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# SHAPE THEOREMS FOR QUADRANGLES*

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A shape of a triangle is a complex number corresponding to an orbit of triangles under the group of the plane direct similarities. A shape of a quadrangle is an ordered pair of complex numbers corresponding to an equivalence class of quadrangles under the same group. We apply the shapes of triangles for the examinations of the shapes of quadrangles. In particular, we obtain some equalities for shapes by the use of some geometric constructions.

Complex numbers are a power tool to study the Euclidean plane. There are several books which consider different applications of complex number in the plane geometry (see [5] and [9]). June Lester created a new complex analytic formalism based on a cross-ratio and introduced the notion of a shape of a triangle. Many applications of this formalism are considered in triangle series (see [6], [7] and [8]). The notion of shape was extended by R. Artzy in [1]. He introduced a shape of polygons and examined some properties of shapes of quadrangles.

In this paper, we apply shapes of triangles for examination of shapes of quadrangles. First, we recall the basic definitions and assertions concerning shapes. According to [6], if **a**, **b** and **c** are three distinct points in the Gaussian plane the number

$$\triangle_{\mathbf{abc}} = \frac{\mathbf{a}-\mathbf{c}}{\mathbf{a}-\mathbf{b}}$$

is called a shape of the oriented triangle. The shape $\triangle_{\mathbf{abc}}$ is real if and only if $\triangle\mathbf{abc}$ is degenerated, i.e. the points **a**, **b** and **c** are collinear. It is clear that $\triangle_{\mathbf{abc}} \in \mathbb{C} \setminus \mathbb{R}$ for any non - degenerate triangle $\triangle\mathbf{abc}$. If $p = \triangle_{\mathbf{abc}}$, then $p' = \triangle_{\mathbf{bca}} = (1-p)^{-1}$ and $p'' = \triangle_{\mathbf{cab}} = 1 - p^{-1}$. In [6], J. Lester proves two main theorems which are a very useful tool for calculation of shapes. We shall recall the parts of these theorems which will be used in our proofs.

**First Shape Theorem.** *Let* **a** *and* **b** *be distinct fixed points and let* **p, q** *and* **r** *be three arbitrary points in the Euclidean plane. If* $\triangle_{\mathbf{pab}} = \lambda, \triangle_{\mathbf{qab}} = \mu, \triangle_{\mathbf{rab}} = \nu$, *then the triangle* $\triangle_{\mathbf{pqr}}$ *has a shape*

$$\triangle_{\mathbf{pqr}} = \frac{(1-\mu)(\lambda-\nu)}{(1-\nu)(\lambda-\mu)}.$$

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Let $\triangle_{\mathbf{abp}} = \lambda'$, $\triangle_{\mathbf{abq}} = \mu'$ and $\triangle_{\mathbf{abr}} = \nu'$, then $\lambda = 1 - \frac{1}{\lambda'}$, $\mu = 1 - \frac{1}{\mu'}$, $\nu = 1 - \frac{1}{\nu'}$. Using First Shape Theorem we obtain another more comfortable representation of the shape of the triangle $\triangle_{\mathbf{pqr}}$

$$\triangle_{\mathbf{pqr}} = \frac{\frac{1}{\mu'}(\frac{1}{\nu'} - \frac{1}{\lambda'})}{\frac{1}{\nu'}(\frac{1}{\mu'} - \frac{1}{\lambda'})} = \frac{\lambda' - \nu'}{\lambda' - \mu'}. \tag{1}$$

**Second Shape Theorem.** *Let* $\triangle\mathbf{abc}$ *be a non - degenerate triangle with shape* $\triangle_{\mathbf{abc}}$*, and let* $\mathbf{p}$*,* $\mathbf{q}$ *and* $\mathbf{r}$ *be three arbitrary points. If* $\triangle_{\mathbf{pcb}} = \lambda$*,* $\triangle_{\mathbf{qac}} = \mu$ *and* $\triangle_{\mathbf{rba}} = \nu$*, then*

$$\triangle_{\mathbf{pqr}} = \frac{\lambda\triangle_{\mathbf{abc}} - (1-\lambda\nu)(1-\nu)^{-1}}{(1-\lambda\mu)(1-\mu)^{-1}\triangle_{\mathbf{abc}} - 1}.$$

As above, if $\triangle_{\mathbf{cbp}} = \lambda'$, $\triangle_{\mathbf{acq}} = \mu'$ and $\triangle_{\mathbf{bar}} = \nu'$, then $\lambda = 1 - \frac{1}{\lambda'}$, $\mu = 1 - \frac{1}{\mu'}$, $\nu = 1 - \frac{1}{\nu'}$ and

$$\triangle_{\mathbf{pqr}} = \frac{(\lambda' - 1)\triangle - (\lambda' + \nu' - 1)}{(\lambda' + \mu' - 1)\triangle - \lambda'}. \tag{2}$$

See [6] for more details. Other applications of shapes are considered in [2] and [3].

According to [1], the shape of an ordered quadrangle **abcd** in the Gaussian plane is the ordered pair $[p, q]$, where $p = \triangle_{\mathbf{abc}}$ and $q = \triangle_{\mathbf{acd}}$. We denote the shape of the quadrangle **abcd** by $S(\mathbf{abcd})$.

It is easy to see that similar triangles with the same orientation have the same shape, i.e. the shape of a triangle is invariant under the group of direct plane similarities. Now we shall prove the same assertion for quadrangles.

**Proposition 1.** *Let* **abcd** *and* $\mathbf{a'b'c'd'}$ *be two convex quadrangles with shapes* $[p, q]$ *and* $[p', q']$*, respectively. Then*
*i. The quadrangles are similar if and only if they have the same orientation and* $p = p'$*,* $q = q'$*.*
*ii. The quadrangles are antisimilar if and only if they have an opposite orientation and* $p' = \overline{p}$*,* $q' = \overline{q}$*.*

**Proof. i.** If **abcd** and $\mathbf{a'b'c'd'}$ are similar, then there exists a similarity $f \in \mathrm{Sim}^+(2)$ which preserves the orientation in the plane and $f(\mathbf{a}) = \mathbf{a}'$, $f(\mathbf{b}) = \mathbf{b}'$, $f(\mathbf{c}) = \mathbf{c}'$, $f(\mathbf{d}) = \mathbf{d}'$. Thus, $\triangle\mathbf{abc}$ is similar to $\triangle\mathbf{a'b'c'}$ and $\triangle\mathbf{acd}$ is similar to $\triangle\mathbf{a'c'd'}$, i. e. $p' = p$ and $q' = q$.
Conversely, if $p' = p$ and $q' = q$, then there is a similarity $f \in \mathrm{Sim}^+(2)$ such that $f(\mathbf{a}) = \mathbf{a}'$, $f(\mathbf{b}) = \mathbf{b}'$ and $f(\mathbf{c}) = \mathbf{c}'$. Suppose that $f(\mathbf{d}) = \mathbf{d}''$. According to i., the quadrangles **abcd** and $\mathbf{a'b'c'd''}$ are similar and then $q = \triangle_{\mathbf{acd}} = \triangle_{\mathbf{a'c'd'}} = \triangle_{\mathbf{a'c'd''}}$. Hence, $\mathbf{d}' = \mathbf{d}''$ and the quadrangles **abcd** and $\mathbf{a'b'c'd'}$ are similar.
The proof of the second assertion is the same.

The proposal of this paper is to give generalizations of the first and second shape theorems. For this we need formulas for calculating the shapes of quadrangles when the order of the vertices is changed.

**Lemma 1.** *Let* **abcd** *be a convex quadrangle with shape* $[p, q]$*, then*
*i.* $$S(\mathbf{bcda}) = \left[\frac{1-pq}{1-p}, \frac{1}{1-pq}\right],\ S(\mathbf{cdab}) = \left[\frac{1}{1-q}, \frac{p-1}{p}\right],$$

$$S(\mathbf{dabc}) = \left[\frac{pq-1}{pq},\, \frac{p(1-q)}{1-pq}\right].$$

*ii.*
$$S(\mathbf{dcba}) = \left[\frac{1-pq}{p(1-q)},\, \frac{pq}{pq-1}\right],\ S(\mathbf{cbad}) = \left[\frac{p}{p-1},\, 1-q\right],$$
$$S(\mathbf{badc}) = \left[1-pq,\, \frac{1-p}{1-pq}\right],\ S(\mathbf{adcb}) = \left[\frac{1}{q},\, \frac{1}{p}\right].$$

**Proof.** Since $p = \dfrac{\mathbf{a}-\mathbf{c}}{\mathbf{a}-\mathbf{b}}$ and $q = \dfrac{\mathbf{a}-\mathbf{d}}{\mathbf{a}-\mathbf{c}}$, then $\triangle_{\mathbf{abd}} = \dfrac{\mathbf{a}-\mathbf{d}}{\mathbf{a}-\mathbf{b}} = pq$. Thus, $\triangle_{\mathbf{bda}} = (1-pq)^{-1}$ and $\triangle_{\mathbf{bad}} = 1-pq$. On the other hand, $\triangle_{\mathbf{bcd}} = \dfrac{\mathbf{b}-\mathbf{d}}{\mathbf{b}-\mathbf{c}} = \dfrac{(1-pq)(\mathbf{b}-\mathbf{a})}{\mathbf{b}-\mathbf{c}} = = (1-pq)(1-p)^{-1}$. Hence, $S(\mathbf{bcda}) = [\triangle_{\mathbf{bcd}},\, \triangle_{\mathbf{bda}}] = \left[\dfrac{1-pq}{1-p},\, \dfrac{1}{1-pq}\right]$. The proof of the rest equalities is analogous.

The first shape theorem can be extended to a formula for calculating the shape of a quadrangle.

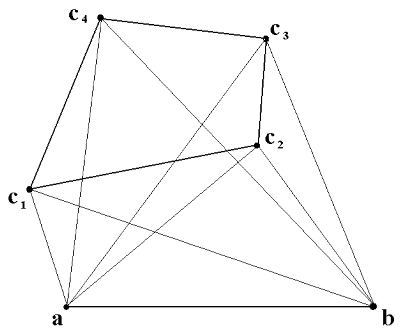


Figure 1

**Theorem 1.** *Let* $\mathbf{a}$ *and* $\mathbf{b}$ *be two distinct points in the plane. For arbitrary points* $\mathbf{c}_i$, $i = 1, 2, 3, 4$, *we suppose that* $\triangle_{\mathbf{abc}_i} = \lambda_i$. *Then, the shape of the quadrangle* $\mathbf{c_1c_2c_3c_4}$ *is*

$$S(\mathbf{c_1},\, \mathbf{c_2},\, \mathbf{c_3},\, \mathbf{c_4}) = \left[\frac{\lambda_3-\lambda_1}{\lambda_2-\lambda_1},\, \frac{\lambda_4-\lambda_1}{\lambda_3-\lambda_1}\right].$$

**Proof.** Since $\dfrac{\mathbf{a}-\mathbf{c}_i}{\mathbf{a}-\mathbf{b}} = \lambda_i$, we have $\mathbf{c}_i = \mathbf{a} - \lambda_i(\mathbf{a}-\mathbf{b})$ (see Figure 1). Then

$$\triangle_{\mathbf{c_1c_2c_3}} = \frac{\mathbf{c_1}-\mathbf{c_3}}{\mathbf{c_1}-\mathbf{c_2}} = \frac{\mathbf{a}-\lambda_1(\mathbf{a}-\mathbf{b})-\mathbf{a}+\lambda_3(\mathbf{a}-\mathbf{b})}{\mathbf{a}-\lambda_1(\mathbf{a}-\mathbf{b})-\mathbf{a}+\lambda_2(\mathbf{a}-\mathbf{b})} = \frac{\lambda_3-\lambda_1}{\lambda_2-\lambda_1}$$

and

$$\triangle_{\mathbf{c_1c_3c_4}} = \frac{\mathbf{c_1}-\mathbf{c_4}}{\mathbf{c_1}-\mathbf{c_3}} = \frac{\mathbf{a}-\lambda_1(\mathbf{a}-\mathbf{b})-\mathbf{a}+\lambda_4(\mathbf{a}-\mathbf{b})}{\mathbf{a}-\lambda_1(\mathbf{a}-\mathbf{b})-\mathbf{a}+\lambda_3(\mathbf{a}-\mathbf{b})} = \frac{\lambda_4-\lambda_1}{\lambda_3-\lambda_1}.$$

This completes the proof.

**Theorem 2.** *Let* $\mathbf{abcd}$ *be a quadrangle with a shape* $[p,\, q]$. *On the sides of the quadrangle* $\mathbf{abcd}$ *construct triangles* $\triangle\mathbf{cda_1}$, $\triangle\mathbf{dab_1}$, $\triangle\mathbf{abc_1}$ *and* $\triangle\mathbf{bcd_1}$ *externally with shapes* $\lambda_1$, $\lambda_2$, $\lambda_3$ *and* $\lambda_4$, *respectively. Then the shape of the quadrangle* $\mathbf{a_1b_1c_1d_1}$

*is*

$$\left[\frac{p-\lambda_1 p(1-q)-\lambda_3}{(1-\lambda_1)p(1-q)+pq\lambda_2}, \quad \frac{\lambda_1 p(1-q)+(1-\lambda_4)(1-p)}{\lambda_1 p(1-q)+\lambda_3-p}\right]$$

**Proof.** It follows from $\triangle_{\mathbf{cda_1}}=\lambda_1$, $\triangle_{\mathbf{dab_1}}=\lambda_2$, $\triangle_{\mathbf{abc_1}}=\lambda_3$ and $\triangle_{\mathbf{bcd_1}}=\lambda_4$ that $\mathbf{a}_1=\mathbf{c}-\lambda_1(\mathbf{c}-\mathbf{d})$, $\mathbf{b}_1=\mathbf{d}-\lambda_2(\mathbf{d}-\mathbf{a})$, $\mathbf{c}_1=\mathbf{a}-\lambda_3(\mathbf{a}-\mathbf{b})$ and $\mathbf{d}_1=\mathbf{b}-\lambda_4(\mathbf{b}-\mathbf{c})$.

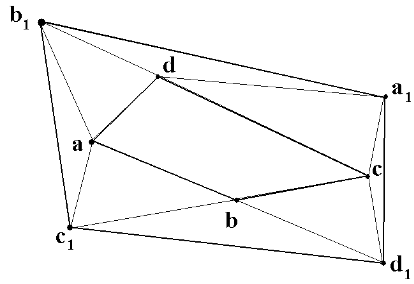


Figure 2

Hence, we find that

$$\text{(3)} \qquad \triangle_{\mathbf{a_1b_1c_1}} = \frac{\mathbf{a_1}-\mathbf{c_1}}{\mathbf{a_1}-\mathbf{b_1}} = \frac{\mathbf{c}-\mathbf{a}-\lambda_1(\mathbf{c}-\mathbf{d})+\lambda_3(\mathbf{a}-\mathbf{b})}{(1-\lambda_1)(\mathbf{c}-\mathbf{d})+\lambda_2(\mathbf{d}-\mathbf{a})} = \frac{1-(1-\lambda_1)\triangle_{\mathbf{dac}}-\lambda_3\triangle_{\mathbf{adb}}}{\lambda_2-(1-\lambda_1)\triangle_{\mathbf{dac}}}.$$

Since $\triangle_{\mathbf{dac}}=1-\triangle_{\mathbf{adc}}=1-\frac{1}{q}$ and $\triangle_{\mathbf{adb}}=1-\triangle_{\mathbf{dab}}=\frac{1}{pq}$(see Figure 2), then replacing in (3) we obtain the first component of the shape of the quadrangle $\mathbf{a_1b_1c_1d_1}$. Similarly

$$\text{(4)} \qquad \triangle_{\mathbf{a_1c_1d_1}} = \frac{\mathbf{a_1}-\mathbf{d_1}}{\mathbf{a_1}-\mathbf{c_1}} = \frac{\mathbf{c}-\mathbf{b}-\lambda_1(\mathbf{c}-\mathbf{d})+\lambda_4(\mathbf{b}-\mathbf{c})}{\mathbf{c}-\mathbf{a}-\lambda_1(\mathbf{c}-\mathbf{d})+\lambda_3(\mathbf{a}-\mathbf{b})} = \frac{1-\lambda_1\triangle_{\mathbf{cbd}}-\lambda_4}{1-\lambda_1\triangle_{\mathbf{cbd}}-(1-\lambda_3)\triangle_{\mathbf{bca}}}.$$

From $\triangle_{\mathbf{cbd}} = \frac{(q-1)p}{1-p}$ and $\triangle_{\mathbf{bca}}=(1-p)^{-1}$, replacing in (4) we get the second component of the shape of the quadrangle $\mathbf{a_1b_1c_1d_1}$ and this completes the proof.

Some particular cases of Theorem 2 are used in [3] to solve of some problems from [4].

**Theorem 3.** *Let* $\mathbf{abcd}$ *be a quadrangle with a shape* $[p,q]$. *Let* $\mathbf{ac}\cap\mathbf{bd}=\mathbf{m}$, $\mathbf{ab}\cap\mathbf{cd}=\mathbf{k}$ *and* $\mathbf{bc}\cap\mathbf{ad}=\mathbf{l}$. *Then*

*i.*
$$\triangle_{\mathbf{abm}}=p.\frac{\operatorname{Im} pq}{|p|^2\operatorname{Im} q+\operatorname{Im} p}, \ \triangle_{\mathbf{bcm}}=\frac{1-pq}{1-p}.\frac{\operatorname{Im} p}{|p|^2\operatorname{Im} q+\operatorname{Im} p},$$

$$\triangle_{\mathbf{cdm}}=\frac{1}{1-q}.\frac{|p|^2\operatorname{Im} q+\operatorname{Im} p-\operatorname{Im} pq}{|p|^2\operatorname{Im} q+\operatorname{Im} p}, \ \triangle_{\mathbf{dam}}=\left(1-\frac{1}{pq}\right).\frac{|p|^2\operatorname{Im} q}{|p|^2\operatorname{Im} q+\operatorname{Im} p},$$

$$\triangle_{\mathbf{abl}} = \frac{p\operatorname{Im} pq - |p|^2 \operatorname{Im} q}{\operatorname{Im} pq - |p|^2 \operatorname{Im} q};$$

*ii.* $\triangle_{\mathbf{mkl}} = \dfrac{q.\operatorname{Im} p(|p|^2\operatorname{Im} q + \operatorname{Im} p) - \operatorname{Im} pq(\operatorname{Im} pq - |p|^2\operatorname{Im} q)}{q.\operatorname{Im} p(|p|^2\operatorname{Im} q + \operatorname{Im} p) - \operatorname{Im} pq(|p|^2\operatorname{Im} q + 2\operatorname{Im} p - \operatorname{Im} pq)} . \dfrac{\operatorname{Im} p - \operatorname{Im} pq}{\operatorname{Im} pq - |p|^2 \operatorname{Im} q};$

*iii. The shape of the quadrangle* **mkld** *is* $[\triangle_{\mathbf{mkl}}, \triangle_{\mathbf{mld}}]$, *where*

$$\triangle_{\mathbf{mld}} = \frac{q.(|p|^2\operatorname{Im} q + \operatorname{Im} p) - \operatorname{Im} pq}{\overline{p}.\operatorname{Im} q(|p|^2\operatorname{Im} q + \operatorname{Im} p) - \operatorname{Im} pq(\operatorname{Im} p + 2|p|^2\operatorname{Im} q - \operatorname{Im} pq)} .(|p|^2\operatorname{Im} q - \operatorname{Im} pq).$$

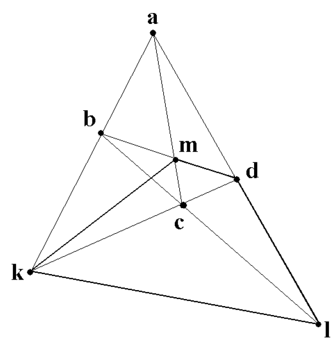


Figure 3

**Proof.** Applying the First Shape Theorem for the triangles $\triangle\mathbf{abb}$, $\triangle\mathbf{abd}$ and $\triangle\mathbf{abm}$ with shapes 1, $pq$ and $\triangle_{\mathbf{abm}}$, respectively we find that

$$\triangle_{\mathbf{bdm}} = \frac{\triangle_{\mathbf{abm}} - 1}{pq - 1}.$$

Since **b**, **d**, **m** are collinear (see Figure 3),then $\triangle_{\mathbf{bdm}} = \overline{\triangle}_{\mathbf{bdm}}$. Hence

$$\frac{\triangle_{\mathbf{abm}} - 1}{pq - 1} = \frac{\overline{\triangle}_{\mathbf{abm}} - 1}{\overline{pq} - 1}. \tag{5}$$

Similarly, applying the First Shape Theorem for the triangles $\triangle\mathbf{aba}$, $\triangle\mathbf{abc}$ and $\triangle\mathbf{abm}$ with shapes 0, $p$ and $\triangle_{\mathbf{abm}}$, respectively and using that the points **a**, **c**, **m** are collinear we get

$$\frac{\triangle_{\mathbf{abm}}}{p} = \frac{\overline{\triangle}_{\mathbf{abm}}}{\overline{p}}. \tag{6}$$

From (5) and (6), eliminating $\overline{\triangle}_{\mathbf{abm}}$ we find the first equality in *i*. The shapes of the triangles $\triangle\mathbf{bcm}$, $\triangle\mathbf{cdm}$ and $\triangle\mathbf{dam}$ are obtained from the shape of the triangle $\triangle\mathbf{abm}$ by cycling the vertices of the quadrangle **abcd** and Lemma 1. Applying the first equality in *i*. for the quadrangle **bacd** with a shape $\left[1 - p, \dfrac{1 - pq}{1 - p}\right]$ we get

$\triangle_{\mathbf{bal}} = (1 - p).\dfrac{\operatorname{Im} pq}{\operatorname{Im} pq - |p|^2\operatorname{Im} q}$. Using that $\triangle_{\mathbf{abl}} = 1 - \triangle_{\mathbf{bal}}$ we find the last equality in *i*. Denote $\triangle_{\mathbf{abm}} = \lambda_1$, $\triangle_{\mathbf{cbk}} = \lambda_2$ and $\triangle_{\mathbf{acl}} = \lambda_3$. Hence $\mathbf{m} = \mathbf{a} - \lambda_1(\mathbf{a} - \mathbf{b})$,

$\mathbf{k} = \mathbf{c} - \lambda_2(\mathbf{c} - \mathbf{b})$ and $\mathbf{l} = \mathbf{a} - \lambda_3(\mathbf{a} - \mathbf{c})$, and replacing in $\triangle_{\mathbf{mkl}} = \dfrac{\mathbf{m} - \mathbf{l}}{\mathbf{m} - \mathbf{k}}$ we find that

$$\triangle_{\mathbf{mkl}} = \frac{\lambda_3 p - \lambda_1}{(1 - \lambda_2)p + \lambda_2 - \lambda_1}. \tag{7}$$

It follows from the second equality in *i*. for the quadrangles **acbd** and **bacd** with shapes $\left[\frac{1}{p}, pq\right]$ and $\left[1 - p, \frac{1 - pq}{1 - p}\right]$ that

$$\triangle_{\mathbf{cbk}} = \lambda_2 = \frac{p(1-q)}{p-1} . \frac{\operatorname{Im} p}{\operatorname{Im} p - \operatorname{Im} pq} \tag{8}$$

$$\triangle_{\mathbf{acl}} = \lambda_3 = q . \frac{\operatorname{Im} p}{\operatorname{Im} pq - |p|^2 \operatorname{Im} q}. \tag{9}$$

Replacing (8), (9) and the first equality from *i* in (7) we get *ii*..
The last assertion in *iii*. follows from the First Shape Theorem for the triangles $\triangle$**abm**, $\triangle$**abl** and $\triangle$**abd** with shapes $\triangle_{\mathbf{abm}}$, $\triangle_{\mathbf{abl}}$ and $\triangle_{\mathbf{abd}} = pq$, respectively. □

## REFERENCES


[1] R. Artzy. Shapes of polygons. *J. Geom.*, **50** (1994), 11–15.

[2] G. Georgiev, R. Encheva, M. Spirova. Shapes of inscribed and circumscribed quadrangles. *Mathematics and Education in Mathematics*, **30** (2001), 225–230.

[3] G. Georgiev, R. Encheva. Studying special quadrangles by shapes. *Annuals of Konstantin Preslavski University, FMI*, **XVC** (2002), 5–21.

[4] V. Gusev, V. Litvinenko, A. Mordkovich. Solving Problems in Geometry. Mir Publishers, Moscow, 1988.

[5] L.-S. Hahn. Complex Numbers and Geometry. MAA, Washington DC, 1994.

[6] J. A. Lester. Triangles I: Shapes. *Aequationes Math.*, **52** (1996), 30–54.

[7] J. A. Lester. Triangles II: Complex triangle coordinates. *Aequationes Math.*, **52** (1996), 215–254.

[8] J. A. Lester. Triangles III: Complex triangle functions. *Aequationes Math.*, **53** (1997), 4–35.

[9] I.M. Yaglom. Complex Numbers in Geometry. Academic Press, New York, 1968.



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# ШЕЙП ТЕОРЕМИ ЗА ЧЕТИРИЪГЪЛНИЦИ

**Георги Христов Георгиев, Радостина П. Енчева**

Шейп на триъгълник е комплексно число, съответстващо на орбитата от триъгълници под действието на групата на подобностите в равнината, запазващи ориентацията. Шейп на четириъгълник е наредена двойка от комплексни числа, съответствуваща на еквивалентен клас от четириъгълници по отношение на същата група. Прилагаме шейпове на триъгълници за изучаване на шейпове на четириъгълници. В частност получаваме тъждества за шейпове чрез използване на накои геометрични конструкции.