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TEMPORAL INTUITIONISTIC FUZZY SETS (REVIEW AND NEW RESULTS)

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In memory of George Gargov

Short remarks on the concepts of Intuitionistic Fuzzy Set and temporal IFS are given. The later concept is illustrated and some of its properties are studied. Some applications are discussed.

1. An introduction or why intuitionistic fuzziness? The present paper contains a review and new results in the area of *Temporal Intuitionistic Fuzzy Sets (TIFSs)*. It is a continuation of [9].

The notion *Intuitionistic Fuzzy Set (IFS)* was introduced (exactly 20 years ago) as extension of the ordinary fuzzy set. By this reason, initial research on IFSs followed step by step the already existing studies on fuzzy sets. Of course, it is not very difficult to extend formally some concepts. It is interesting to show that the corresponding extension has specific properties not found in the basic concept. Just when the author convinced himself that the so-constructed sets really have their worthy properties, he discussed his ideas with George Gargov (7 April 1947 – 9 Nov. 1996), who offered their name, because the way of fuzzification contains ideas of intuitionism (see, e.g. [11]). Gargov encouraged the author to publish the results in [1, 2].

Of course the question *"Are there adequate examples of the IFSs?"* immediately arose. The answer is *"yes"*. Let us start with a rather routine example (following [9]) that illustrates the necessity as of ordinary fuzzy sets as well as of IFSs. Imagine someone just arriving from a trip in a foreign country. How shall we answer his/her question about the weather today if the day has started with a beautiful sunrise, and some hours later it was already raining cats and dogs. The ordinary logic cannot help us. And we cannot answer *"it was sunny"* only due to the sun in the morning, as we cannot answer *"it was rainy"* just for the rain. To provide the most accurate possible answer, we must have stayed the whole day, a chronometer in hand, and measure that sun has been observed in $S\%$ of the daytime, in $Q\%$ it has been cloudy and in the rest $(100 - S - Q)\%$ the sun could have been seen through the clouds yet, not brightly shining. And whereas the fuzzy set theory gives us the means to determine that *"Yes, it has been sunny in $S\%$ of* *the day",* the apparatus of the IFSs helps us for the more comprehensive estimation: *"It has been sunny in S% of the day and cloudy – in Q%".*

Thus, plausibility and correctness can be achieved by means of the more accurate statements that the IFSs theory offers. Just like we demand more exhaustive answers than simply *"yes"* or *"no"* that propositional calculus provides us, and we find them in the fuzzy sets, by analogy, we regard the intuitionistic fuzzy sets a higher level of accuracy. However, no matter which evaluation technique we use, our appraisal will only render the results of the whole day, disregarding the observations of the parts.

Of course, we can construct a lot of other examples ranging from politics to artificial intelligence. For example, for every country we can estimate how stable the government is, on the basis of the people a) who voted for the government party, b) who voted for an oppositional party, i.e., against the government party and c) who did not vote. Now, we can prepare analogous estimations for different countries and for different time-moments.

For this aim we need to involve some temporal components in our investigation, in order to have a detailed impact on the estimated events. These temporal elements, provided by the temporal logics, shall be the object of research in this paper.

2. Short remarks on IFSs. As we noted above, the IFSs (see, e.g., [7]) are extensions of the ordinary fuzzy sets. All results that are valid for the fuzzy sets can be applied for the IFSs, too. All investigations, using the apparatus of the fuzzy sets, can be described in the terms of the IFSs, also.

On the other hand, over the IFSs there have been defined not only operations similar to the ordinary fuzzy set operations, but also operators that cannot be defined or do not have sense in the case of ordinary fuzzy sets.

Let a set E be given. An IFS A in E is an object of the following form:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where functions $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ determine the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Let us define for every $x \in E$:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x).$$

Therefore, function π determines the degree of uncertainty.

Obviously, for every ordinary fuzzy set $\pi_A(x) = 0$ for each $x \in E$ and these sets have the form:

$$\{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle | x \in E\}.$$

Let a universe E be given. One of the geometrical interpretations of the IFSs uses figure F on Fig. 1.

For every two IFSs A and B a lot of relations and operations are defined (see, e.g. [7]), the most important of which are:

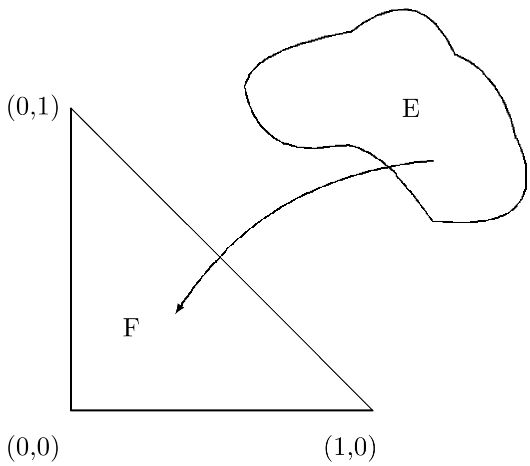

Fig. 1.

$$
\begin{array}{lll}
A \subset B & iff & (\forall x \in E)(\mu_A(x) \leq \mu_B(x) \& \nu_A(x) \geq \nu_B(x)); \\
A \supset B & iff & B \subset A; \\
A = B & iff & (\forall x \in E)(\mu_A(x) = \mu_B(x) \& \nu_A(x) = \nu_B(x)); \\
\overline{A} & = & \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in E\}; \\
A \cap B & = & \{\langle x, min(\mu_A(x), \mu_B(x)), max(\nu_A(x), \nu_B(x))\rangle | x \in E\}; \\
A \cup B & = & \{\langle x, max(\mu_A(x), \mu_B(x)), min(\nu_A(x), \nu_B(x))\rangle | x \in E\}; \\
A + B & = & \{\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x), \nu_A(x).\nu_B(x)\rangle \mid x \in E\}; \\
A.B & = & \{\langle x, \mu_A(x).\mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x).\nu_B(x)\rangle \mid x \in E\}; \\
A@B & = & \{\langle x, (\frac{\mu_A(x) + \mu_B(x))}{2}, \frac{(\nu_A(x) + \nu_B(x))}{2}\rangle | x \in E\}.
\end{array}
$$

The last operation is defined firstly in [10] by T. Buhaescu.

The operations and relations above are defined similarly to these from the fuzzy set theory. More interesting are the modal operators that can be defined over the IFSs.

Let A be an IFS and let $\alpha, \beta \in [0, 1]$.

We shall define the following operators (see, e.g., [7]):

$$\Box A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle | x \in E\};$$

$$\Diamond A = \{\langle x, 1 - \nu_A(x), \nu_A(x)\rangle | x \in E\};$$

$$D_\alpha(A) = \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \nu_A(x) + (1-\alpha).\pi_A(x)\rangle | x \in E\};$$

$$F_{\alpha,\beta}(A) = \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \nu_A(x) + \beta.\pi_A(x)\rangle | x \in E\}, \text{ where } \alpha + \beta \leq 1;$$

$$G_{\alpha,\beta}(A) = \{\langle x, \alpha.\mu_A(x), \beta.\nu_A(x)\rangle | x \in E\}.$$

$$H_{\alpha,\beta}(A) = \{\langle x, \alpha.\mu_A(x), \nu_A(x) + \beta.\pi_A(x)\rangle | x \in E\},$$

$$H^*_{\alpha,\beta}(A) = \{\langle x, \alpha.\mu_A(x), \nu_A(x) + \beta.(1 - \alpha.\mu_A(x) - \nu_A(x))\rangle | x \in E\},$$

$$J_{\alpha,\beta}(A) = \{\langle x, \mu_A(x) + \alpha.\pi_A(x), \beta.\nu_A(x)\rangle | x \in E\},$$

$$J^*_{\alpha,\beta}(A) = \{\langle x, \mu_A(x) + \alpha.(1 - \mu_A(x) - \beta.\nu_A(x)), \beta.\nu_A(x)\rangle | x \in E\}.$$

If we have an ordinary fuzzy set A, then

$$\Box A = A = \Diamond A,$$

while for a proper IFS A:

$$\Box A \subset A \subset \Diamond A$$

and

$$\Box A \neq A \neq \Diamond A.$$

Also the following equalities are valid for each IFS A:

$$\Box \overline{A} = \overline{\Diamond A},$$

$$\Diamond \overline{A} = \overline{\Box A}.$$

In the modal logic both operators $\Box$ and $\Diamond$ are related to the last two connections, but no other relations between them have been observed. In the IFS-case, we can see that operators D_α and $F_{\alpha,\beta}$ ($\alpha, \beta \in [0,1]$ and $\alpha + \beta \leq 1$) are their direct extensions, because:

$$\Box A = D_0(A) = F_{0,1}(A) = H_{0,1} = H^*_{0,1},$$

$$\Diamond A = D_1(A) = F_{1,0}(A) = J_{1,0} = J^*_{1,0}.$$

These equalities show the deeper interaction between the two ordinary modal logic operators.

Two analogues of the topological operators can be defined over the IFSs, too: *"closure" operator* C and *"intersection" operator* I:

$$C(A) = \{\langle x, \sup_{y \in E} \mu_A(y), \inf_{y \in E} \nu_A(y)\rangle | x \in E\},$$

$$I(A) = \{\langle x, \inf_{y \in E} \mu_A(y), \sup_{y \in E} \nu_A(y)\rangle | x \in E\}.$$

It is very interesting to note that the IFS-interpretations of both operators coincide, respectively, with the IFS-interpretations of the logic quantifiers $\exists$ and $\forall$ (see, e.g. [7]).

An IFS A is called:

- an *intuitionistic fuzzy tautological set* if for all $x \in E$:

$$\mu_A(x) \geq \nu_A(x),$$

- an *intuitionistic fuzzy sure set* if for all $x \in E$: $\mu_A(x) > \frac{1}{2}$ (and therefore, $\nu_A(x) < \frac{1}{2}$).

Survey of the research on intuitionistic fuzzy sets is given in [12].

3. Short remarks on Temporal IFSs. Let E be a universe, and T be a non-empty set. We will call the elements of T "time-moments". Basing on the definition of an IFS, we will define now another type of an IFS (see, e.g., [3, 7]).

We define a *Temporal IFS (TIFS)* as:

$$A(T) = \{\langle x, \mu_A(x,t), \nu_A(x,t)\rangle | \langle x, t\rangle \in E \times T\},$$

where:

(a) $A \subset E$ is a given set,

(b) $\mu_A(x,t) + \nu_A(x,t) \leq 1$ for every $\langle x, t\rangle \in E \times T$,

(c) $\mu_A(x,t)$ and $\nu_A(x,t)$ are the degrees of membership and non-membership, respectively, of the element $x \in E$ at the time-moment $t \in T$.

In some cases it is suitable to use the following more extended form of the above set

$$A(T) = \{\langle\langle x, t\rangle, \mu_A(x,t), \nu_A(x,t)\rangle | \langle x, t\rangle \in E \times T\},$$

but here it is not necessary.

For brevity we will write A instead of $A(T)$ when this does not cause confusions.

Obviously, every ordinary IFS can be regarded a TIFS for which T is a singleton set.

All operations and operators on the IFSs can be defined for the TIFSs.

Suppose that we have two TIFSs:

$$A(T) = \{\langle x, \mu_A(x,t), \nu_A(x,t)\rangle | \langle x, t\rangle \in E \times T'\},$$

and

$$B(T) = \{\langle x, \mu_B(x,t), \nu_B(x,t)\rangle | \langle x, t\rangle \in E \times T''\},$$

then we can define the above operations ($\cap$, $\cup$, etc.) and the topological (C and I) and modal ($\Box$ and $\Diamond$) operators by the same way.

The specific operators over TIFSs are (see [3, 7, 8])

$$C^*(A(T)) = \{\langle x, \sup_{t \in T} \mu_{A(T)}(x,t), \inf_{t \in T} \nu_{A(T)}(x,t)\rangle | x \in E\},$$

$$I^*(A(T)) = \{\langle x, \inf_{t \in T} \mu_{A(T)}(x,t), \sup_{t \in T} \nu_{A(T)}(x,t)\rangle | x \in E\}.$$

The later operators are analogous of the previous topological operators C and I, but now they are related to the temporal compoments of the TIFS. For example, if the universe E corresponds to the countries in Europe, for a fixed country we can obtain the boundary values for the above discussed degrees, through operators C^* and I^*.

We have the following important equalities for every TIFS $A(T)$:

(**a**) $C^*\ (C^*\ (A(T))) = C^*\ (A(T))$, (**g**) $\overline{C^*\ (\overline{A(T)}\ )} = I^*\ (A(T))$,

(**b**) $C^*\ (I^*\ (A(T))) = I^*\ (A(T))$, (**h**) $\overline{I^*\ (\overline{A(T)}\ )} = C^*\ (A(T))$,

(**c**) $I^*\ (C^*\ (A(T))) = C^*\ (A(T))$, (**i**) $C^*\ (\square A(T)) = \square C^*\ (A(T))$,

(**d**) $I^*\ (I^*\ (A(T))) = I^*\ (A(T))$, (**j**) $C^*\ (\diamondsuit A(T)) = \diamondsuit C^*\ = (A(T))$,

(**e**) $C(C^*\ (A(T))) = C^*\ (C(A(T)))$, (**k**) $I^*\ (\square A(T)) = \square I^*\ (A(T))$,

(**f**) $I(I^*\ (A(T))) = I^*\ (I(A(T)))$, (**l**) $I^*\ (\diamondsuit A(T)) = \diamondsuit I^*\ = (A(T))$.

For every two TIFSs $A(T)$ and $B(T)$:

(a) $C^*(A(T') \cap B(T'')) \subset C^*(A(T')) \cap C^*(B(T''))$,

(b) $C^*(A(T') \cup B(T'')) = C^*(A(T')) \cup C^*(B(T''))$,

(c) $I^*(A(T') \cap B(T'')) = I^*(A(T')) \cap I^*(B(T''))$,

(d) $I^*(A(T') \cup B(T'')) \supset I^*(A(T')) \cup I^*(B(T''))$.

Let us have the TIFS

$$A(T) = \{\langle x, \mu_A(x,t), \nu_A(x,t)\rangle | \langle x, t\rangle \in E \times T\}.$$

For every $x \in E$ we can construct the set

$$T(A, x) = \{\langle t, \mu_A(x,t), \nu_A(x,t)\rangle | t \in T\}.$$

It is directly seen that $T(A, x)$ is an IFS, but now over universe T. Let us call it *"Temporal support"* of A.

If we return to our example from section 1 about the status of the sun at the past day, the apparatus of the TIFS will give us possibility to trace the changes of the weather status for the whole observed time-period (see, e.g., Fig. 2).

If the universe E is a set of different towns, A is the set of towns of a fixed country, then the ordinary IFS A can be interpreted as a set of the estimations of the degrees of sunny weather and of the degree of cloudiness (e.g., for a fixed time-moment) for all towns of the given country. Set $T(A, x)$ can be interpreted as the set of the above estimations, but now, only about the fixed town $x \in A$ and for the different time-moments from time-scale T. Finally, set $A(T)$ corresponds to the set of the estimations of the degrees of sunny weather and of the degree of cloudiness for all towns of the given country and for all time-moments from time-scale T.

Now, operators C^* and I^* defined over set $A(T)$ will determine the highest and the lowest degrees, respectivery, of the above discussed estimations for each town $x \in A$, while they, defined over set $T(A, x)$, will determine the highest and the lowest degrees of these estimations for the fixed town $x \in A$. On the other hand, operators C and I, defined oves set $A(T)$ will determine the highest and the lowest degrees, respectively, of the same estimations but about all towns from A in general, i.e., for the fixed country.

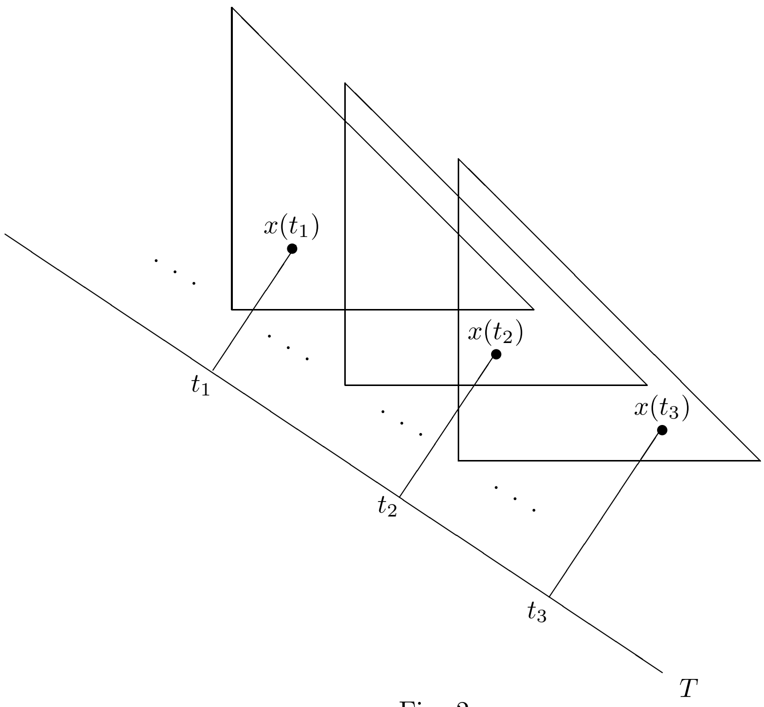

Fig. 2.

4. New results for the TIFSs. As it was discussed in section 2, modal operators can be extended. By analogy with section 2, we can define not only operators $\Box$ and $\Diamond$ over TIFSs, but also the other modal operators (D_α, etc.), e.g.,

$$F_{\alpha,\beta}(A(T)) = \{\langle x, \mu_A(x,t) + \alpha.\pi_A(x,t), \nu_A(x,t) + \beta.\pi_A(x,t)\rangle | x \in E, t \in T\},$$

where$\alpha + \beta \leq 1$,

$$G_{\alpha,\beta}(A(T)) = \{\langle x, \alpha.\mu_A(x,t), \beta.\nu_A(x,t)\rangle | x \in E, t \in T\},$$

etc.

Theorem 1. *For every TIFS A and for every two real numbers $\alpha, \beta \in [0,1]$:*

(a) $C^*(F_{\alpha,\beta}(A(T))) \subset F_{\alpha,\beta}(C^*(A(T)))$, *where* $\alpha + \beta \leq 1$;

(b) $I^*(F_{\alpha,\beta}(A(T))) \supset F_{\alpha,\beta}(I^*(A(T)))$, *where* $\alpha + \beta \leq 1$;

(c) $C^*(G_{\alpha,\beta}(A(T))) = G_{\alpha,\beta}(C^*(A(T)))$,

(d) $I^*(G_{\alpha,\beta}(A(T))) = G_{\alpha,\beta}(I^*(A(T)))$,

(e) $C^*(H_{\alpha,\beta}(A(T))) \subset H_{\alpha,\beta}(C^*(A(T)))$,

(f) $I^*(H_{\alpha,\beta}(A(T))) \supset H_{\alpha,\beta}(I^*(A(T)))$,

(g) $C^*(J_{\alpha,\beta}(A(T))) \subset J_{\alpha,\beta}(C^*(A(T)))$,

(h) $I^*(J_{\alpha,\beta}(A(T))) \supset J_{\alpha,\beta}(I^*(A(T)))$,

(i) $C^*(H^*_{\alpha,\beta}(A(T))) \subset H^*_{\alpha,\beta}(C^*(A(T)))$,

(j) $I^*(H^*_{\alpha,\beta}(A(T))) \supset H^*_{\alpha,\beta}(I^*(A(T)))$,

(k) $C^*(J^*_{\alpha,\beta}(A(T))) \subset J^*_{\alpha,\beta}(C^*(A(T)))$,

(l) $I^*(J^*_{\alpha,\beta}(A(T))) \supset J^*_{\alpha,\beta}(I^*(A(T)))$.

Proof. (l) $I^*(J^*_{\alpha,\beta}(A(T)))$

$$\begin{aligned}
&= I^*(\{\langle x, \mu_{A(T)}(x,t) + \alpha.(1 - \mu_{A(T)}(x,t) - \beta.\nu_{A(T)}(x,t)), \beta.\nu_{A(T)}(x,t)\rangle | x \in E, \\
&\quad t \in T\}) \\
&= \{\langle x, \inf_{t\in T} (\mu_{A(T)}(x,t) + \alpha.(1 - \mu_{A(T)}(x,t) - \beta.\nu_{A(T)}(x,t))), \\
&\quad \sup_{t\in T} (\beta.\nu_{A(T)}(x,t))\rangle | x \in E\} \\
&= \{\langle x, \inf_{t\in T} (\mu_{A(T)}(x,t) + \alpha.(1 - \mu_{A(T)}(x,t) - \beta.\nu_{A(T)}(x,t))), \\
&\quad \beta. \sup_{t\in T} \nu_{A(T)}(x,t)\rangle | x \in E\} \\
&\supset \{\langle x, \inf_{t\in T} \mu_{A(T)}(x,t) + \alpha.(1 - \inf_{t\in T} \mu_{A(T)}(x,t) - \beta. \sup_{t\in T} \nu_{A(T)}(x,t), \\
&\quad \beta. \sup_{t\in T} \nu_{A(T)}(x,t)\rangle | x \in E\} \\
&= J^*_{\alpha,\beta}(\{\langle x, \inf_{t\in T} \mu_{A(T)}(x,t), \sup_{t\in T} \nu_{A(T)}(x,t)\rangle | x \in E\}) \\
&= J^*_{\alpha,\beta}(I^*(A(T))). \qquad \diamondsuit
\end{aligned}$$

By analogy with Theorem 1.10.6 [7] we can formulate and prove the following

Theorem 2. *Let A, B be two proper TIFSs, for which there exist $y, z \in E$, and two time-moments $t_1, t_2 \in T$ so that $\mu_A(y,t_1) > 0$ and $\nu_B(z,t_2) > 0$. If $C(C^*(A(T))) \subset I(I^*(B(T)))$, then there are real numbers $\alpha, \beta, \gamma, \delta \in [0,1]$, such that $J_{\alpha,\beta}(A(T)) \subset H_{\gamma,\delta}(B(T))$.*

Proof. Let $C(C^*(A(T))) \subset I(I^*(B(T)))$. Therefore

$$0 < \mu_{A(T)}(y,t_1) \leq \max_{x\in E} \max_{t\in T} \mu_{A(T)}(x,t) \equiv K \leq k \equiv \min_{x\in E} \min_{t\in T} \mu_{B(T)}(x,t)$$

and

$$\min_{x\in E} \min_{t\in T} \nu_{A(T)}(x,t) \equiv L \geq l \equiv \max_{x\in E} \max_{t\in T} \nu_{B(T)}(x,t) \geq \nu_{B(T)}(z,t_2) > 0.$$

Let

$$a = \max_{x\in E} \max_{t\in T} \pi_{A(T)}(x,t) > 0 \text{ and } b = \max_{x\in E} \max_{t\in T} \pi_B(x,t) > 0,$$

because A and B are proper IFSs. Let

$$\alpha = \min(1, \frac{k-K}{2a}),\ \beta = \frac{L+l}{2L},\ \gamma = \frac{K+k}{2k},\ \delta = \min(1, \frac{L-l}{2b}).$$

Then

$$J_{\min(1,\frac{k-K}{2a}),\frac{L+l}{2L}}(A(T))$$

$$= \{< x, \mu_{A(T)}(x,t) + \min(1, \frac{k-K}{2a}).\pi_{A(T)}(x,t), \frac{L+l}{2L}.\nu_{A(T)}(x,t) > |x \in E\},$$

$$H_{\frac{k+K}{2k},\min(1,\frac{L-l}{2b})}(B(T))$$

$$= \{\langle x, \frac{k+K}{2k}.\mu_{B(T)}(x,t), \nu_{B(T)}(x,t) + \min(1, \frac{L-l}{2b}).\pi_{B(T)}(x,t)\rangle | x \in E\}.$$

From

$$\mu_{A(T)}(x,t) + \min(1, \frac{k-K}{2a}).\pi_{A(T)}(x,t) \le K + \frac{k-K}{2a}.a = \frac{k+K}{2} \le \frac{k+K}{2k}.\mu_{B(T)}(x,t)$$

and

$$\nu_{B(T)}(x,t) + \min(1, \frac{L-l}{2b}).\pi_{B(T)}(x,t) \le l + \frac{L-l}{2b}.b = \frac{L+l}{2} \le \frac{L+l}{2L}.\nu_{A(T)}(x,t)$$

it follows that $J_{\alpha,\beta}(A(T)) \subset H_{\gamma,\delta}(B(T))$. $\square$

Now, we can return to the extended modal operators. They can be used for obtaining of more detailed estimations. For our example, this detailization can be expressed in the following three forms.

Let apart from us (as observers of the sun), there be one or more other observers, called "experts'. Let them be able to detailize the degree of uncertainty, that we report, in the cases when we cannot ourselves decide whether it is sunny or cloudy (for instance, when sun is shining through the cloud, but not enough brightly). The experts' opinion can be also in IF-form. Let them determine the uncertain for us time-period as sunny in $\alpha\%$ and as cloudy in $\beta\%$ (they can also be hesitant about the sun). Then we can use operator $F_{\alpha,\beta}$ and we can decrease our degree of uncertainty, increasing both other degrees.

Using the experts' estimations, we can change our estimations in pessimistic (by operators $H_{\alpha,\beta}$ or $H^*_{\alpha,\beta}$) or in optimistic (by operators $J_{\alpha,\beta}$ or $J^*_{\alpha,\beta}$) directions.

If we have more than one expert, we can calculate their estimations in different ways (see, e.g. [4, 5, 7]), including the possibility to use ordinary and temporal intuitionistic fuzzy graphs (see [6, 7, 13]).

All this shows that the TIFSs give us the possibility to make more detailed estimations of real processes flowing at time.

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ТЕМПОРАЛНИ ИНТУИЦИОНИСТКИ РАЗМИТИ МНОЖЕСТВА
(обзор и нови резултати)

Красимир Т. Атанасов

Дадени са кратки бележки за понятията интуиционистки размито множество (ИРМ) и темпорално ИРМ. Последното понятие се илюстрира с няколко примера и се изследват някои от неговите свойства.