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PROPERTIES OF THE (3,2)-LANGUAGES RECOGNIZED BY (3,2)-SEMIGROUP AUTOMATA

Violeta Manevska, Donco Dimovski

The aim of this paper is to examine properties of (3,2)-languages which is recognized by finite (3,2)-semigroup automata.
Key words: (3,2)-semigroup, semigroup automata, (3,2)-semigroup automata, non-deterministic (3,2)-automata, languages, (3,2)-languages

Introduction. Here we recall the necessary definitions and known results. From now on, let B be a nonempty set and let $(B,\cdot)$ be a semigroup, where $\cdot$ is a binary operation.

A semigroup automaton is a triple $(S,(B,\cdot),f)$, where S is a set, $(B,\cdot)$ is a semigroup, and $f : S \times B \to S$ is a map satisfying
$f(f(s,x),y) = f(s,x\cdot y)$ for every $s \in S$, $x, y \in B$.

The set S is called the set of **states** of $(S,(B,\cdot),f)$ and f is called the **transition function** of $(S,(B,\cdot),f)$.

A nonempty set B with the (3,2)-operation $\{\} : B^3 \to B^2$ is called **a (3,2)-semigroup** iff the following equality

$$\{\{xyz\}t\} = \{x\{yzt\}\}$$

is an identity for every $x, y, z, t \in B$. It is denoted with the pair $(B,\{\ \})$.

Example 1. Let $B = \{a, b\}$. Then the (3,2)-semigroup $(B,\{\ \})$ is given by Table 1.

$\{\ \}$	
aaa	(b,a)
aab	(a,a)
aba	(a,a)
abb	(b,a)
baa	(a,a)
bab	(b,a)
bba	(b,a)
bbb	(a,a)

Table 1

This example of (3,2)-semigroup is generated by an appropriate computer program.

A (deterministic) (3,2)-semigroup automaton is a triple $(S,(B,\{\ \}),f)$, where S is a set, $(B,\{\ \})$ is a (3,2)-semigroup, and $f{:}S\times B^2\to S\times B$ is a map satisfying

$f(f(s,x,y),z)=f(s,\{xyz\})$ for every $s\in S$, $x,y,z,\in B$.

The set S is called the set of **states** of $(S,(B,\{\ \},f)$ and f is called the **transition function** of $(S,(B,\{\ \}),f)$.

Example 2. Let $(B,\{\ \})$ be an (3,2)-semigroup given by Table 1 from Example 1 and $S=\{s_0,s_1,s_2\}$. A (3,2)-semigroup automaton $(S,(B,\{\ \}),f)$ is given by Table 2.

f	(a,a)	(a,b)	(b,a)	(b,b)
s_0	(s_1,b)	(s_2,b)	(s_2,b)	(s_1,b)
s_1	(s_1,b)	(s_0,a)	(s_2,b)	(s_1,b)
s_2	(s_2,b)	(s_0,b)	(s_1,b)	(s_2,b)

Table 2

This example of (3,2)-semigroup automaton is generated by computer.

Let $(Q,[\ ])$ be a free (3,2)-semigroup with a basis B constructed in [1].

Any subset $L^{(3,2)}$ of the universal language $Q^*=\cup_{p\geq 1}Q^p$, where Q is a free (3,2)-semigroup with a basis B, is called **a (3,2)-language** on the alphabet B.

A (3,2)-language $L^{(3,2)}\subseteq Q^*$ is called **recognizable** if there exists:

(1) an (3,2)-semigroup automaton $(S,(B,\{\ \}),f)$, where the set S is finite;

(2) an initial state $s_0\in S$;

(3) a subset $T\subseteq S$; and

(4) a subset $C\subseteq B$ such that

$L^{(3,2)}=\{w\in Q^*|\overline{\varphi}(s_0,(w,1),(w,2))\in T\times C\}$,

where $(S,(Q,[\ ]),\overline{\varphi})$ is the (3,2)-semigroup automaton constructed in [2] for the (3,2)-semigroup automaton $(S,(B,\{\ \}),f)$.

We also say that the (3,2)-semigroup automaton $(S,(B,\{\ \}),f)$ **recognizes** $L^{(3,2)}$, or that $L^{(3,2)}$ **is recognized** by $(S,(B,\{\ \}),f)$.

Example 3. Let $(S,(B,\{\ \}),f)$ be a (3,2)-semigroup automaton given in Example 2. We construct the (3,2)-semigroup automaton $(S,(Q,[\ ]),\overline{\varphi})$ for the (3,2)-semigroup automaton $(S,(B,\{\ \}),f)$. A (3,2)-language $L^{(3,2)}$, which is recognized by the (3,2)-semigroup automaton $(S,(Q,[\ ]),\overline{\varphi})$, with initial state s_0 and terminal state (s_2,b) is

$L^{(3,2)}=\{w\in Q^*|w=w_1w_2\dots w_q,\ q\geq 3,$ where $w_l=\begin{cases}(u_1^n,i),n\geq 3,u_\alpha\in Q\\(a^*b^*)^*\end{cases}$,

$l\in\{1,2,\dots,q\}$, and:

a) If $i=1$, then:

a1) $(u_1^n,1)=a$, where $\psi_{p-1}(u_1)\dots\psi_{p-1}(u_n)=a^tb^ja^rb^h$ and $t+r=2k, t+j+r+h=n$, $t,j,r,h,k\in\{0,1,2,\dots\}$,$k\geq 1$;

a2) $(u_1^n, 1) = b$, where $\psi_{p-1}(u_1)\ldots\psi_{p-1}(u_n) = a^t b^j a^r b^h$ and $t + r = 2k + 1, t + j + r + h = n$, $t, j, r, h, k \in${0,1,2,...},$k \geq 1$;

b) If $i = 2$, then $(u_1^n, 2) = a$, where $\psi_{p-1}(u_1)\ldots\psi_{p-1}(u_n) = (a^*b^*)^*$ and $\psi_p(w_1)\ldots\psi_p(w_q) = b^*(ab^*)^{2k+1}$ }.

2. A nondeterministic (3,2)-semigroup automaton. A nondeterministic (3,2)-semigroup automaton is a triple $(S,(B,\{\}),g)$, wh ere S is a set, $(B,\{\ \})$ is a (3,2)-semigroup, $P(S\times B)$ is a subset of $S\times B$ and $g{:}S\times B^2\to P(S\times B)$ is a map satisfying $g(g(s,x,y),z)=g(s,\{xyz\})$ for every $s\in S$, $x,y,z,\in B$. We denote $g(X, y) = \cup_{(t,a)\in X} g(t, a, y)$ for $X \in S \times B$.

Theorem. *Let $L_1^{(3,2)}$ and $L_2^{(3,2)}$ be (3,2)-languages which is recognized by (3,2)-semigroup automata $(S_1,(B,\{\}),f_1)$ and $(S_2,(B,\{\}),f_2)$, with initial state s s_0' and s_0'', and sets of terminal states $T_1 \times C_1$ and $T_2 \times C_2$. Then:*
a) $L_1^{(3,2)}\cup L_2^{(3,2)}$;
b) $\overline{L}_1^{(3,2)}$;
c) $L_1^{(3,2)}\times L_2^{(3,2)}$;
d) $L_1^{(3,2)}L_2^{(3,2)}$;
e)$L_1^{(3,2)}L_1^{(3,2)}\ldots L_1^{(3,2)}$ *are recognizable (3,2)-languages with nondeterministic (3,2)-semigroup automaton $(S,(B,\{\})\ ,g)$.*

Proof. In each case we show how to construct a nondeterministic (3,2)-semigroup automaton fro m one or two given (3,2)-semigroup automata. Without loss of generality, we may assume that S_1 a nd S_2 are disjoint sets and e is a empty or dead letter.

a) We construct a nondeterministic (3,2)-semigroup automaton $(S,(B,\{\}),g)$ as follows:

$S{=}S_1\cup S_2\cup\{s_0\}$, where s_0 is a new initial state not in S_1 or S_2;

$$g(s,x,y) = \begin{cases} f_1(s,x,y), & s \in S_1 \\ f_2(s,x,y), & s \in S_2 \\ (s_0, e), & s = s_0 \end{cases}$$

$$f_1(s,x,e) = f_1(s,e,x) = (s,x); f_2(s,x,e) = f_2(s,e,x) = (s,x);$$
$$g(s_0,x,e) = f(s_0,e,x) = (s_0,x);$$
$$g(s_0,e,e) = \{(s_0',e),(s_0'',e)\};$$

for every $x, y \in B$;

$$T = T_1 \cup T_2;\ C = C_1 \cup C_2.$$

That is, $(S,(B,\{\}),g)$ begins any computation by nondeterministically choosing to enter e ither with $g(s_0,e,e) = (s_0',e)$ or $g(s_0,e,e) = (s_0'',e)$, and thereafter $(S,(B,\{\}),g)$ imitates either $(S_1,(B,\{\}),f_1)$ or $(S_2,(B,\{\}),f_2)$.

Let $L^{(3,2)}$ be a (3,2)-language which is recognized by nondeterministic (3,2)-semigroup automat on $(S,(B,\{\}),g)$.Then

$w \in L^{(3,2)} \Longleftrightarrow w \in Q^*$ and $\overline{\varphi}(s_0,(w,1),(w,2)) \in T \times C \Longleftrightarrow w\ inQ^*$ and

$$\overline{\varphi}(s_0,(w,1),(w,2)) = \begin{cases} \overline{\varphi}(s_0',(w,1),(w,2)) = \overline{\varphi_1}(s_0',(w,1),(w,2)) \in T_1 \times C_1 \\ \overline{\varphi}(s_0'',(w,1),(w,2)) = \overline{\varphi_2}(s_0'',(w,1),(w,2)) \in T_2 \times C_2 \end{cases}.$$

Hence $L^{(3,2)}$ $=\{w \in Q^* | \overline{\varphi}(s_0,(w,1),(w,2)) = \overline{\varphi_1}(s'_0,(w,1),(w,2)) \in T_1 \times C_1$ or $\overline{\varphi}(s_0,(w,1),(w,2)) = \overline{\varphi_2}(s''_0,(w,1),(w,2)) \in T_2 \times C_2\}$
$=\{w \in Q^* | \overline{\varphi_1}(s'_0,(w,1),(w,2)) \in T_1 \times C_1\} \cup \{w \in Q^* | \overline{\varphi_2}(s''_0,(w,1),(w,2)) \in T_2 \times C_2\}$
$=L_1^{(3,2)} \cup L_2^{(3,2)}$.

b) A complementation of a (3,2)-language $L^{(3,2)}$ is the (3,2)-language $\overline{L}^{(3,2)} = Q^* \backslash L^{(3,2)}$.

We construct a nondeterministic (3,2)-semigroup automaton $(S,(B,\{\}),g)$ with initial stat e s_0 and terminal states $(S \times B) \backslash (T \times C)$.

$w \in L^{(3,2)} \iff w \in Q^*$ and

$$\overline{\varphi}(s_0,(w,1),(w,2)) \in (S \times B) \backslash (T \times C) \iff w \in Q^* \backslash L^{(3,2)}.$$

So
$\overline{L}^{(3,2)}$ $=\{w \in Q^* | \overline{\varphi}(s_0,(w,1),(w,2)) \notin T \times C\} = Q^* \backslash L^{(3,2)}$.

c) We construct a nondeterministic (3,2)-semigroup automaton $(S,(B,\{\}),g)$ which recogniz es (3,2)-language $L_1^{(3,2)} \times L_2^{(3,2)}$ as follows:
$S = S_1 \times S_2$; $B = B_1 \times B_2$; $C = C_1 \times C_2$; $T = T_1 \times T_2$;
$g = f_1 \times f_2$ defined by $(f_1 \times f_2)(s_1,s_2),(x_1,y_1)(x_2,y_2)) =$
$= (\pi_1(f_1(s_1,x_1,x_2)), \pi_1(f_2(s_2,y_1,y_2)), \pi_2(f_1(s_1,x_1,x_2)), \pi_2(f_2(s_2,y_1,y_2)))$,
where π_i, $i = 1,2$ is a i-th projection of the argument.

Let $L^{(3,2)}$ is a (3,2)-language which is recognized by nondeterministic (3,2)-semigroup automat on $(S,(B,\{\}),g)$.

$w \in L^{(3,2)} \iff w = (w',w'') \in Q_1^* \times Q_2^*$ and
$\overline{\varphi}((s'_0,s''_0),(w',1),(w'',1),(w',2),(w'',2))) \in (T_1 \times T_2) \times (C_1 \times C_2)$
$\iff (w',w'') \in Q_1^* \times Q_2^*$ and
$\overline{\varphi}((s'_0,s''_0),(w',1),(w'',1),(w',2),(w'',2))) = (\pi_1(\overline{\varphi_1}(s'_0,(w',1),(w',2))),$
$\pi_1(\overline{\varphi_2}(s''_0,(w'',1),(w'',2))), \pi_2(\overline{\varphi_1}(s'_0,(w',1),(w',2))), \pi_2(\overline{\varphi_2}(s''_0,(w'',1),(w'',2)))$
$\in (T_1 \times T_2) \times (C_1 \times C_2)$
$\iff w' \in Q_1^*$ and $(\pi_1(\overline{\varphi_1}(s'_0,(w',1),(w',2))), \pi_2(\overline{\varphi_1}(s'_0,(w',1),(w',2)))) \in T_1 \times C_1$
$w'' \in Q_2^*$ and $(\pi_1(\overline{\varphi_2}(s''_0,(w'',1),(w'',2))), \pi_2(\overline{\varphi_2}(s''_0,(w'',1),(w'',2)))) \in T_2 \times C_2$
$\iff w' \in Q_1^*$ and $\overline{\varphi_1}(s'_0,(w',1),(w',2)) \in T_1 \times C_1$
$w'' \in Q_2^*$ and $\overline{\varphi_2}(s''_0,(w'',1),(w'',2)) \in T_2 \times C_2$.
Hence $L^{(3,2)}$ $=\{w \in Q_1^* \times Q_2^* | w = (w',w'')$ and
$\overline{\varphi}((s'_0,s''_0),(w',1),(w'',1),(w',2),(w'',2))) \in (T_1 \times T_2) \times (C_1 \times C_2)\}=$
$\{w \in Q_1^* \times Q_2^* | w = (w',w'')$ and $\overline{\varphi_1}(s'_0,(w',1),(w',2)) \times \overline{\varphi_2}(s''_0,(w'',1),(w'',2)) \in (T_1 \times C_1) \times (T_2 \times C_2)\}=$
$\{w' \in Q_1^* | \overline{\varphi_1}(s'_0,(w',1),(w',2)) \in T_1 \times C_1\} \times \{w'' \in Q_2^* | \overline{\varphi_2}(s''_0,(w'',1),(w'',2)) \in T_2 \times C_2\} = L_1^{(3,2)} \times L_2^{(3,2)}$.

d) We construct a nondeterministic (3,2)-automaton $(S,(B,\{\}),g)$ with initial state s'_0 and sets of terminal states $T_2 \times C_2$ as follows:
$S = S_1 \cup S_2$; $f_1(s,e,e) = (s,e)$, for all $s \in S_1$,
$f_2(s,e,e) = (s,e)$, for all $s \in S_2$, $f_1(s,x,e) = (s''_0,e)$, for $(s,x) \in T_1 \times C_1$,
$f_1(s,x,e) = f_1(s,e,x) = (s,x)$, for $(s,x) \in (B \times S_1) \backslash (T_1 \times C_1)$,
$f_2(s,x,e) = f_2(s,e,x) = (s,x)$, for $x \in B$, $s \in S_2$,

$g(s,x,y)=\begin{cases} f_1(s,x,y), & s\in S_1 \\ f_2(s,x,y), & s\in S_2 \end{cases}$.

In this way, $(S,(B,\{\}),g)$ operates by simulating $(S_1,(B,\{\}),f_1)$ for a while, and then "jumping" nondeterministically from a terminal state of $(S_1,(B,\{\}),f_1)$ with $f_1(s,x,e)=(s''_0,e)$, for $(s,x)\in T_1\times C_1$, to the initial state of $(S_2,(B,\{\}),f_2)$. Thereafter, $(S,(B,\{\}),g)$ imitates $(S_2,(B,\{\}),f_2)$.

Let $L^{(3,2)}$ is a (3,2)-language which is recognized by nondeterministic (3,2)-automaton $(S,(B,\{\}),g)$.

$w\in L^{(3,2)} \Longleftrightarrow w=w'w''\in Q^*, |w'|,|w''|>1$ and $\overline{\varphi}(s'_0,(w,1),(w,2))\in T_2\times C_2$
$\Longleftrightarrow w=w'w''\in Q^*$ and $\overline{\varphi}(s'_0,w'w'')=\overline{\varphi}(\overline{\varphi}(s'_0,w'),w'')=\overline{\varphi}(\overline{\varphi_1}(s'_0,w'),w'')$.

Let $\overline{\varphi_1}(s'_0,w')=(s,x)\in T_1\times C_1$. By $f_1(s,x,e)=(s''_0,e)$, for $(s,x)\in T_1\times C_1$, we go in initial state s''_0 of (3,2)-semigroup automaton $(S_2,(B,\{\}),f_2)$. Then
$\overline{\varphi}(s''_0,e,w'')=\overline{\varphi}(s''_0,w'')=\overline{\varphi_2}(s''_0,w'')\in T_2\times C_2$.

Hence
$L^{(3,2)}=\{w\in Q^*|w=w'w''$, for $|w'|,|w''|>1$ and
$\overline{\varphi}(s'_0,w'w'')=\overline{\varphi}(\overline{\varphi}(s''_0,w'),w'')=\overline{\varphi}(s''_0,e,w'')=\overline{\varphi_2}(s''_0,e,w'')\in T_2\times C_2\}$
$=L_1^{(3,2)}L_2^{(3,2)}$.

e) We construct a nondeterministic (3,2)-semigroup automaton $(S,(B,\{\}),g)$ with initial s tate s'_0 and sets of terminal states $T_1\times C_1$ as follows:

$S=S_1$;
$g(s,x,e)=(s'_0,e)$, for $(s,x)\in T_1\times C_1$,
$g(s,x,e)=g(s,e,x)=(s,x)$, for $(s,x)\in(S_1\times B)\backslash(T_1\times C_1)$,
$g(s,x,y)=f_1(s,x,y)$, for ($s\in S_1$ and $x,y\in B$.

Let $L^{(3,2)}$ is a (3,2)-language which is recognized by nondeterministic (3,2)-automaton $(S,(B,\{\}),g)$.

$w\in L^{(3,2)}\Longleftrightarrow w=w_1w_2\dots w_t\in Q^*, |w_1|,|w_2|,\dots,|w_t|>1$ and $\overline{\varphi}(s'_0,(w,1),(w,2))\in T_2\times C_2 \Longleftrightarrow\ w=w_1w_2\dots w_t\in Q^*$ and $\overline{\varphi}(s'_0,w_1w_2\dots w_t)=\overline{\varphi}(\overline{\varphi}(s'_0,w_1),w_2\dots w_t)=\overline{\varphi}(\overline{\varphi_1}(s'_0,w_1),w_2\dots w_t)$.

Let $\overline{\varphi_1}(s'_0,w_1)=(s,x)\in T_1\times C_1$. By $g(s,x,e)=(s'_0,e)$, for $(s,x)\in T_1\times C_1$, we go again in initial state s'_0 of (3,2)-semigroup automaton $(S_1,(B,\ \}),f_1)$. Then
$\overline{\varphi}(s'_0,e,w_2\dots w_t)=\overline{\varphi}(s'_0,w_2\dots w_t)=\overline{\varphi}(\overline{\varphi}(s'_0,w_2),w_3\dots w_t)=$
$\overline{\varphi}(\overline{\varphi_1}(s'_0,w_2),w_3\dots w_t)$.
$\overline{\varphi_1}(s'_0,w_2)\in T_1\times C_1$, so with $g(s,x,e)=(s'_0,e)$, for $(s,x)\in T_1\times C_1$, we go again in initial state s'_0 of (3,2)-semigroup automaton $(S_1,(B,\{\ ),f_1)$. This procedure continues for other (3,2)-subwords $w_2\dots w_t$. Finally, we have $\overline{\varphi}(s'_0,w_t)=\overline{\varphi_1}(s'_0,w_t)\in T_1\times C_1$.
Hence $L^{(3,2)}=\{w\in Q^*|w=w_1w_2\dots w_t, |w_1|,|w_2|,\dots,|w_t|>1$ and
$\overline{\varphi}(s'_0,w_1w_2\dots w_t)=\overline{\varphi}(\overline{\varphi_1}(s'_0,w_1),w_2\dots w_t)=\overline{\varphi}(s'_0,e,w_2\dots w_t)=\overline{\varphi}(s'_0,w_2\dots w_t)=\dots=\overline{\varphi}(s'_0,w_t)=\overline{\varphi_1}(s'_0,w_t)=\in T_1\times C_1\}=L_1^{(3,2)}L_1^{(3,2)}\dots L_1^{(3,2)}$.

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Violeta Manevska
Faculty of Tehnical Sciences
Bitola, Macedonia
e-mail: violeta.manevska@uklo.edu.mk

Donco Dimovski
Institute of Mathematics
Faculty of Mathematics and Natural Sciences
Skopje, Macedonia
e-mail: donco@iunona.pmf.ukim.edu.mk

СВОЙСТВА НА (3,2)-ЕЗИЦИ, РАЗПОЗНАВАНИ ОТ (3,2)-ПОЛУГРУПОВИ АВТОМАТИ

Виолета Маневска, Донко Димовски

Целта на статията е да изследва свойствата на (3,2)-езици, които се разпознават от крайни (3,2)-полугрупови автомати.