МАТЕМАТИКА И МАТЕМАТИЧЕСКО ОБРАЗОВАНИЕ, 2004
MATHEMATICS AND EDUCATION IN MATHEMATICS, 2004
Proceedings of the Thirty Third Spring Conference of the Union of Bulgarian Mathematicians
Borovets, April 1–4, 2004

VERTICAL MODALITIES OF LANGUAGE OF eRATL LOGIC AND EXISTENCE OF CHILDREN, PARENT AND ANCESTORS*

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Let Δ_1 and Δ_2 be two maximally consistent set, and $\perp$ be the canonical child relation over maximally consistent set. We are going to show that for every formula $\varphi \in \Delta_2$ there must be corresponding Fx $\varphi \in \Delta_1$ and analogously for Xh and Fh. Our next objective is to show that whenever we have Fx $\varphi \in \Delta_1$ then there exists a maximally consistent set of formulas Δ_2 such that $\varphi \in \Delta_2$. We are going to prove the similar statements for vertical modalities Fh and Xh too.

1. Introduction. To prove the completeness theorem of enriched with abstractions of time linear temporal logic eRATL, which is discrete, we have to use a method that involves building a canonical model. The idea may be formally described as follows: The objective of proving completeness is to show that any given formula A that is known to be consistent must also be satisfiable. One way of proving satisfiability is to construct a model for A. **First step**: We have to decide on a suitable set of worlds (nodes). We construct a node for every combination of formulas that could be true at the same time. Each node is a consistent set of formula. **Second step**: We have to decide suitable relations for access over the set of nodes. For this purpose we have to arrange how modal operators of eRATL work as intended.The difficult part of that step is to prove that there exists a maximally consistent set of formulas, which is related to the first set and contains a suitable witness formula in the case of an existential modality. **Third step**: We have to show that a formula A belongs to a given maximally consistent set of formulas iff A is satisfied at the corresponding node in our canonical model.

In the present paper, we are going to consider a part of the second step of proving satisfiability of a given formula A.

2. Syntax and semantics of enriched temporal logic eRATL.

Definition 2.1. *The set of well-formed formulas of eRATL is the smallest set such that propositional letters, true and false are formulas and if φ and ψ are formulas, so are $(\neg\varphi)$, $(\varphi\wedge\psi)$, $(\varphi\vee\psi)$, $(\varphi \to \psi)$, $(\varphi \leftrightarrow \psi)$,* (Xh$\varphi$), (Fh$\varphi$), (Gh$\varphi$), (Xp$\varphi$), (Fp$\varphi$), (Gp$\varphi$), (X$\varphi$), (F$\varphi$), (G$\varphi$), (Fx$\varphi$), *and* (Gx$\varphi$), *where the modal operators* X, F *and* G *are taken from the basic temporal logic. The informal definition of* F, G, X, Xp, Fp, Gp *are the same as in* LTLp. *The following operators are the new vertical operators. There are five*

*__2000 Mathematics Subject Classification:__ 03B44, 03B45, 03B60.
Key words: temporal logics, modal logics, tree.

operators to refer to the states on the next lower or the higher level of abstraction, where Fxφ : φ *is true at some state of the next lower level in the tree structure;* Gxφ : φ *is true at all states of the next lower level in the tree structure;* Xhφ : φ *is true at the next higher level in the tree structure;* Fhφ : φ *is true at some higher level in the tree structure;* Ghφ : φ *is true at all higher levels in the tree structure. The following notation expresses that the same modality is applied n times to a formula,* $n \in N^+ : \mathrm{X^n}\varphi = \underbrace{X \dots X}_{\mathrm{n-times}}\varphi$.

The semantics of formulas in eRATL will be given by a definition of truth of a formula in a model. Models are based on trees of type α, i.e. the frames for this modal logic are trees of type α. Using [2] we can obtain the following definition.

Definition 2.2. [using 2] *A tree is a pair* $\tau = (\mathrm{T}, \mathrm{R_T})$*, where T is a set and* $\mathrm{R_T}$ *is an irreflexive binary relation over T satisfying the following conditions:*

- *For every* $\mathrm{t} \in \mathrm{T}$ *there exists at most one* $\mathrm{t}' \in \mathrm{T}$ *with* $(\mathrm{t}', \mathrm{t}) \in \mathrm{R_T}$.
- *There exists a unique* $\mathrm{r} \in \mathrm{T}$ *such that* $\{\mathrm{t} \in \mathrm{T} \mid (\mathrm{r}, \mathrm{t}) \in \mathrm{R_T^*}\} = \mathrm{T}$.

The elements of T *are called nodes. The element r from second condition is called the root of* τ.$\mathrm{R_T}$ *is called the child relation,* $(\mathrm{R_T^{-1}})^+$ *is the ancestor relation and* $\mathrm{R_T^{-1}} \circ \mathrm{R_T}$ *is the brotherhood relation. The brotherhood relation* $\mathrm{R_T^{-1}} \circ \mathrm{R_T}$ *is an equivalence relation over the set of nodes excluding the root. For a node* t $[\mathrm{t}]_{\mathrm{R_T^{-1}} \circ \mathrm{R_T}} = \{\mathrm{t}' \in \mathrm{T} \mid (\mathrm{t}, \mathrm{t}') \in \mathrm{R_T^{-1}} \circ \mathrm{R_T}\}$ *is the set of brothers of* t*. If* t *is the root then* $[\mathrm{t}]_{\mathrm{R_T^{-1}} \circ \mathrm{R_T}}$ *is the empty set, where* $\mathrm{R_T^+}$ *is a transitive closure of* $\mathrm{R_T}$ *and* $\mathrm{R_T^*}$ *is a transitive reflexive closure of* $\mathrm{R_T}$.

Definition 2.3. *A tree of type* α *is a tuple* $\tau = (\mathrm{T}, \mathrm{R_T}, \mathrm{R}, \mathrm{Rn})$*, where* $(\mathrm{T}, \mathrm{R_T})$ *is a tree,* $\mathrm{R} \subseteq \mathrm{R_T^{-1}} \circ \mathrm{R_T}$*, and in a pair* $([\mathrm{t}]_{\mathrm{R_T^{-1}} \circ \mathrm{R_T}}, \mathrm{R})$ *the relation* R *is irreflexive, transitive and connected for every* $\mathrm{t} \in \mathrm{T}$*. If* $\mathrm{t_1 R t_2}$*, then* $\mathrm{t_1}$ *is called a left brother of* $\mathrm{t_2}$*, and* $\mathrm{t_2}$ *is called the right brother of* $\mathrm{t_1}$*. The relation* $\overline{\mathrm{R} \circ \mathrm{R}}$ *is called the neighborhood relation and* $\mathrm{Rn} \equiv \overline{\mathrm{R} \circ \mathrm{R}} \subseteq \mathrm{R}$.

Definition 2.4. *A tree of type* α *model is a pair* $\mathrm{M} = (\tau, \mathrm{V})$*, where* τ *is a tree of type* α *and* V *as a valuation, which maps propositional letter to subsets of the set of nodes in* τ.

We think of V(P) as the set of nodes in τ at which the atomic proposition P is true. We can extend this notation to complex formulas in the following definition.

Definition 2.5. *The notion of a formula of eRATL being true in a tree of type* α *model* M=(τ,V) *at a node* t$\in$T *is defined inductively as follows:*

- M, t$\models$P iff t$\in$V(P) for P; •M,t$\models$*true*;
- M, t$\models \neg\varphi$ iff M, t $\not\models \varphi$; •M, t$\models \varphi \wedge \psi$ iff M, t$\models \varphi$ and M, t$\models \psi$;
- M, t$\models$F φ iff $(\exists \mathrm{t}')(\mathrm{t\ R\ t}')$ M, t$' \models \varphi$;
- M, t$\models$Fp φ iff $(\exists \mathrm{t}')(\mathrm{t}'\ \mathrm{R\ t})$ M, t$' \models \varphi$;
- M, t$\models$X φ iff $(\exists \mathrm{t}')(\mathrm{t\ Rn\ t}')$ M, t$' \models \varphi$;
- M, t$\models$Xp φ iff $(\exists \mathrm{t}')(\mathrm{t}'\ \mathrm{Rn\ t})$ M, t$' \models \varphi$;
- M, t$\models$Fx φ iff $(\exists \mathrm{t}')(\mathrm{t}\ R_T\ \mathrm{t}')$ M, t$' \models \varphi$;
- M, t$\models$Xh φ iff $(\exists \mathrm{t}')(\mathrm{t}R_T^{-1}\mathrm{t}'$ and t is not the root of τ) M, t$'\models \varphi$;
- M, t$\models$Fh φ iff $(\exists \mathrm{t}')(\mathrm{t}\ (R_T^{-1})^{+}\mathrm{t}')$ M, t$' \models \varphi$;

Definition 2.6. *The dual operators of our language are defined as follows:*

false=$\neg$*true*	$\varphi \vee \psi = \neg(\neg\varphi \wedge \neg\psi)$	Gh $\varphi = \neg$Fh $\neg\varphi$
Gp $\varphi = \neg$Fp $\neg\varphi$	G $\varphi = \neg$F $\neg\varphi$	Gx $\varphi = \neg$Fx $\neg\varphi$

Definition 2.7.
(1) *A formula φ has a tree of type α model* $\mathrm{M} = (\tau, \mathrm{V})$ *iff there is a node* t *in τ such that* M, t $\models \varphi$ *holds. A formula is called satisfiable if it has a tree of type α model.*

(2) *A formula φ is called valid iff it is satisfied at every node in every tree of type α model.*

The axioms of eRATL are:

(K1) X (A→B)→(X A→X B)	(K2) Xp (A→B)→(Xp A→Xp B)
(K3) Xh (A→B)→(Xh A→Xh B)	(K4) G (A→B)→(G A→G B)
(K5) Gp (A→B)→(Gp A→Gp B)	(K6) Gh (A→B)→(Gh A→Gh B)
(K7) Gx (A→B)→(Gx A→Gx B)	(D1) G A↔ ¬F ¬A
(D2) Gp A↔ ¬Fp ¬A	(D3) Gh A↔ ¬Fh ¬A
(D4) Gx A↔ ¬Fx ¬A	(B1) A →GFp A
(B2) A →GpF A	(B3) (A∧X *true*)→XXp A
(B4) (A∧Xp *true*)→XpX A	(B5) A →GxFh A
(B6) (A∧Xh *true*)→XhFx A	(41) G A→GG A
(42) GG A→(G A∨X *true*)	(43) G A→GG ¬Xp ¬ A
(X1) X A↔(¬X ¬A∧X *true*)	(X2) Xp A↔(¬Xp ¬A∧Xp *true*)
(X3) Xh A↔(¬Xh ¬A∧Xh *true*)	(FX1) F A→XhFx A
(FX2) Xh FxA→(A∨Fp A∨FA)	(FX3) (F∧X *true*)↔(XA ∨XFA)
(FX4) (Fp A∧Xp *true*)↔(Xp A ∨XpFp A)	(FX5) Fh A↔(Xh A ∨XhFh A)

The rules of eRATL are: MP, N, and US, and all propositional tautologies.

Theorem 2.8. *If* Γ *is a* G*-operator then* $\Gamma^{\mathrm{n}}(\mathrm{A} \wedge \mathrm{B}) \leftrightarrow (\Gamma^{\mathrm{n}}\mathrm{A} \wedge \Gamma^{\mathrm{n}}\mathrm{B})$ *is a theorem, where* A *and* B *are formulas and* $n \in \mathrm{N}$.

Proof.

(A∧B)→A	propositional tautology	(1)
G(A∧B)→G A	(1)+N+K4	(2)
G(A∧B)→G B	as (1), (2)	(3)
A→(B→(A∧B))	propositional tautology	(4)
(G A∧G B)→G A∧B))	(4)+N+K4	(5)
G(A∧B)↔(G A∧G B)	(2), (3), (5)	(6)

In our proof we repeat the application of N and K5. N with respect to an iterated G-operator is a valid rule in our proof system. The distribution laws have been formulated only for two propositions A and B. It is easy this law be generalized to any number of propositions: $\mathrm{Gp}(\mathrm{A}_1 \wedge ... \wedge \mathrm{A}_\mathrm{n}) \leftrightarrow (\mathrm{GpA}_1 \wedge \cdots \wedge \mathrm{A}_\mathrm{n})$ is a theorem, if $\mathrm{A}_1, \ldots, \mathrm{A}_\mathrm{n}$ are any finite number of formulas. This result can be obtain by requiring $n-1$ instance of the derived theorem (10). □

Theorem 2.9. *The formula* $\vdash \mathrm{FhA} \leftrightarrow (\mathrm{XhA} \vee \cdots \vee \mathrm{Xh}^{\mathrm{n}}\mathrm{A} \vee \mathrm{Xh}^{\mathrm{n}}\mathrm{FhA})$ *is theorem for all* $n \in \mathrm{N}$.

Proof. The formula can be proved by induction over index n.

n=1 Fh A$\leftrightarrow$(Xh A$\vee$XhFh A)	FX5	(1)
Fh A$\leftrightarrow$(Xh A$\vee$...$\vee$Xh^n A$\vee$Xh^n Fh A)	ind. hypothesis	(2)
Xh^{n}Fh A$\leftrightarrow$(Xh^n (Xh A$\vee$XhFh A)	(1)	(3)
Xh^{n}Fh A$\leftrightarrow$(Xh$^{n+1}\vee$Xh^{n+1}Fh A)	(3)	(4)
Fh A$\leftrightarrow$(Xh A$\vee$...$\vee$Xh$^n\vee$Xh^{n+1} A$\vee$Xh^{n+1}Fh A)	(2), (4)	(5) $\square$

3. Maximally consistent sets.

Definition 3.1. *A consistent set of formulas Δ is called maximally consistent iff every proper extension of Δ is inconsistent.*

The Definition 3.1 is not the only way of defining maximally consistent sets of formulas. Hughes and Cresswell [1] define maximally consistent sets as consistent sets containing either φ or $\neg\varphi$ for every formula φ. The following two lemmas are standard results in modal logic.

Lemma 3.2. [1] *Let Δ be a maximally consistent set and let φ, φ_1, and φ_2 be formulas. Then the following statements are true:*

- if $\vdash \varphi$ then $\varphi \in \Delta$;
- $\neg\varphi \in \Delta$ iff $\varphi \notin \Delta$;
- $\varphi_1 \wedge \varphi_2 \in \Delta$ iff $\varphi_1 \in \Delta$ *and* $\varphi_2 \in \Delta$;
- $\varphi_1 \vee \varphi_2 \in \Delta$ iff $\varphi_1 \in \Delta$ *or* $\varphi_2 \in \Delta$;
- if $\varphi_1 \in \Delta$ *and* $\varphi_1 \to \varphi_2 \in \Delta$ *then also* $\varphi_2 \in \Delta$;

Lemma 3.3. (Lindenbaum's Lemma) *Every consistent set of formulas can be extended to a maximally consistent set.*

Definition 3.4. *Let Δ_1 and Δ_2 be maximally consistent sets. The canonical child relation $\perp$ over maximally consistent sets is defined as follows: $\Delta_1 \perp \Delta_2$ iff Gx $\varphi \in \Delta_1$ implies $\varphi \in \Delta_2$ for all formulas φ.*

We refer to vertical accessibility relation over maximally consistent sets of formulas as the child relation. The next theorem shows some consequence of Definition 3.4.

Theorem 3.5. *Let Δ_1 and Δ_2 be maximally consistent sets and let φ be a formula, then:*

- *$\varphi \in \Delta_1$ and $\Delta_1 \perp \Delta_2$ imply Xh $\varphi \in \Delta_2$;*
- *$\varphi \in \Delta_1$ and $\Delta_1 \perp^+ \Delta_2$ imply Fh $\varphi \in \Delta_2$;*
- *$\varphi \in \Delta_2$ and $\Delta_1 \perp \Delta_2$ imply Fx $\varphi \in \Delta_1$.*

Proof. The proof of these statements follows all most directly from the Definition 3.4.

•	$\varphi \in \Delta_1$		(1)
	GxXh $\varphi \in \Delta_1$	(1)+B5	(2)
	Xh $\varphi \in \Delta_2$	from definition of $\perp$	(3)
•	$\varphi \in \Delta_1$		(1)
	$\Delta_1 \perp^+ \Delta_2$		(2)
	$\Delta_1 \perp^n \Delta_2$	$\exists n \in$N + (2)	(3)
	Xh$^n\varphi \in \Delta_2$	from case one of the theorem	(4)
	Fh $\varphi \in \Delta_2$	Theorem 2.9	(5)
•	$\varphi \in \Delta_2$		(1)
	$\neg\varphi \notin \Delta_2$	Δ_2 is max.consistent + (1)	(2)
	Gx $\neg\varphi \notin \Delta_1$		(3)
	$\neg$Gx $\neg\varphi \in \Delta_1$	Δ_1 is max.consistent	(4)
	Fx $\varphi \in \Delta_1$	D4	(5) $\square$

4. Vertical accessability relations. Let Δ_1 and Δ_2 be two maximally consistent sets with $\Delta_1 \perp \Delta_2$.

Theorem 4.1. *Let Δ_1 be a maximally consistent set. If $Fx\ \varphi \in \Delta_1$ then there exists a maximally consistent set Δ_2 such that $\Delta_1 \perp \Delta_2$ and $\varphi \in \Delta_2$.*

Proof. Let first define a set $\Delta_3 = \{\varphi\} \cup \{\psi \mid \text{Gx}\ \psi \in \Delta_1\}$ that contains all formulas which are bound to be part of any maximally consistent set. Our aim is to find a set Δ_2 with $\varphi \in \Delta_2$, and therefore the formula φ must be contained in any such set. When we have Gx $\psi \in \Delta_1$, Δ_2 will have to contain ψ if we want $\Delta_1 \perp \Delta_2$. Let us assume that Δ_3 is not consistent: $\exists \psi_1, \ldots, \psi_n$ such that Gx $\psi_i \in \Delta_1$, $i = \overline{1, n}$ and $\vdash \neg(\psi_1 \wedge ... \wedge \psi_n \wedge \varphi)$.

$\vdash \neg(\psi_1 \wedge \cdots \wedge \psi_n \wedge \varphi) \equiv \vdash (\psi_1 \wedge \cdots \wedge \psi_n) \to \neg\varphi$		(1)
$\vdash \text{Gx}\ (\psi_1 \wedge \cdots \wedge \psi_n) \to \text{Gx}\ \neg\varphi$	1+N+K7	(2)
$\vdash (\text{Gx}\ \psi_1 \wedge \cdots \wedge \text{Gx}\ \psi_n) \to \text{Gx}\ (\psi_1 \wedge \cdots \wedge \psi_n)$	from Theorem 2.8	(3)
$\vdash (\text{Gx}\ \psi_1 \wedge \cdots \wedge \text{Gx}\ \psi_n) \to \text{Gx}\ \neg\varphi$	(2)+(3)	(4)
$\text{Gx}\ \neg\varphi \in \Delta_1$		(5)
$(5) \equiv \text{Fx}\ \varphi \notin \Delta_1$	D4	(6)

(6) contradicts assumption Fx $\varphi \in \Delta_1$, hence Δ_3 must be a consistent set of formulas. Δ_3 is a consistent set. We can apply Lemma 3.3, and we can extend Δ_3 to a maximally consistent set Δ_2. Δ_3 fulfils requirements by: $\varphi \in \Delta_1$ and $\Delta_1 \perp \Delta_2$. $\square$

Theorem 4.2. *Let Δ_2 be a maximally consistent set. If Xh $\varphi \in \Delta_2$ then there exists a maximally consistent set Δ_1: $\Delta_1 \perp \Delta_2$ and $\varphi \in \Delta_1$.*

Proof. Let first define a set $\Delta_3 = \{\psi \mid \text{Xh}\ \psi \in \Delta_2\}$ and this implies that $\varphi \in \Delta_3$. Let us assume that Δ_3 is not consistent.

$\exists \psi_1, \ldots, \psi_n : \text{Xh}\ \psi_1, \ldots, \text{Xh}\ \psi_n \in \Delta_2$ and $\vdash \neg(\psi_1 \wedge \cdots \wedge \psi_n)$		(1)
$\text{Xh}\ (\psi_1 \wedge \cdots \wedge \psi_n) \in \Delta_2$	Theorem 2.8	(2)
$\vdash \neg\text{Xh}\ \neg\neg(\psi_1 \wedge \cdots \wedge \psi_n)$	N+(1)	(3)

(3) $\equiv \vdash \neg\text{Xh}\ (\psi_1 \wedge \cdots \wedge \psi_n)$ contradicts $\Delta_2 \not\ni \neg\text{Xh}\ (\psi_1 \wedge \cdots \wedge \psi_n)$. We have proved that $\varphi \in \Delta_1$. Let Δ_1 be a maximally consistent set with $\Delta_3 \subseteq \Delta_1$. Let us assume that $\Delta_1 \not\perp \Delta_2 : \exists \xi :$ Gx $\xi \in \Delta_1$ but $\xi \notin \Delta_2$.

$\neg\xi \in \Delta_2$	Δ_2 is maximally consistent	(4)
Xh *true* $\in \Delta_2$	from Theorem 3.5	(5)
$\text{Fx}\ \neg\xi \in \Delta_2$	(B6) + (4) + construction of Δ_3	(6)
$\text{Fx}\ \neg\xi \in \Delta_1$	$\Delta_3 \subseteq \Delta_1$	(7)
$\text{Gx}\ \xi \notin \Delta_1$	(D4)	(8)

(8) contradicts assumption Gx $\xi \in \Delta_1 \Rightarrow \Delta_1 \perp \Delta_2$. $\square$

Now we have to formalize the notion of a set of formulas having a level.

Definition 4.3. *Let Δ be a maximally consistent set and let $n \in \mathrm{N}_0$. The set Δ has level n iff* $\text{Xh}^{\text{n}}\text{Gh}$ *false* $\in \Delta$. *A formula has a level iff it is a conjunction of the form* $\varphi \wedge \text{Xh}_{\text{n}}\text{Gh}$ *false*, $n \in \mathrm{N}_0$.

Theorem 4.4. *Let Δ_2 be a maximally consistent set with level. If* $\text{Fh}\varphi \in \Delta_2$ *then there exists a maximally consistent set $\Delta_1 : \Delta_1 \perp^* \Delta_2$ and $\varphi \in \Delta_1$.*

Proof. Let Δ_2 has n level.

$\mathrm{Xh}_n\mathrm{Gh}\ \textit{false} \in \Delta_2,\ n \in \mathrm{N}_0$ (1)
$\mathrm{Fh}\ \varphi \in \Delta_2$ (2)
$\mathrm{Fh}\ \varphi \leftrightarrow (\mathrm{Xh}\ \varphi \vee \cdots \vee \mathrm{Xh}^n\varphi \vee \mathrm{Xh}^n\mathrm{Fh}\ \varphi)$ from Theorem 2.9 (3)
$\mathrm{Xh}\ \varphi \vee \cdots \vee \mathrm{Xh}^n\varphi \in \Delta_2$ and $\mathrm{Xh}^n\mathrm{Fh}\ \varphi \in \Delta_2$ (3) (4)

We will prove that (4) is not possible. Let us assume that $\mathrm{Xh}^{\mathrm{n}}\mathrm{Fh}\varphi \in \Delta_2$

$\mathrm{Xh}^n(\mathrm{Fh}\ \varphi \wedge \mathrm{Gh}\ \textit{false}) \in \Delta_2$ (1) + (4) (5)
$\mathrm{Fh}\ \varphi \wedge \mathrm{Gh}\ \textit{false}$ is inconsistent (6)
$\neg\mathrm{Xh}_n(\mathrm{Fh}\ \varphi \wedge \mathrm{Gh}\ \textit{false})$ is a theorem (6) + N (7)
(5)+(7) contradict our assumption (8)
$\exists m = \overline{1,n} : \mathrm{Xh}_m\varphi \in \Delta_2$ (4)+(8) (9)

By Theorem 4.2 $\exists\ \Delta_3, \ldots, \Delta_{m+2} : \Delta_{m+2} \perp \Delta_{m+1} \perp \cdots \Delta_3 \perp \Delta_2$ and $\mathrm{Xh}_{m+2-i}\varphi \in \Delta_i,\ i = \overline{3, m+2} \Rightarrow \varphi \in \Delta_{m+2}$. If we define $\Delta_1 = \Delta_{m+2} \Rightarrow \Delta_1 \perp^m \Delta_2, \Delta_1 \perp^+ \Delta_2$, and $\varphi \in \Delta_1$. □

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ВЕРТИКАЛНИ МОДАЛНОСТИ И СЪЩЕСТВУВАНЕ НА СИНОВЕ, РОДИТЕЛ И ПРЕДШЕСТВЕНИЦИ В ЛОГИКАТА eRATL

Ирена Л. Атанасова

Нека Δ_1 и Δ_2 да бъдат две пълни множества, и $\perp$ да бъде канонична релация над пълното множество. Ще докажем, че за всяка формула $\varphi \in \Delta_2$ формулата $\mathrm{Fx}\varphi \in \Delta_1$ (аналогично за Xh и Fh). Следващата ни цел е да докажем, че всеки път, когато $\mathrm{Fx}\varphi \in \Delta_1$, то съществува пълно множество от формули Δ_2 такова, че $\varphi \in \Delta_2$. Ще докажем подобни твърдения и за вертикалните модалности Fh и Xh.