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# AN APPLICATION OF COMPUTER GRAPHICS TO GEOMETRY

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By moving line through a fixed point we cut a plane figure (square, rectangle, trapezium or polygon which sides are circles arcs) and rotate the line. Then we prove that the length function of the section is only a continuos function, but the area function is exactly once differentiable function in the vertices of the plane figure. Using the program *MAPLE 6* we visualize the corresponding surfaces and results.

We consider a square $OABC$ and a straight line $g = MN$ where $M(m, 0)$ and $N(0, n)$. The straight line has an equation $MN : \dfrac{x}{m} + \dfrac{y}{n} = 1$. We intersect the square with the line, then we rotate the line around the point $M(m, 0)$, and want to investigate what will happen when the line goes through the vertex $C$. Here we consider the case $m > 1$.

We denote by $P$ the intersection point of the line $g$ and the segment $AB$ and by $Q$ the intersection point of $g$ with the segment $BC$. It is easy to find their coordinates:

$$P\left[1; \frac{n}{m}(m-1)\right], \qquad Q\left[\frac{m}{n}(n-1); 1\right].$$

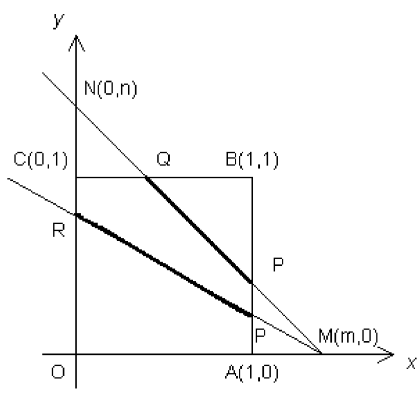
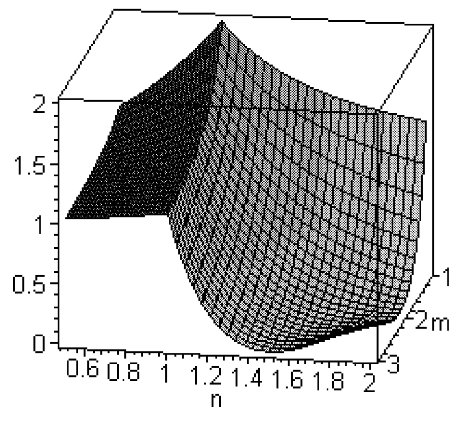



Figure 1




Picture 1

MSC 51M04, 53A05, 5204, 52A38.

For the lengths $PQ$ and $PN$ we have:

$$PQ^2(m,n) = (m+n-mn)^2 \frac{(m^2+n^2)}{m^2n^2}.$$

$$PN^2(m,n) = \frac{m^2+n^2}{m^2n^2}$$

The line $g$ goes through the vertex $C$ if $n = 1$. We have:

**Assertion 1.** $PQ^2(m, n = 1) = PN^2(m, n = 1) = PC^2$.

For the derivative of the length functions with respect to $n$ we have

**Assertion 2.** $(PQ^2)'(m, n = 1) = (PN^2)'(m, n = 1)$.

Thus it can be proved the following

**Theorem 1.** *The length of the section of a moving line through a fixed point is only a continuos function in the vertices of the square, but not differentiable.*

This theorem is proved by Prof. G. Stanilov in [1].

Now we want to investigate also the area functions for the corresponding sections in the vertices of the square. We find

$$S_{OAPQCO}(m,n) = -\frac{(mn-m-n)^2}{2mn}, \qquad S_{OAPNO} = \frac{(2m-1)\,n}{2m}$$

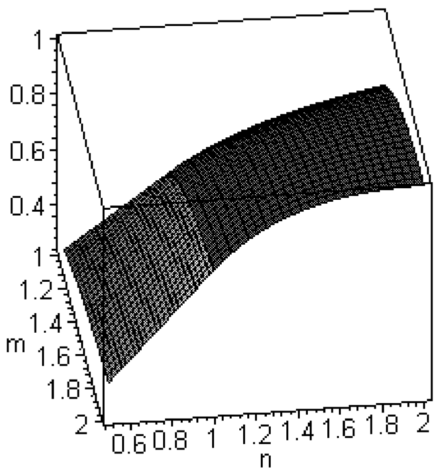
Then we have

**Assertion 3.** $S_{OAPQCO}(m, n = 1) = S_{OAPNO}(m, n = 1)$*;*

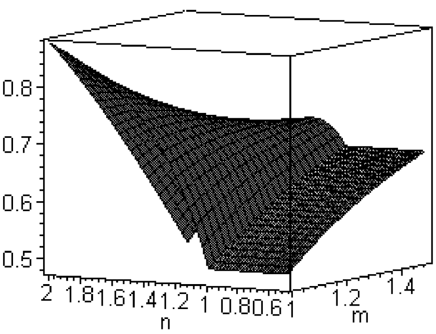
**Assertion 4.** $S'_{OAPQCO}(m, n = 1) = S'_{OAPNO}(m, n = 1)$*;*

**Assertion 5.** $S''_{OAPQCO}(m, n = 1) \neq S''_{OAPNO}(m, n = 1)$.

**Theorem 2.** *The area of the section of a square with a moving line through a fixed point* $(M(m))$ *is exactly once differentiable function in the vertices of the square.*




Picture 2




Picture 3

Using the programme *MAPLE* these theorems are visualized by pictures 1 and 2.

**Remark 1.** The theorems 1 and 2 hold also for any $m < 0$ and $0 < m < 1$.

**Remark 2.** Both theorems hold also for:

a) any rectangular trapezium;

b) any rectangle.

We omit the proofs for these cases. But we will prove Theorem 1 for more general case, namely, we consider the polygon $OABC$ (Fig. 2) which consists of a segment $OA$, an arc $AB$, an arc $BC$ and an arc $CO$, where:

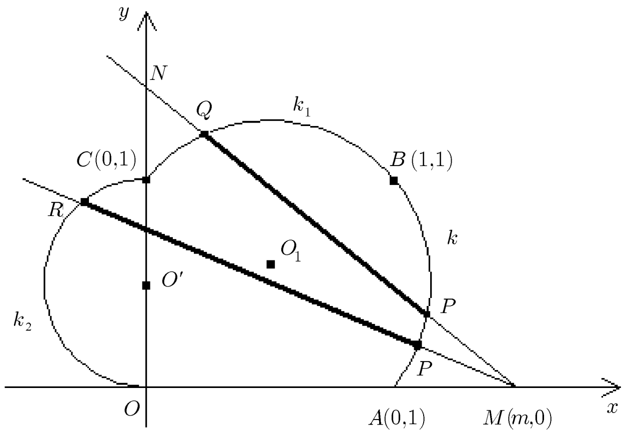



Figure 2

the arc $AB$ is a part of the circle $k$ with radius $0.5\sqrt{5}$ and center $O'(0; 0.5)$;
the arc $BC$ is a part of the circle $k_1$ with radius $\sqrt{0.5}$ and center $O_1(0.5; 0.5)$;
the arc $CO$ is a part of the circle $k_2$ with radius 0.5 and center $O'(0; 0.5)$.

The moving line $MN$ cut the sides of the polygon in the points:

$$P = MN \cap AB, \qquad Q = MN \cap BC, \qquad R = MN \cap CO.$$

These circles have equations:

$$k : x^2 + (y - 0.5)^2 = \frac{5}{4},$$

$$k_1 : (x - 0.5)^2 + (y - 0.5)^2 = 0.5,$$

$$k_2 : x^2 + (y - 0.5)^2 = 0.25.$$

The coordinates of the intersection points are:

$$P(x(P), y(P)), Q(x(Q), y(Q)), R(x(R), y(R)) \text{ where}$$

$$x(P) = \frac{mn\,(2n-1) + \sqrt{D}}{2\,(m^2 + n^2)},$$

$$y(P) = \frac{n}{m}[m - x(P)], D = m^2[5m^2 + 4m^2(n + 1 - n^2)];$$

$$x(Q) = \frac{m(2n^2 + m - n) - \sqrt{D_1}}{2(m^2 + n^2)},$$

$$y(Q) = \frac{n}{m}[m - x(Q)], D_1 = m^2[(m - n)^2 + 4mn(m + n - mn)];$$

$$x(R) = \frac{mn(2n - 1 - \sqrt{D_2})}{2(m^2 + n^2)},$$

$$y(R) = \frac{n}{m}[m - x(R)], D_2 = m^2n(n + 4m^2 - 4m^2n)$$

We find the following length functions:

$$PQ^2(m, n) = \frac{(\sqrt{D} + \sqrt{D_1} - m^2)^2}{4(m^2 + n^2)};$$

$$PR^2(m, n) = \frac{(\sqrt{D} + \sqrt{D_2})^2}{4(m^2 + n^2)}.$$

Then we have:

**Assertion 6.** $PQ^2(m, n = 1) = PR^2(m, n = 1)$*;*

**Assertion 7.** $(PQ^2)'(m, n = 1) = (PR^2)'(m, n = 1)$.

Thus the Theorem 1 holds also for the polygon $OABC$ (Fig. 2).

Assertions 6 and 7 are visualized by Picture 3.

Having in mind such results Prof. G. Stanilov has stated the following

**Conjecture.** *Let it be given an arbitrary plane convex polygon with sides which are sufficiently differentiable curves. We consider a straight line $g$ which will be rotated around a fixed point $M$. Then for the length functions $PQ$ and $PR$ is holds Theorem* 1*, and for the area functions it holds Theorem* 2*.*

## REFERENCES


[1] G. Stanilov, P. Boychev, J. Cankov. Mittels Computer zu mathematischen Entdeckungen. Beitraege zum Mathematikunterricht 2001, Vortraege auf der 35. Tagung der Didaktik der Mathematik vom 5. bis 9. März in Ludwigsburg, S.592–595.

[2] G. Stanilov, L. Stanilova. Mittels Computer zu mathematischen Entdeckungen. Technology in Mathematics Teaching.Proc. of ICTMT 5 in Klagenfurt, Schriftenreihe Didaktik der Mathematik v. 26 oebv&hpt, Vienna 2002, 227–230.



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## ЕДНО ПРИЛОЖЕНИЕ НА КОМПЮТЪРНАТА ГРАФИКА В ГЕОМЕТРИЯТА

**Добри Ст. Добрев**

Пресичаме равнинна фигура (квадрат, правоъгълник, трапец или изпъкнал полигон със страни дъги от окръжности) с подвижна права, минаваща през фиксирана точка, след което въртим правата около тази точка. Доказваме, че функцията дължина на сечението е само непрекъсната функция, обаче лицето на сечението, ограничено от фигурата и правата, е точно еднократно гладка функция във върховете на фигурата. С помощта на програмата *Maple* визуализираме получените повърхнини и резултати.