(T > x)$ we have

$$\begin{aligned} P(T > x) &= P(h(t) < \kappa(t) < g(t),\ \forall t \in (0, x]) = \\ &= P\left[\left(\bigcap_{i=1}^{\infty} \kappa(\min(v_i, x)) \leq i - 1\right) \cap \left(\bigcap_{i=1}^{\infty} \kappa(w_i) \geq \min(i, h)\right)\right] \end{aligned} \tag{2.1}$$

The expression in (2.1.) can be transformed into

$$(2.2.)\quad P(T>x)=P\left[\left(\bigcap_{i=1}^{\infty}\kappa(\min(v_i,x))\le i-1\right)\cap\overline{\overline{\left(\bigcap_{i=1}^{\infty}\kappa(w_i)\ge\min(i,h)\right)}}\right]=$$

$$=P\left(\bigcap_{i=1}^{\infty}\kappa(\min(v_i,x))\le i-1\right)-P\left[\left(\bigcap_{i=1}^{\infty}\kappa(\min(v_i,x))\le i-1\right)\cap\left(\bigcup_{i=1}^{\infty}\kappa(w_i)<\min(i,h)\right)\right]=$$

$$=P\left(\bigcap_{i=1}^{\infty}\kappa(\min(v_i,x))\le i-1\right)-P\left[\left(\bigcap_{i=1}^{\infty}\kappa(\min(v_i,x))\le i-1\right)\cap\left(\bigcup_{i=1}^{\infty}\kappa(w_i)\le\min(i-1,h-1)\right)\right],$$

where the upper bar denotes the complement of the event staying under this bar. For any integer k, such that $v_{k-1}<x\le v_k$ we have $\min(v_i,x)=x$ for $i\ge k$. Therefore for $i\ge k$ follows $(\kappa(\min(v_i,x))\le i-1\equiv(\kappa(x)\le i-1)\supset(\kappa(x)\le k-1)$ and

$$(2.3)\qquad \bigcap_{i=1}^{\infty}(\kappa(\min(v_i,x))\le i-1)\equiv\bigcap_{i=1}^{k}(\kappa(\min(v_i,x))\le i-1)$$

By analogy with the above for $i\ge h$ we have:

$$(2.4)\qquad \begin{gathered}(\kappa(w_i)\le\min(i-1,h-1))\equiv(\kappa(w_i)\le h-1)\subset(\kappa(w_h)\le h-1)\text{ and}\\ \bigcup_{i=1}^{\infty}(\kappa(w_i)\le\min(i-1,h-1))\equiv\bigcup_{i=1}^{h}(\kappa(w_i)\le\min(i-1,h-1))\equiv\bigcup_{i=1}^{h}(\kappa(w_i)\le i-1)\end{gathered}$$

Combining (2.2), (2.3) and (2.4) we get

$$(2.5)\quad P(T>x)=P\left(\bigcap_{i=1}^{k}(\kappa(\min(v_i,x))\le i-1\right)-P\left(\bigcap_{i=1}^{k}(\kappa(\min(v_i,x))\le i-1)\cap\bigcup_{i=1}^{h}(\kappa(w_i)\le i-1)\right)$$

The event $B=\left(\bigcap_{i=1}^{k}(\kappa(\min(v_i,x))\le i-1)\cap\bigcup_{i=1}^{h}(\kappa(w_i)\le i-1)\right)$ on the right hand side of (2.5) can be written as $B=\bigcup_{i=1}^{h}A\cap\left(\kappa(w_i)\le i-1\right)$ where $A=\bigcap_{i=1}^{k}\left(\kappa(\min(v_i,x))\le i-1\right)$. Applying the inclusion exclusion formula to the event B we get

$$(2.6)\qquad \begin{aligned}P(B)=P\left(\bigcup_{i=1}^{h}A\cap(\kappa(w_i)\le i-1)\right)&=\sum_{i=1}^{h}P(A\cap(\kappa(w_i)\le i-1))-\\ -\sum_{1\le i<j\le h}&P(A\cap(\kappa(w_i)\le i-1)\cap(\kappa(w_i)\le j-1))+\ldots+\\ +(-1)^{m-1}\sum_{1\le i_1<i_2<\ldots<i_m\le h}&P(A\cap(\kappa(w_{i_1})\le i_1-1)\cap(\kappa(w_{i_2})\le i_2-1))\cap\ldots\\ \cap(\kappa(w_{i_m})\le i_m-1))+\ldots+P(A\cap&(\kappa(w_1)\le 0)\cap(\kappa(w_2)\le 1)\cap\ldots\cap(\kappa(w_h)\le h-1))\end{aligned}$$

For $1\le i<k$ $\min(v_i,x)=v_i$ and $\min(v_k,x)=x$, so the general term in (2.6.) can be expressed in the form:

$$(2.7)\ P(\kappa(v_1)\le 0,\kappa(v_2)\le 1,\ldots,\kappa(v_{k-1})\le k-2,\kappa(x)\le k-1,\kappa(w_{i_1}\le i_1-1,\ldots,\kappa(w_{i_m})\le i_m-1)$$

From the definition of h and k it is clear that $h\le k$.

Construct the sequences $\tau_1(i_1,\ldots,i_m),\ldots,\tau_{k-1}(i_1,\ldots,i_m)$, briefly τ_j, $j=1,\ldots,k-1$, through the following table:

$$\underbrace{\begin{array}{c|c|c|c}\tau_1&\tau_2&\ldots&\tau_{i_1-1}\\\hline 0&0&\ldots&0\end{array}}_{i_1-1}\quad\underbrace{\begin{array}{c|c|c}\tau_{i_1}&\ldots&\tau_{i_2-1}\\\hline w_{i_1}&\ldots&w_{i_1}\end{array}}_{i_2-i_1}\quad\begin{array}{c|c}\tau_{i_1}&\ldots\\\hline w_{i_2}&\ldots\end{array}\quad\underbrace{\begin{array}{c|c|c}\tau_{i_{i_m}}&\ldots&\tau_{k-1}\\\hline w_{i_m}&\ldots&w_{i_m}\end{array}}_{k-i_m}$$

and the sequence $z_j(i_1,\dots,i_m)=\max(\tau_j(i_1,\dots,i_m),v_j)$, $j=1,2,\dots,k-1$, briefly z_j. Substitute $z_k(i_1,\dots,i_m)=x$.

It can be easily seen that

$$z_i < x,\ i=1,\dots,k-1 \tag{2.8}$$

In terms of the new notations (2.7.) can be transformed to:

$$\begin{aligned} &P\big(\kappa(v_1)\le 0,\kappa(v_2)\le 1,\dots,\kappa(v_{k-1})\le k-2,\kappa(x)\le k-1,\kappa(w_{i_1})\le i_1-1,\dots,\\ &\kappa(w_{i_m}\le i_m-1\big)=P\left(\left(\bigcap_{j=1}^{k-1}\kappa(z_j(i_1,\dots,i_m)\le j-1\right)\cap(\kappa(x)\le k-1)\right)=\\ &=P\left(\bigcap_{j=1}^{k}\{\kappa(\min(z_j(i_1,\dots,i_m),x)\le j-1)\}\right)\end{aligned} \tag{2.9}$$

Now we will use the following result from [1] and [5]

$$P\left(\bigcap_{j=1}^{k}\{\kappa(\min(z_j(i_1,\dots,i_m),x)\le j-1)\}\right)=e^{-x}\sum_{l=0}^{k-1}(-1)^l\delta_l\sum_{j=0}^{k-1-l}\frac{x^j}{j!} \tag{2.10}$$

if $t_1<t_2<\ldots<t_{k-1}<x<t_k$, where δ_l is the determinant

$$\delta_l(t_1,\dots,t_l)=\begin{vmatrix} \frac{t_1}{1!} & 1 & 0 & 0 & \dots & 0 & 0\\ \frac{t_2^2}{2!} & \frac{t_2}{1!} & 1 & 0 & \dots & 0 & 0\\ \frac{t_3^3}{3!} & \frac{t_3^2}{2!} & \frac{t_3^1}{1!} & 1 & \dots & 0 & 0\\ \dots & \dots & \dots & \dots & \dots & \dots & \dots\\ \frac{t_l^l}{l!} & \frac{t_l^{l-1}}{(l-1)!} & \frac{t_l^{l-2}}{(l-2)!} & \dots & \dots & \frac{t_l^2}{2!} & \frac{t_l}{1!}\end{vmatrix}\quad \text{for } l\ge 1$$

and $\delta_0\equiv 1$.

Finally from (2.5), (2.6), (2.9) and (2.10) we obtain

$$\begin{aligned} P(T>x)=e^{-x}\Bigg\{&\sum_{l=0}^{k-1}(-1)^l\delta_l(v_1,\dots,v_l)\sum_{j=0}^{k-1-l}\frac{x^j}{j!}+\\ &+\sum_{m=1}^{h}(-1)^m\sum_{l\le i_1<i_2<\ldots<i_m\le h}\ \sum_{l=0}^{k-1}(-1)^l\delta_l(z_1(i_1,\dots,i_m),\dots,z_l(i_1,\dots,i_m))\sum_{j=0}^{k-1-l}\frac{x^j}{j!}\Bigg\}\end{aligned} \tag{2.11}$$

Clearly, when $h=0$ the formula (2.11) coincides with (2.10).

If $h\ge 1$, then (2.11) can be simplified. Let us first show that the expressions before δ_0 are equal to zero. We have:

$$\begin{aligned} &e^{-x}\left[\sum_{j=0}^{k-1}\frac{x^j}{j!}+\sum_{m=1}^{h}(-1)^m\sum_{1\le i_1<\ldots<i_m\le h}1\sum_{j=0}^{k-1}\frac{x^j}{j!}\right]=e^{-x}\sum_{j=0}^{k-1}\frac{x^j}{j!}\left(1+\sum_{m=1}^{h}(-1)^m\binom{h}{m}\right)\\ &=e^{-x}\left(\sum_{j=0}^{k-1}\frac{x^j}{j!}\right)\sum_{m=0}^{h}(-1)^m\binom{h}{m}=e^{-x}\left(\sum_{j=0}^{k-1}\frac{x^j}{j!}\right)(1-1)^h=0\end{aligned}$$

So for $h \geq 1$ the formula (2.11) can be written in the form

$$
\begin{gathered}
P(T > x) = e^{-x} \left\{ \sum_{l=1}^{k-1} (-1)^l \delta_l(v_1, \ldots, v_l) \sum_{j=0}^{k-1-l} \frac{x^j}{j!} + \right. \\
\left. + \sum_{m=1}^{h} (-1)^m \sum_{l \leq j_1 < j_2 < \ldots < j_m \leq h} \sum_{l=1}^{k-1} (-1)^l \delta_l(z_1(i_1, \ldots, i_m), \ldots, z_l(i_1, \ldots, i_m)) \sum_{j=0}^{k-1-l} \frac{x^j}{j!} \right\}
\end{gathered} \tag{2.12}
$$

Multiply the both sides of (2.12) with e^x and exchange the order of the summation with respect to the indices l and m. We have

$$
\begin{gathered}
e^x P(T > x) = \sum_{l=1}^{k-1} (-1)^l \delta_l(v_1, \ldots, v_l) \sum_{j=0}^{k-1-l} \frac{x^j}{j!} + \\
\sum_{l=1}^{k-1} (-1)^l \sum_{m=1}^{h} (-1)^m \sum_{1 \leq i_1 < \cdots < i_m < h} \delta_l(z_1(i_1, \ldots, i_m), \ldots, z_l(i_1, \ldots, i_m)) \sum_{j=0}^{k-1-l} \frac{x^j}{j!}
\end{gathered}
$$

Let us fix the number $l : 1 \leq l \leq h-1$ and consider all the terms $(-1)^m \delta_l(z_1(i_1, \ldots, i_m), \ldots, z_l(i_1, \ldots, i_m))$ in the second sum so that the first t indices smaller than l are fixed, i.e. $1 \leq i_1^0 < \cdots < i_t^0 \leq l$, and the other $m-t$ indices are not fixed but are after l. Then $\delta_l(z_1(i_1^0, \ldots, i_t^0, i_{t+1}, \ldots, i_m), \ldots, z_l(i_1^0, \ldots, i_t^0, i_{t+1}, \ldots, i_m))$ does not change its value and that allows us to write

$$
\begin{gathered}
(-1)^l \sum_{j=0}^{k-1-l} \frac{x^j}{j!} \delta_l(z_1(i_1^0, \ldots, i_t^0, i_{t+1}, \ldots, i_m), \ldots, z_l(i_1^0, \ldots, i_t^0, i_{t+1}, \ldots, i_m)) \sum_{m=t}^{t+h-l} (-1)^m \sum_{\substack{1 \leq i_1 < \cdots < i_m \leq h \\ i_1 = i_1^0 \\ \vdots \\ i_t = i_t^0 \\ i_{t+1} > 1}} 1 = \\
(-1)^l \sum_{j=0}^{k-1-l} \frac{x^j}{j!} \delta_l(z_1(i_1^0, \ldots, i_t^0, i_{t+1}, \ldots, i_m), \ldots, z_l(i_1^0, \ldots, i_t^0, i_{t+1}, \ldots, i_m)) \sum_{m=t}^{t+h-l} (-1)^m \binom{h-l}{m-t} = \\
(-1)^l \sum_{j=0}^{k-1-l} \frac{x^j}{j!} \delta_l(z_1(i_1^0, \ldots, i_t^0, i_{t+1}, \ldots, i_m), \ldots, z_l(i_1^0, \ldots, i_t^0, i_{t+1}, \ldots, i_m))(1-1)^{h-l} = 0
\end{gathered} \tag{2.13}
$$

The last equation can be written because $h - l \geq 1$. In the case $t = 0$, i.e. $i_1 > l$ it is true that $z_1(i_1, \ldots, i_m) = v_1, \ldots, z_l(i_1, \ldots, i_m) = v_l$ and if we transform the expressions containing $\delta_l(v_1, \ldots, v_l)$ they are again zero. In fact this terms with fixed l satisfy the following:

$$(-1)^l\delta_l(v_1,\dots,v_l)\sum_{j=0}^{k-1-l}\frac{x^j}{j!}\Big(1+\sum_{m=1}^{h-1}(-1)^m\sum_{\substack{1\le i_1<\cdots<i_m\le h\\ i_1>l}}1\Big)=$$

(2.14)
$$=(-1)^l\delta_l(v_1,\dots,v_l)\sum_{j=0}^{k-1-l}\frac{x^j}{j!}\left[\binom{h-l}{0}+\sum_{m=1}^{h-1}(-1)^m\binom{h-l}{m}\right]=$$
$$=\left((-1)^l\delta_l(v_1,\dots,v_l)\sum_{j=0}^{k-1-l}\frac{x^j}{j!}\right)(1-1)^{h-l}=0$$

Combining (2.12), (2.13) and (2.14) we can write:

(2.15)
$$P(T>x)=e^{-x}\left\{\sum_{l=1}^{k-1}(-1)^l\delta_l(v_1,\dots,v_l)\sum_{j=0}^{k-1-l}\frac{x^j}{j!}+\right.$$
$$\left.+\sum_{l=h}^{k-1}(-1)^l\sum_{m=1}^{h}(-1)^m\sum_{1\le j_1<\cdots<j_m\le h}\delta_l(z_1(i_1,\dots,i_m),\dots,z_l(i_1,\dots,i_m))\sum_{j=0}^{k-1-l}\frac{x^j}{j!}\right\}$$

The last formula we derived under the assumptions $v_i<w_i$ for $i=1,2,\dots$. The following lemma allows us to remove the condition $v_i<w_i$ for $i=1,2,\dots$.

Lemma. *If for some j $(j=1,\dots,h)$ $v_j\ge w_j$, then $P(T>x)$ in formula (2.15) is equal to zero.*

Proof. The vanishing of $P(T>x)$ is obvious from the following:

(2.16)
$$P(T>x)=P\left(\left\{\bigcap_{i=1}^{k}\kappa(0,\min(v_i,x))\le i-1\right\}\cap\left\{\bigcap_{i=1}^{h}\kappa(0,w_i)\ge i\right\}\right)=$$
$$=P\Big(\{\xi_1+\cdots+\xi_j\ge v_j\}\cap\{\xi_1+\cdots+\xi_j\}\cap\{\bigcap_{\substack{j=1\\ i\ne j}}^{k}(\xi_1+\cdots+\xi_j\ge v_j)\cap\{\bigcap_{\substack{j=1\\ i\ne j}}^{h}(\xi_1+\cdots+\xi_j\le w_i)\}\Big)$$

From the assumption $v_j\ge w_j$ follows that the event $\{\xi_1+\cdots+\xi_j\ge v_j\}\cap\{\xi_1+\cdots+\xi_j\le w_j\}$ is the impossible event. Then from the equality (2.16) we obtain:

(2.17)
$$P(T>x)=P(\oslash)=0$$

The formal proof of (2.17) from the formula (2.15) will be dropped.

From all the above we can formulate the following theorem.

Theorem 1. *The probability that the Poisson process trajectory will not cross the boundaries $g(t)$ and $h(t)$ till the moment x $(x<\infty)$, is*

$$P(T>x)=e^{-x}\left\{\sum_{l=1}^{k-1}(-1)^l\delta_l(v_1,\dots,v_l)\sum_{j=0}^{k-1-l}\frac{x^j}{j!}+\right.$$
$$\left.+\sum_{l=h}^{k-1}(-1)^l\sum_{m=1}^{h}(-1)^m\sum_{1\le j_1<\cdots<j_m\le h}\delta_l(z_1(i_1,\dots,i_m),\dots,z_l(i_1,\dots,i_m))\sum_{j=0}^{k-1-l}\frac{x^j}{j!}\right\}$$

Using Theorem 1 in the particular case $g(x)=\infty$, i.e. $v_i=0$, $i=1,2,\dots$ it can be derived

Theorem 2. *The probability that the Poisson process trajectory will not cross the lower boundary $h(t)$ till the moment x ($x < \infty$), is*

$$P(t_1 < w_1, t_2 < w_2, \ldots, t_n < w_n) = 1 - \sum_{j=1}^{n} e^{-w_j}\delta_{j-1}(w_{j-1}, \ldots, w_1) \tag{2.18}$$

Consequently, the probability that the trajectory of the Poisson process will cross for the first time $h(t)$ on the level $n+1$ can be calculated with the formula

$$P(t_1 < w_1, t_2 < w_2, \ldots, t_n < w_n, t_{n+1} > w_{n+1}) = e^{-w_{n+1}}\delta_n(w_n, w_{n-1}, \ldots, w_1)$$

The last result was found by A. Wald and J. Wolfowitz (1939).

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АБСОРБИРАНЕ НА ПОАСОНОВ ПРОЦЕС МЕЖДУ ДВЕ ГРАНИЦИ

Цветан Игнатов, Добрин Марчев, Татяна Маджарова

В тази статия разглеждаме абсорбиране на поасонов процес между две криви граници. Изведена е формула за вероятността траекторията на поасоновия процес да не напуска областта между горната и долна граници до фиксиран момент.