МАТЕМАТИКА И МАТЕМАТИЧЕСКО ОБРАЗОВАНИЕ, 2002
MATHEMATICS AND EDUCATION IN MATHEMATICS, 2002
Proceedings of Thirty First Spring Conference of
the Union of Bulgarian Mathematicians
Borovets, April 3–6, 2002

SEARCHING FOR EXTREMAL SELF-DUAL CODES OBTAINED FROM HADAMARD MATRICES OF ORDER 44 WITH AUTOMORPHISMS OF ORDER 7

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The aim of this work is to find new extremal [76, 38, 14] and [72, 36, 14] self-dual binary codes from parts of Hadamard 2-(43, 22, 11) designs with automorphisms of order 7. The result of this search is negative.

1. Introduction. We refer to [1], [3], [4] for the basic concepts and notations concerning Hadamard matrices and combinatorial designs, and to [6] and [7] for more information about the construction of extremal self-dual binary codes from designs.

Still, there are open cases for single-even [72, 36, 14] and doubly-even [72, 36, 16] extremal self-dual codes. Baartmans and Yorgov [2] have discovered one single-even [76, 38, 14] extremal self-dual code. Conway and Pless [5] have proved that the primes which could divide the automorphism group order of a doubly-even [72,36,16] code, are 23, 17, 11, 7, 5, 3, 2. In [7] 23, 17, 11 have been rejected. Hence, we have the following possible prime divisors: 7, 5, 3, 2.

The aim of the present work is to examine the 'children' of extremal self-dual [88,44,16] codes resulting from Hadamard matrices of order 44 with automorphisms of order 7. A generator matrix of such a code is described in [7]. This work is an attempt to find extremal self-dual codes of lengths 72 and 76 using the Hadamard matrices and the extremal self-dual [88,44,16] codes obtained by Topalova [8].

Construction tips are discussed in section 2. The searching program gave a negative result.

2. Construction of extremal self-dual codes. Let us denote by G_{88} the generator matrix of an [88,44,16] extremal self-dual code, which is obtained from a Hadamard matrix of order 44,

$$G_{88} = (A' \;\; I_{44}),$$

where

$$A' = \begin{pmatrix} A & u_{43}^t \\ u_{43} & 0 \end{pmatrix},$$

where u_{43} is the all-one vector of length 43, A is an incidence matrix of a Hadamard design, such that $v = 4t - 1 = 43, k = 2t = 22, \lambda = t = 11$.

To obtain codes of length 76, we need to delete 6 columns from I_{44}. Next, we need to consider all $\binom{44}{6}$ choices to delete 6 columns from A'. For each combination we must erase 6 rows from G_{88} in 44 choose 6 different ways. Then for each case we should check, if every codeword's weight is even. Then we should check whether the obtained set of codewords are self-orthogonal.

Could we do it better? Yes - after we choose the columns to be deleted we can test whether some of the remaining vectors have even weight, say their number is R, then we never test them for self-orthogonality. Hence we need to consider only

$$\begin{pmatrix} \text{all rows of the generator matrix of [88,44,16]} - R \\ 6 - R \end{pmatrix}$$

ways to choose a set of those vectors which must be tested for self-orthogonality.

As far as the rows of the generator-matrix are even, we need to test for even weight and self-orthogonality only the deleted parts of each row, i.e. vectors of length 6.

When we search for extremal self-dual codes of lenght 72 we work in the same way, but we need to replace 6 with 8 everywhere.

Unfortunately, no self-dual codes of lenghts 72 and 76 were obtained this way from the Hadamard matrices of order 44 with automorphisms of order 7.

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ТЪРСЕНЕ НА ЕКСТРЕМАЛНИ САМОДУАЛНИ КОДОВЕ ПОЛУЧАВАЩИ СЕ ОТ АДАМАРОВИ МАТРИЦИ ОТ РЕД 44 С АВТОМОРФИЗМИ ОТ РЕД 7

Росен Стоянов Златарски

Целта на тази работа е да се открият екстремални [76,38,14] и [72,36,14] самодуални двоични кодове от части от Адамарови 2-(43,22,11) дизайни с автоморфизми от ред 7. Резултатът от търсенето е отрицателен.