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MATHEMATICAL CAPITAL: AN ECONOMIC APPROACH

Constantinos Tsamadias, George Dimakos

Mathematics is a central branch in the “tree” of sciences. It constitutes the hard “core” in the area of sciences and it is a basic “tool” in scientific research. The dynamic interaction between mathematics and other sciences, technology and real world is established. This paper introduces the concepts of individual and social mathematical capital and focuses on their economic meaning. It offers a tentative theoretical framework with the aim to provide a basis for theoretical and empirical research which can lead to estimates of the economic value of mathematical capital.

I. Introduction. During the last fifty years, the contribution of technology, human capital and knowledge on economic evolution has been theoretically defended and empirically verified in the field of economics (see: Solow [15], Schultz [13, 14], Becker [3], Lucas [7], Romer [12], et al). In addition, the impact of education on productivity, economic development, incomes, income distribution, among other economic sectors has also been both theoretically and empirically analyzed and estimated [see: Mincer [9], Psacharopoulos [11], et al), as well as critically debated (see: Abramovitz [1], Bowen [3], Arrow [2], Spence [16], et al).

We live in a global economy, where technologies of information and communication, as well as knowledge and learning are the “engines” for the whole process of economic and social development. The modern economic growth on a global scale depends mainly upon the diffusion of a body of knowledge concerning new techniques of production and management. The OECD [10, p. 2], recognizing the growing significance of the knowledge-based economy, defines it as one “directly based on the production, distribution and use of knowledge and information”. Concurrently, it notes that “knowledge and information tend to be abundant; what is scarce is the capacity to use them in meaningful ways” [10, p. 11]. While all this is true, it is also true that knowledge only reaches its full potential to create economic value when it becomes embedded in organizational processes and routines. Unlike physical goods that are consumed as they are used, providing decreasing returns over time, knowledge provides increasing returns as it is used. The more knowledge is used, the more valuable it becomes, creating thereby a self-reinforcing cycle. Stiglitz characteristically emphasizes that “it is the process of embodying knowledge in people (learning) and things (applications) that is costly in time and resources” [17, p. 9)]. In the knowledge economy, the basis of competitive advantage is the effective utilization of knowledge resources [5]. The new economy of knowledge is defined as possessing the following characteristics: highly intensive knowledge, rapid diffusion of information, extensive innovation networks, flexible and adaptive to new market opportunities, high

levels of education, skill and training, as well as firms' linkages across complementary assets and competencies [8].

The role of mathematics in education and training throughout history has been central. Recently, in a world driven by numbers, the scientific research has used increasingly mathematical knowledge, while the new high technology has embodied it. There is no doubt that the evolution of other sciences has been strengthened by mathematics. Thus, these, having freely borrowed the tools that the mathematics have provided to help them develop, have reciprocated by providing to mathematics with new problems which have generated new sources of inspiration. Also technology, has had a profound effect on mathematics. The new knowledge economy, the so-called digital economy, and/or the new society of knowledge have as a kernel the mathematical knowledge and the mathematical technology. It can be argued that the mathematical knowledge and technology are determining factors on the economic, social, cultural and political development of individuals and social groups [18].

Accordingly, the purpose of this paper is to elaborate the useful concept of the individual and social mathematical capital from the perspective of an economic approach. In doing so, it offers a tentative theoretical framework, with the aim to provide a basis for estimating the economic value of mathematical capital, and for suggesting research on the dynamic interaction between the mathematical capital and the economy in general.

In this context, section II, provides an answer to the question what is mathematics. Section III, introduces the concept of mathematical capital and suggests its economic meaning. The last section, offers some concluding remarks.

II. What is Mathematics? What is mathematics? How was it created? By whom has it been developed and practiced? What is its role in the history of scientific thinking? What is its relationship with other disciplines? These are some of the old-standing questions that have come under much serious and critical thinking, and to which new answers have been given while the debate continuing.

Given the fact that there is such a richness in the definition of the concept of mathematics from the ancient to the modern times, it is not prudent for the purposes of this paper to attempt to give a single, precise, and comprehensive definition of mathematics. However, it is sufficient to note the following in this context. Mathematics contrary to a widespread opinion among non-scientists, is not a closed and perfect edifice. While mathematics is a self-contained microcosm it also has the potentiality of mirroring and modeling all the processes of thought and perhaps all of science. In other words, mathematics is an outward-looking body of knowledge. It continuously collaborates with observational science and laboratory science. Through its power to quantify and to organize knowledge, it makes possible the use of that knowledge in problems on a vastly bigger scale than that of the scientists' own laboratories. It has always had, and continues to an ever-increasing degree to have, great usefulness. One could even go so far as to say that mathematics was necessary for man's conquest of nature and for the development of the human race through the shaping of the modes of thinking.

Like other sciences, mathematics has been subject to great changes during the past fifty years. Not only has its subject matter vastly increased, not only has the emphasis on what were considered the central problems changed but the tone and the aims of mathematics have to some extent been transmuted. There is no doubt that many great

triumphs of physics, astronomy, biology and other “exact” sciences arose in significant measure from mathematics.

III. Mathematical capital and its economic meaning. In economics, we refer to “*capital*” as resources which have been produced in an economic system through a production process and which is utilized for the production of other goods.

As a “*mathematical capital of an individual*”, we define all inherent and acquired mathematical abilities, as well as all acquired mathematical knowledge (logic, foundations and structure, methodologies, techniques, critical thought), experiences, skills and effectiveness in mathematical applications.

As a “*mathematical capital of a social group*”, we define the sum of the overall mathematical capital of the social group’s members and the mathematical tradition and culture of the group.

The inherent mathematical ability/skill/talent is one of the seven attributes composing the “spectrum of human’s intelligence” [6]. The acquired ability is derived through the processes of education, research and experience.

The economic characteristics of mathematical capital, as well as its function in the economy, can be summarized as follows:

1. The research in mathematics is the main mechanism of production and accumulation of the individual/social mathematical capital.
2. The education in mathematics is a mechanism of both production and accumulation of the individual and, by extension, of the social mathematical capital. It is also the main mechanism of the diffusion of the mathematical capital to individuals of society.
3. The rate of production and accumulation of the acquired part of the mathematical capital for each individual and social group depends, in general, on the mathematical tradition, the social, economic, cultural environment and, mainly on the quality of research, education and training systems. The exploitation of mathematical capital occurs at different levels of effectiveness.
4. The research and the education in mathematics can be considered as consumption but mainly as investment for both the individual and the society¹. The consumption element and the investment element provide utility (now or later) and contribute to the discounted stream of utility enjoyed by the economic agent. The production and accumulation of the mathematical capital from every individual and/or society require investment in both time and economic resources. An individual or family decision unit selects an amount of investment in mathematical capital (or other self-investment) in order to maximize an objective function subject to some constraints. The objective function may be the lifetime income, appropriately discounted, or it may be a utility, a measure of well-being. The constraints include the limits imposed by a family’s own financial resources, its capacity to borrow outside funds, and limits upon the time the individual (and, in earlier years, the parents as well) can devote to mathematical education or research. Costs include out-of-pocket payments plus earnings forgone. Benefits include the increase in expected life-time earnings, as well as non-pecuniary returns, such

¹As it is well known, consumption and investment are important concepts in economics.

Whereas consumption refers to the purchase or use of goods and services which bring immediate but short-lived benefits, investment, refers to the acquisition of assets which yield benefits over a long period of time. All expenditure can be classified as either consumption or investment, although the borderline is not always precise.

as improved working conditions, job security and the consumption benefits of mathematical education, or research expected from future leisure activities. Optimal investment in mathematical capital occurs when the discounted value of the costs incurred equals the discounted value of the benefits expected. A society selects an amount of investment in mathematical capital in order to maximize an objective social welfare function subject to some constraints. Optimal social investment to mathematical capital occurs when the discounted social cost incurred equals the discounted value of the benefits expected.

The empirical estimation of the rate of returns on private and social investment in mathematical capital is an open problem.

5. The investment in mathematical capital, despite its long-term character, is accompanied by limited uncertainty, due to its continuing usefulness on both individual and social basis.

6. The individual and social investment in mathematical capital is progressively reduced during an individual's life, tending to zero at the stage of retirement from work. Other things equal, the earlier the investment, the longer will be the expected stream of benefits. Hence, maximization of lifetime utility implies that most formal mathematical education and training will occur during one's youth. The return on mathematical capital has a long-term character.

7. The mathematical capital reserves/ stock borne by each individual cannot be transferred, cannot be bought, and cannot be inherited in its entirety.

8. The quantity and quality of mathematical capital contributes to the improvement of individual productivity and to the total productivity of society. As regards the channels through which mathematical capital increases productivity, the following must be stressed: (a) Mathematical capital enhances the ability of an individual to perform standard tasks (factor efficiency) and learn to perform new tasks, (b) enhances the ability to receive and process new information, (c) enhances ability to evaluate and adjust to changing circumstances (ability to deal with disequilibria), (d) it increases ability to communicate and coordinate activities with one another, (e) it reduces subjective uncertainty and unnecessary anxiety on the one hand, whereas, on the other hand, it favors the establishment of a more critical approach towards the status quo, fostering, in parallel, the probability of accepting new technologies and practices and (f) helps to bring about new innovations in production processes and to develop new products.

9. The mathematical capital is a resource, which increases rather than diminishes with use. It has little cost to generate and diffuse. Once mathematical knowledge is discovered and made public, there is essentially little marginal cost to adding more users.

10. The mathematical capital which is not used diminishes with the passage of time, or as the individual grows older, or with evolution and changes in sciences, technology and production. The mathematical capital which is not continuously replenished is to be quickly diminished.

11. The differences in quantity and quality of individual and social mathematical capital affect the inequities between individuals and societies and determine, to a high degree, the hierarchies.

12. The benefits from mathematical research and education can be classified into private (accruing to the individual and the family), direct narrow-social (accruing to so-

ciety at large) and indirect or wide-social (including externalities² and spillovers³). The benefits could also be categorized into micro-level (monetary/private returns on investment in mathematical education and narrow social returns) and macro-level (market and non-market wide social benefits, positive externalities and spillovers).

13. Mathematical capital embodied in new products and services has become a source of wealth creation.

14. The mathematical capital contributes to economic growth and development, and vice versa, that is, economic development contributes to mathematical capital growth.

The empirical evidence on the contribution of mathematical capital to the economic growth and development remains an open problem.

IV. Conclusion. In this article, we defined the concept of individual and social mathematical capital, suggested its economic meaning, and showed the central role of mathematical capital in the new knowledge-based economy. In stressing the importance of mathematical capital as an economic resource, we highlighted the process of production, accumulation, transmission and diffusion of mathematical capital in economy. It follows from this analysis that the mathematical capital has to be recognized and utilized accordingly by the scientific, economic, social and political elite as a strategically important resource.

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²Externalities (positive) are those benefits to society that are above and beyond the private benefits realized by the individual "decision-maker", that is, the student or the family. These are above and beyond both private monetary benefits, and the private non-monetary consumption benefits, both of which are captured by the "decision-maker" and taken into account whenever the decision is made.

³Spillovers are a type of externality, since they are a benefit that is not captured by the decision-making unit within which occurs the mathematical educational process.

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Constantinos Tsamadias
 Mathematician – Economist (Ph. D)
 28, Agion Anargiron
 11146 Galatsi, Athens, Greece

George Dimakos
 Mathematician – Economist (Ph. D)
 Assistant Professor
 University of Athens, Greece

МАТЕМАТИЧЕСКИ КАПИТАЛ: ИКОНОМИЧЕСКИ ПОДХОД

Константинос Цамадиас, Георг Димитракос

Математиката е централен клон на “дървото” на науките. Тя съставлява твърдото “ядро” в научната област и е основно “средство” в научното изследване. Установено е динамично взаимодействие между математиката и другите науки, технологията и реалния свят. Тази статия борави с понятията от индивидуалния и математическия капитал, като се фокусира върху тяхното икономическо значение. Тя предлага експериментална теоретическа рамка с цел да осигури база за теоретично и емпирично изследване, което може да доведе до оценка на икономическата стойност на математическия капитал.