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APPLICATION OF THE SUBJECTIVE QUANTITATIVE CRITERIA METHOD FOR OPERATIONAL PROGNOSIS

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Sometimes we face situations where decision-making, based on classical optimization methods, is impossible. Such problems exists where objects, actions and processes are described fuzzily. In similar situations, complaining of the lack of methods, unmotivated decisions are made. Actually, there are some solutions, and one of them is to apply a method of subjective quantitative criteria. This method is based on expert evaluations, that is, the human intellect is being used as a measuring instrument. Influence of subjective extraneous factor may be so distinct, that referring to a single expert opinion is too risky. Therefore methods of processing expert group evaluations are being discussed in this article. The article contains thoughts on choosing the criteria of evaluation, compilation and processing of submitted evaluations, and an application example with conclusions.

1. Introduction. In more complicated decision-making situations methods of situational control [1], based on classification of fuzzily described objects are used. As alternative, methods of subjective quantitative criteria [2] are used. These methods are based on expert evaluations, that is, the human intellect is being used as a measuring instrument [3]. Perhaps the easiest task is to process evaluations of the same single expert. Referring to a single expert opinion may be too risky, as there are situations, where influence of subjective extraneous factor is quite distinct. Especially it happens in unique (rare, non-typical) situations. Therefore improved methods of processing group expert evaluations (presented from several persons) based on qualitative criteria evaluations are being discussed in this article.

2. Criteria of Evaluation. For evaluation of an object, action or process A procedures are executed:

1. Develop set of goals that influence a given problem or evaluation scenario A. Set $A = f(s_i)\,|, i = \overline{1,n}$ of goals has to be full, but minimal; the goals must not overlap [4].

2. For evaluation of every goal its indicator (criterion) h_{s_i} is chosen. This indicator has to be quantitative, that is, measurable, numerable or subjective ratable.

3. Complex indicator (criterion) e_A is formed for evaluation of A:

$$e_A = f(a_1, \ldots, a_m, h_{s_1}, \ldots, h_{s_n}), \qquad (1)$$

where a_i $i = \overline{1,m}$ - unknown coefficients; $f(\cdot)$ form is unknown in general case.

In case of a linear dependence:

$$e_A \;=\; \sum_{i=1}^{n} g_i h_{s_i}. \tag{2}$$

An alternative way is to use the other, quite universal [7] formula:

$$e_A = a_0 \;+\; \sum_{i=1}^{n} a_i h_{s_i} \;+\; \sum_{i<j} a_{ij} h_{s_i} \;+\; \ldots \;+ \sum_{i_1<i_2<\ldots<i_n} a_{i_1 i_2 \ldots i_n} h_{s_1} h_{s_2} \ldots h_{s_n} \tag{3}$$

Here coefficients a_i are not normalized. Applying criterion (3), the values of h_{s_i} are usually submitted (evaluated) by an expert directly, and the values of coefficients a_i are calculated from other evaluations (of the same expert). Methods of experiment planning can be used for that purpose. A bit more complicated (for small experiments) way is to use generalized evaluations, expressed [5] in fuzzy integral (*fuzzy expected value*):

$$e_A = \sup_{\alpha\in[0,1]} \min(\alpha, g_\alpha) \tag{4}$$

$$g_\alpha = \frac{1}{\lambda}\left(\prod_{i=\Theta_\alpha} (\lambda g_i + 1) - 1\right) \tag{5}$$

$$\Theta_\alpha = \{S_i \,|h_{s_i} \geq \alpha\} \tag{6}$$

where λ – coefficient of fuzzy measure g_λ from formula (5), when $\Theta_\alpha = \{S_i\}$, $-1 \leq \lambda \leq \infty$; e_A, g_i, $h_{s_i} \in [0, 1]$ has the same meaning as in formula (2).

3. Compiling Expert Evaluations. In most cases method of *questionnaire* is considered as a priority method of compiling evaluations with precondition that an expert and a knowledge engineer cannot interpret questions differently. From our point of view it is achievable when a knowledge engineer himself both prepares questionnaire questions and investigates expert (joint methods of interview and questionnaire). From psychological point of view a more convenient way is where an expert is asked to point out:

1) Interval (*a, b*), where the value of the criterion is, by the opinion of an expert;

2) Subjective probability *p*, under which the expert is right.

In that case the evaluation of indication *g* is:

$$h_g = ((a, b), p). \tag{7}$$

After interviewing of the experts, an array of indistinct numbers is received

$$\left\{ h_{jA}, g_{ji}, h_{jS_i} \,\Big|_{i=\overline{1,n},\;\; j=\overline{1,k}} \right\}, \tag{8}$$

where $j = \overline{1, k}$ – index of the expert; h_{jA} – evaluation of e_A by the j-th expert. Each item from the array (8) is written specifying 4 characteristic points of its membership function:

$$g_{ji}(x) \quad = \quad \left({}^{c}g_{ji},\; {}^{a}g_{ji},\; {}^{b}g_{ji},\; {}^{d}g_{ji} \right). \tag{9}$$

Experiments has shown that expert evaluations are often inadequate and it is mostly dependent on two reasons [6]:
1) Non-linearity of expert's subjective efficiency function; 2) Shortage of expert experience. Trying to eliminate the mistakes of the first type, arising from non-linearity of expert's subjective efficiency function, a precondition is formulated: expert turn for non-risk can be supposed to be constant, therefore a correction procedure can be performed by using classical lottery methods. For elimination the above mentioned mistakes of the second type, the evaluation ranking can be used. The ranking means forming weight coefficients for evaluation of each expert. The expert is invited to submit both $\{g_i,\, h_{s_i}\}$ and e_A evaluations (in the questionnaire). Submitting e_A evaluation $h_{jA}(x)$, an expert specifies a Paretto set of A evaluations; while submitting $\{g_{ji}(x),\, h_{js_i}(x)\}$, − an expert specifies a certain item of this set. The $e_{jA}(x)$ can be calculated by applying formulas (2), or (3), or (4). In an ideal case the indistinct numbers $h_{jA}(x)$ and $e_{jA}(x)$ would coincide. The degree of their coincidence can be the expression of expert ability to evaluate in chime (coefficient β_j).

$$\beta_j \;=\; \frac{2\oint\limits_X \left(e_{jA}\left(x\right) \wedge h_{jA}\left(x\right)\right) \circ g}{\oint\limits_X e_{jA}\left(x\right) \circ g + \oint\limits_X h_{jA}\left(x\right) \circ g}, \tag{10}$$

where $\oint\limits_X f(x) \circ g = \sup\limits_{\alpha \in [0,1]} (\alpha \wedge g(H_\alpha f(x)))$ – indistinct integral $f(x)$ by indistinct measure $g(\cdot)$; $H_\alpha = \{x : h(x) \geq \alpha\}$; and $\wedge$ means "minimum".

It can be proved that the distinctness of an evaluation prognosis can be characterized with coefficient

$$\gamma_j \;=\; \frac{\min\limits_j \left(\oint\limits_X e_{jA}\left(x\right) \circ g + \oint\limits_X h_{jA}\left(x\right) \circ g\right)}{\oint\limits_X e_{jA}\left(x\right) \circ g + \oint\limits_X h_{jA}\left(x\right) \circ g}. \tag{11}$$

Generalized expert weight coefficients are:

$$\alpha_j \;=\; \beta_j \gamma_j,\ j = \overline{1, k}; \tag{12}$$

Then generalized indistinct evaluation of an object, process or action A is characterized with:

$$H_A(x) \;=\; \frac{1}{\sum\limits_{j=1}^{k} \alpha_j} \sum_{j=1}^{k} \alpha_j h_{jA}\left(x\right). \tag{13}$$

Similar results are achieved by applying robust data evaluation methods.

4. An Application. The methodology, explained above, has been used for evaluation of implementation effectiveness of new technologies in a transportation company "Lithuanian Railways". Unfortunately we cannot present the results here because of the huge amount of data array. Instead, an application example is presented, in which an expert commission is trying to evaluate essay A in mark $h \in \{0.1,\, 0.2,\, \ldots,\, 1.0\}$. Let the $K = \{s_1,\, s_2,\, s_3\}$ be indicators of A quality. Evaluations of the indicators by the experts are listed in Table 1. Expert generalized evaluations h_{jA} are also shown there. Table

1, as well as the questionnaire, contains some redundant (check) evaluations h_{jA}. In an ideal case (for a well experienced expert) h_{jA} should be equal to generalized evaluation e_{jA}, achieved on $\{g_i,\, h_{js_i}\}$ base. For simplicity purpose let us confine on processing the 1-th (j=1) expert's data. Applying formula (2), resulted $e_{1A} \cong h_{1A} = 0.8$, for the first expert.

j	h_{jA}		s_1 spelling	s_2 punctuation	s_3 contents
1	0.8	g_1	0.7	0.5	0.6
		h_{1s_i}	0.8	0.7	0.9
2	0.7	g_2	0.7	0.6	0.7
		h_{2s_i}	0.7	0.6	0.8
3	0.9	g_3	0.8	0.5	0.7
		h_{3s_i}	0.8	0.7	0.9
4	0.8	g_4	0.8	0.6	0.6
		h_{4s_i}	0.8	0.7	0.7
5	0.8	g_5	0.7	0.7	0.7
		h_{5s_i}	0.8	0.7	0.9

Table 1. Unprocessed data of expert evaluations

Unfortunately, as it was mentioned before, in most of cases formula (2) is not adequate, – and in this analyzed example, too. Assuredly, if it would be in the table $h_{1s_1}= 0.1$ (almost complete illiteracy) or $h_{1s_3} = 0.1$ (an essay almost without contents), the $e_{1A}\cong$ 0.5. That is absolutely unreal evaluation. Applying formula (3), the column of g_j is not needed in the Table 1 any more. Replacement is performed with calculated coefficients $\{a\}$, which can be found applying experiment planning methods. For instance, a full factorial experiment 2^n can be performed, for finding a set of coefficients $\{a\}$. For illustration Table 2 shows the results. In this example, after calculations we obtain:

$$e_{1A} \;=\; 0.1(h_{1s_1} + h_{1s_2} + h_{1s_3} + h_{1s_1}h_{1s_3}) + 0.6h_{1s_1}h_{1s_2}h_{1s_3}. \tag{14}$$

If the values of h_{1s_i}, $i = \overline{1,3}$ from the Table 2 are inserted, result is $e_{1A} \cong 0.6$.

Attempt No.	h_{1s_1}	h_{1s_2}	h_{1s_3}	h_{1A}
1	0	0	0	0
2	0	0	1	0.1
3	0	1	0	0.1
4	0	1	1	0.2
5	1	0	0	0.1
6	1	0	1	0.3
7	1	1	0	0.2
8	1	1	1	1

Table 2. Experiment for finding a set $\{a\}$ of coefficients

Assuredly, Table 2 shows that the expert demand for an essay concordance, that is, the essay should meet all the criteria. In a case of applying formula (18), $\alpha_1 = 0.6$. If we use formula (4) for the same data, then $e_{1A}= 0.8$. In this case $e_{1A}= h_{1A}$. But it should be noticed that the result, achieved using formula (4) would be the same if h_{1s_2}were $h_{1s_2} \in [0, 0.7]$. In our case such a criterion is unacceptable, but in other cases, where criteria, having lower marks, are unimportant, formula (4) may be effective.

6. Conclusions

1. It is very important that the evaluations, performed by experts in a way of submitting criteria evaluations, are "comfortable" for the experts both from mathematical and psychological point of view. A good example of "comfortable" evaluation is when the expert specifies an interval of possible criterion value, and a probability, under which he resumes the evaluation of the criterion is correct. The submitted evaluation can be easily changed with fuzzy number, possessing "trapezoid" type dependence function.

2. Expert evaluations of objects, actions or processes are often inadequate: evaluations are dispersed unequally due to different expert experience (competence) and precaution (conservatism). It can be compensated with additional weight coefficients and by applying the methods of a subjective utility theory.

3. A complex evaluation criterion often cannot be expressed by combination of linear partial criteria.

4. The efficient expressions for indistinct criteria calculations are achieved by using fuzzy integrals of appropriate dependence functions.

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ПРИЛАГАНЕ НА МЕТОДА НА СУБЕКТИВНИ КАЧЕСТВЕНИ КРИТЕРИИ ЗА ОПЕРАЦИОНАЛНИ ПРОГНОЗИ

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Понякога вземането на решение, основано на класически оптимизационни методи, е невъзможно. Такива задачи възникват там където обектите, действията или процесите са описани неточно. В подобни ситуации, липсата на методи води до вземане на немотивирани решения. Всъщност такива методи съществуват и един от тях е методът на субективните качествени критерии. Този метод се основава на експертни оценки, т.е. човешкият интелект се използва като инструмент за оценяване. Влиянието на субективните външни фактори може да бъде толкова различно, че да е твърде рисковано основаването само на една експертна оценка. Дискутирани са методи за обработване на групи от експертни оценки. В статията се обсъждат и въпроси като избор на подходящ критерий и обработване на предложените оценки. Представен е един пример и са направени съответните изводи.