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SOME SELF-AFFINE SETS IN THE EUCLIDEAN PLANE AND THEIR FRACTAL DIMENSIONS*

Radostina P. Encheva, Georgi H. Georgiev

We discuss the box and the Hausdorff dimensions of two types of self-affine sets in the Euclidean plane. The first type is the class of self-affine curves generated by self-affine zippers. The second one is a total disconnected fractal obtained by two contracting affinities. These affinities are related to every obtuse triangle.

1. Introduction. An affine transformation of the Euclidean plane $\mathbb{R}^2$ is a bijective mapping $A : \mathbb{R}^2 \to \mathbb{R}^2$ which preserves the set of all lines in $\mathbb{R}^2$ and the affine ratio of every three collinear points. All affine transformation of $\mathbb{R}^2$ form a group with respect to a composition of mappings. An affine transformation $A : \mathbb{R}^2 \to \mathbb{R}^2$ is a contraction if $|A(z_1) - A(z_2)| \leq |z_1 - z_2|$ for any $z_1, z_2 \in \mathbb{R}^2$ and there exist $z_1^0, z_2^0 \in \mathbb{R}^2$ for which $|A(z_1^0) - A(z_2^0)| < |z_1^0 - z_2^0|$. Every such transformation is called a contracting affine transformation or, shortly, a contracting affinity. A product of a contracting affinity and an Euclidean motion is also a contracting affinity. A scaling transformation of $\mathbb{R}^2$ is an affine transformation given by the equalities

$$x' = ax, \quad y' = by, \tag{1}$$

where a and b are nonzero real constants. The scaling transformation defined by (1) is a contracting if and only if $0 < a^2 \leq 1$, $0 < b^2 \leq 1$ and $a^2 + b^2 < 2$. In this paper we consider contracting affinities as compositions of contracting scaling transformations, rotations and translations. A finite family of contracting affinities of $\mathbb{R}^2$ $\{A_1, \ldots, A_m\}$ with $m \geq 2$ is a particular case of an iterated function system or, shortly, IFS. For such IFS, there is an invariant set F (or an attractor F) with the properties: $F \subset \mathbb{R}^2$ is a non-empty compact subset, and $F = \bigcup_{i=1}^{m} A_i(F)$. The attractor F is called a self-affine set when all A_i are contracting affinities.

In the next section we study the box dimension and the Hausdorff dimension of some self-affine curves which are generated by special iterated function systems called self-affine zippers. In Section 3, we calculate the Hausdorff dimension of a total disconnected fractal. This fractal is an attractor of IFS defined by two contracting affinities. Every obtuse triangle determines such pair of contracting affinities.

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2. Box and Hausdorff dimension of some self-affine zippers in $\mathbb{R}^2$. Let $\mathcal{A} = \{A_1, A_2, \ldots, A_m\}$ be a finite set of contracting affine mappings of $\mathbb{R}^2$ into itself. If there are $m+1$ different points in $\mathbb{R}^2$ $p_0, p_1, \ldots, p_m$ such that $A_i(p_0) = p_{i-1}$ and $A_i(p_m) = p_i$ for $i = 1, 2, \ldots, m$, then the iterated function system $\mathcal{A}$ is called a self-affine zipper with signature (0,0,...,0). The points $p_0, p_1, \ldots, p_m$ are base points of $\mathcal{A}$. In [1, 2, 3] many properties of the self-similar zippers are proved. From Theorem 1.2 in [1] it follows that the attractor F of the iterated function system (IFS) $\mathcal{A}$ is a Jordan arc with endpoints p_0 and p_m if and only if $A_i(F) \cap A_j(F) = \emptyset$ for $|i-j| > 1$ and $Card(A_i(F) \cap A_j(F)) = 1$ for $|i-j| = 1$.

In this section we deal with Hausdorff and Box dimension of the set $F = \bigcup_{i=1}^{m} A_i(F)$ which is, in general, a fractal.

Theorem 1. *Let $\mathcal{A} = \{A_1, A_2, \ldots, A_m\}$ be a self-affine zipper in $\mathbb{R}^2$. If the linear part of A_i is represented by the matrix $T_i = \begin{pmatrix} a_i & 0 \\ 0 & c_i \end{pmatrix}$, where $0 < c_i < 1$ for $i = 1, 2, \ldots, m$, $\sum_{i=1}^m a_i = 1$ and F is the attractor of $\mathcal{A}$, then*

$$\dim_B F = 1 + \frac{\log(c_1 + \cdots + c_m)}{\log m}.$$

Proof. According to Lemma 1.1 in [1], there exists a continuous mapping $\gamma : [0,1] \to F$ with the following property: for a certain real function $f : [0,1] \to \mathbb{R}$ it is fulfilled

$$F = \operatorname{graph} f = (x, f(x)) = \gamma(x).$$

We denote by $\mathbf{i} = (i_1, i_2, \ldots, i_m) \in \{1, 2, \ldots, m\}^k$ for $k \geq 1$, a k-term sequence with $1 \leq i_j \leq m$. The part of F over the interval $I_{\mathbf{i}}$ of the x-axis is the affine image $A_{i_1} \circ A_{i_2} \circ \cdots \circ A_{i_k}(F)$. We have that $T_{i_1} \circ T_{i_2} \circ \cdots \circ T_{i_k} = \begin{pmatrix} a_{i_1} a_{i_2} \ldots a_{i_k} & 0 \\ 0 & c_{i_1} c_{i_2} \ldots c_{i_k} \end{pmatrix}$. Hence, $T_{i_1} \circ T_{i_2} \circ \cdots \circ T_{i_k}(F)$ is contained in a rectangle of height $hc_{i_1} c_{i_2} \ldots c_{i_k}$, where h is the height of F. Since $A_{i_1} \circ A_{i_2} \circ \cdots \circ A_{i_k}(F)$ is an image of $T_{i_1} \circ T_{i_2} \circ \cdots \circ T_{i_k}(F)$ under a translation, the height of $A_{i_1} \circ A_{i_2} \circ \cdots \circ A_{i_k}(F)$ is the same. If q_1, q_2, q_3 are three non-collinear points chosen from $p_0, p_1, \ldots, p_m$, then $T_{i_1} \circ T_{i_2} \circ \cdots \circ T_{i_k}(F)$ contains the points $T_{i_1} \circ T_{i_2} \circ \cdots \circ T_{i_k}(q_j)$, $j = 1, 2, 3$. So that, the height of the obtained triangle is at least $dc_{i_1} c_{i_2} \ldots c_{i_k}$, where d is the vertical distance from q_2 to the segment $[q_1, q_3]$. We denote by $R_f[I_{\mathbf{i}}] = \sup_{x_1, x_2 \in I_{\mathbf{i}}} |f(x_1) - f(x_2)|$ the range of the function f over $I_{\mathbf{i}}$. Thus,

$$dc_{i_1} c_{i_2} \ldots c_{i_k} \leq R_f[I_{\mathbf{i}}] \leq hc_{i_1} c_{i_2} \ldots c_{i_k}.$$

Summing over all m^k intervals $I_{\mathbf{i}}$ for fixed $k \geq 1$ we get

$$d(c_1 + c_2 + \cdots + c_m)^k \leq \sum_{\mathbf{i} \in \{1, \cdots, m\}^k} R_f[I_{\mathbf{i}}] \leq h(c_1 + c_2 + \cdots + c_m)^k \tag{2}$$

From Proposition 11.1 ([4], p. 161) it follows that

$$m^k \sum_{\mathbf{i} \in \{1, \ldots, m\}^k} R_f[I_{\mathbf{i}}] \leq N_{m^{-k}} \leq 2m^k + m^k \sum_{\mathbf{i} \in \{1, \ldots, m\}^k} R_f[I_{\mathbf{i}}],$$

where N_δ is the number of squares of the $\delta-$mesh that intersect $F = \operatorname{graph} f$. Using (2), we obtain

$$m^k d(c_1+c_2+\cdots+c_m)^k \le N_{m^{-k}} \le 2m^k + m^k h(c_1+c_2+\cdots+c_m)^k.$$

Now, applying the equivalent definition (see [4], p.43) of the box dimension, we can conclude that

$$\dim_B F = \lim_{k\to\infty} \frac{\log N_{m^{-k}}}{k\log m} = 1 + \frac{\log(c_1+\cdots+c_m)}{\log m}\cdot$$

Theorem 2. *Let $\mathcal{A} = \{A_1, A_2, \ldots, A_m\}$ be a self-affine zipper in $\mathbb{R}^2$. If the linear part of A_i is represented by the matrix $T_i = \begin{pmatrix} a_i & b_i \\ b_i & a_i \end{pmatrix}$ for $i = 1,2,\ldots,m$, where $\sum_{i=1}^m a_i = 1$, $\sum_{i=1}^m b_i = 0$ and F is the attractor of $\mathcal{A}$, then $\dim_{\mathcal{H}} F = 1$.*

Proof. We have that $F = \mathrm{graph} f = \{(x, f(x)) = \gamma(x),\ f : [0,1] \to \mathbb{R}\}$. If $\mathrm{proj}\, F$ is the orthogonal projection of F over the segment $[0,1]$, then $\dim_{\mathcal{H}} F \ge \dim_{\mathcal{H}} \mathrm{proj}\, F = \dim_{\mathcal{H}}[0,1] = 1$. Let R^{φ} be a rotation of angle φ around the origin. We obtain that $T_i = R^{-\pi/4} \circ T_i' \circ R^{\pi/4}$, where $T_i' = \begin{pmatrix} a_i - b_i & 0 \\ 0 & a_i + b_i \end{pmatrix}$. Since the Hausdorff dimension is an invariant under a rotation, $\dim_{\mathcal{H}} F = \dim_{\mathcal{H}} F'$, where F' is the attractor of the iterated function system $\{A_1', A_2', \ldots, A_m'\}$ with $A_i' = R^{\pi/4} \circ A_i \circ R^{-\pi/4}$, $i = 1,2,\ldots,m$. Applying Theorem 1, we get $\dim_{\mathcal{H}} F = \dim_{\mathcal{H}} F' \le \dim_B F' = 1 + s\dfrac{s\log\sum_{i=1}^m (a_i+b_i)}{\log m} = 1$ and this completes the proof.

Proposition 1. *Let $\triangle p_0p_3p$ be an obtuse triangle in $\mathbb{R}^2$ with an obtuse angle $\sphericalangle p_0pp_3$ and let $p_1 \in [p_0, p]$, $p_2 \in [p_3, p]$ be interior points on the sides of the triangle. There exists a self-affine zipper with base points p_0, p_1, p_2, p_3 and an attractor F_1, such that $dim_{\mathcal{H}} F_1 = 1$.*

Proof. First, we consider the case when $\triangle p_0p_3p$ is an isosceles triangle. Since the Hausdorff dimension is invariant under a similarity, we may put $p_0 = (0,0)$ and $p_3 = (1,0)$. Then, $p = (1/2, a)$, $0 < a < 1/2$. We construct the mappings

$$A_1\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \dfrac{k}{2\lambda} & \dfrac{ka}{\lambda} \\ \dfrac{ka}{\lambda} & \dfrac{k}{2\lambda} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix},$$

$$A_2\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 - \dfrac{l}{2\lambda} - \dfrac{k}{2\lambda} & \dfrac{l-k}{\lambda}a \\ \dfrac{l-k}{\lambda}a & 1 - \dfrac{l}{2\lambda} - \dfrac{k}{2\lambda} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} \dfrac{k}{2\lambda} \\ \dfrac{k}{\lambda}a \end{pmatrix},$$

$$A_3\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \dfrac{l}{2\lambda} & -\dfrac{l}{\lambda}a \\ -\dfrac{l}{\lambda}a & \dfrac{l}{2\lambda} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 1 - \dfrac{l}{2\lambda} \\ \dfrac{l}{\lambda}a \end{pmatrix},$$

where $\lambda = |p_0 - p| = \sqrt{1/4 + a^2}$, $k = |p_0 - p_1|$, $l = |p_3 - p_2|$. Since A_i are contracting affine transformations of $\mathbb{R}^2$, $A_i(p_0) = p_{i-1}$ and $A_i(p_3) = p_i$ for $i = 1,2,3$, the set $\mathcal{A} = \{A_1, A_2, A_3\}$ is a self-affine zipper with base points p_0, p_1, p_2, p_3. The linear part

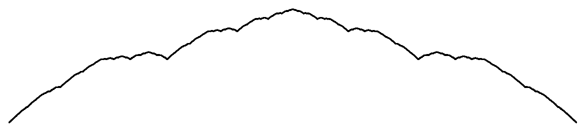

Fig. 1. The attractor F_1 for $p = (0.5, 0.2)$, $k = l = 0.3$

of A_i is represented by the matrix in the form of $T_i = \begin{pmatrix} a_i & b_i \\ b_i & a_i \end{pmatrix}$ for $i = 1, 2, 3$. As $a_1 + a_2 + a_3 = \dfrac{k}{2\lambda} + (1 - \dfrac{l}{2\lambda} - \dfrac{k}{2\lambda}) + \dfrac{l}{2\lambda} = 1$ and $b_1 + b_2 + b_3 = \dfrac{k}{\lambda}a + \dfrac{l-k}{\lambda}a - \dfrac{l}{\lambda}a = 0$, from Theorem 2 it follows that $\dim_{\mathcal{H}} F_1 = 1$, where $F_1 = \bigcup_{i=1}^{3} A_i(F_1)$.

If the triangle $\triangle p_0p_3p$ is not isosceles, then there exists a contracting affinity g of $\mathbb{R}^2$, such that $g(p_0, p_3, p') = p_0, p_3, p$, where $\triangle p_0p_3p'$ is an isosceles obtuse triangle with obtuse angle $\sphericalangle p_0pp_3'$. Thus, g is a Lipshitz transformation, i.e. $|g(p) - g(q)| \leq |p - q|$. Using Corollary 2.4 [4] we get $1 \leq \dim_{\mathcal{H}} F_1 \leq \dim_{\mathcal{H}} g^{-1}(F_1) = 1$. □

The attractor F_1 is plotted in Figure 1 under the conditions $p = (0.5,\ 0.2)$, $k = l = 0.3$.

Let $p_0 = (0, 0)$, $p_1 = (a, b)$, $p_2 = (1, 0)$ be points in the Euclidean plane. Suppose that $a^2 + b^2 < a$. From here $0 < a < 1$ and $|b| < 1/2$. Then, we consider the affine mappings of $\mathbb{R}^2$ defined by

$$A_1 \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a & b/m \\ b & -a/n \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix},$$

(3)

$$A_2 \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1-a & -b/r \\ -b & -(1-a)/l \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix},$$

where $m \geq n > 1$, $r \geq l > 1$ are real numbers.

Proposition 2. *The set $\mathcal{A} = \{A_1, A_2\}$, where A_i, $i = 1, 2$ are affine mappings of $\mathbb{R}^2$ defined by* (3)*, is a self-affine zipper with base points p_0, p_1, p_2. If $F_2 = A_1(F_2) \cup A_2(F_2)$ is the attractor of $\mathcal{A}$, then F_2 is a Jordan arc and $1 \leq \dim_{\mathcal{H}} F_2 \leq s$, where s is the unique solution of the equation $(a^2 + b^2)^{s/2} + ((1-a)^2 + b^2)^{s/2} = 1$.*

Proof. It is clear that $A_1(p_0) = p_0$, $A_2(p_2) = p_2$ and $A_1(p_2) = A_2(p_0) = p_1$. If

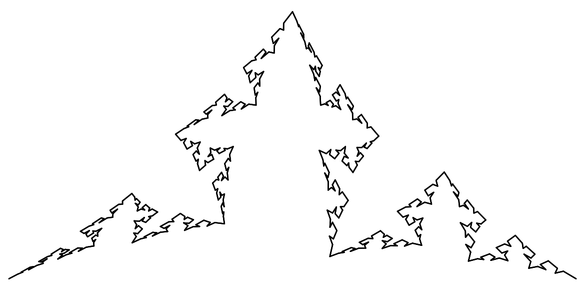

Fig. 2. The attractor F_2 for $p_1 = (0.5,\ 0.47)$, $m = n = 1.7$, $r = l = 1.2$

$p, q \in \mathbb{R}^2$ are any points, then we get

$$|A_1(p) - A_1(q)| \leq c_1|p-q|, |A_2(p) - A_2(q)| \leq c_2|p-q|, \tag{4}$$

where $0 < c_1 = \sqrt{a^2+b^2} < 1$ and $0 < c_2 = \sqrt{(1-a)^2+b^2} < 1$. Thus, A_1 and A_2 are contracting affine transformations of $\mathbb{R}^2$. Consequently, $\mathcal{A} = \{A_1, A_2\}$ is a self-affine zipper with base points p_0, p_1, p_2 and $\dim_{\mathcal{H}} F_2 \geq 1$. From (4) we have that A_1 and A_2 are Lipshitz transformations with Lipshitz constants $c_1 = \sqrt{a^2+b^2}$ and $c_2 = \sqrt{(1-a)^2+b^2}$. Applying Proposition 9.6 ([4], p. 135), we obtain that $\dim_{\mathcal{H}} F_2 \leq s$, where $c_1^s + c_2^s = 1$.

F_2 is a Jordan arc with endpoints p_0 and p_2 if $\mathrm{Card}(A_1(F_2) \cap A_2(F_2)) = 1$. We find that $A_1(p_1) = (a^2 + b^2/m,\, ab(1-1/n))$ and $A_2(p_1) = ((1-a)a - b^2/r + a,\, (1-a)b(1-1/l))$. The inequalities $0 < ab(1-1/n) < b$ and $0 < (1-a)b(1-1/l) < b$ imply that the points $A_1(p_1)$ and $A_2(p_1)$ are inner points in the triangle $\triangle p_0p_1p_2$. Since $a^2+b^2 < a$, $a^2 + b^2/m < a^2+b^2 < a$ and $(1-a)a - b^2/r + a > a$. So that, the triangles $\triangle p_0p_1A_1(p_1)$ and $\triangle p_1p_2A_2(p_1)$ have not overlaps. Continuing in this way we get that the fractal F_2 has no overlaps. This means that $\mathrm{Card}(A_1(F_2) \cap A_2(F_2)) = 1$. □

The attractor F_2 is plotted in Figure 2.

3. Total disconnected affine set as an attractor of IFS. Let $\mathcal{A} = \{A_1, A_2\}$ be an iterated function system of two contracting affine transformations of $\mathbb{R}^2$. If $F = A_1(F) \cup A_2(F)$ is the invariant set of $\mathcal{A}$ and $A_1(F) \cap A_2(F) = \emptyset$, then F is totally disconnected.

In this section we find an explicit formula for the Hausdorff dimension of the totally disconnected fractal F which is obtained by a fixed triangle $\triangle p_0p_1p$. Recently, Kenneth Falconer and Jun Miao [5] gave an explicit formula for the Hausdorff dimension of the attractor F of the IFS $A_i = T_i + v_i$, $i = 1, 2, \ldots, N$, and of affine contractions where T_i are upper triangular matrices and $\|T_i\| < 1/2$ for all $i = 1, \ldots, N$. In this case the Hausdorff dimension of F is expressed by the diagonal entries of the T_i.

Let $p_0 = (0,0)$, $p_1 = (1,0)$ and $p = (a,b)$ be the vertices of the positively-oriented obtuse triangle $\triangle p_0p_1p$ with an obtuse angle $\measuredangle p_1 p\, p_0$. This means that $0 < a < 1$, $0 < b < 1/2$ and $a^2 + b^2 < a$. Let $p_2 \in [p_0, p]$ and $p_3 \in [p_1, p]$ be interior points on the sides of the triangle, i. e. $0 < k = |p_0 - p_2| < |p_0 - p|$ and $0 < l = |p_1 - p_3| < |p_1 - p|$. We consider the affine mappings A_1 and A_2 given by

$$(5)\qquad \begin{gathered} A_1\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \dfrac{ak}{\lambda} & \dfrac{bk}{\lambda} \\ \dfrac{bk}{\lambda} & \dfrac{ak}{\lambda} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}, \\ A_2\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \dfrac{(1-a)l}{\mu} & \dfrac{-bl}{\mu} \\ \dfrac{-b.l}{\mu} & \dfrac{(1-a)l}{\mu} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} \dfrac{1-(1-a)l}{\mu} \\ \dfrac{bl}{\mu} \end{pmatrix}, \end{gathered}$$

where $\lambda = |p_0 - p| = \sqrt{a^2+b^2}$, $\mu = |p_1 - p| = \sqrt{(1-a)^2+b^2}$. Both A_1 and A_2 are contractive and $A_1(p_0) = p_0$, $A_1(p_1) = p_2$, $A_2(p_1) = p_1$, $A_2(p_0) = p_3$.

Proposition 3. *Let A_i, $i = 1, 2$ be the affine mappings defined by* (5) *and F be the attractor of the IFS $\mathcal{A} = \{A_1, A_2\}$. If $\dfrac{k}{\lambda} \leq 1/2$ and $\dfrac{l}{\mu} \leq 1/2$, then* $\dim_{\mathcal{H}} F = \max\{s_1,\, s_2\}$, *where*

$$|(a-b)k/\lambda|^{s_1} + |(1-a+b)l/\mu|^{s_1} = 1$$

and

$$|(a+b)k/\lambda|^{s_2} + |(1-a-b)l/\mu|^{s_2} = 1.$$

Proof. Using the rotations $R^{\pi/4}$ and $R^{-\pi/4}$, we get $A'_1 = R^{\pi/4} \circ A_1 \circ R^{-\pi/4}$ and $A'_2 = R^{\pi/4} \circ A_2 \circ R^{-\pi/4}$, where the linear part of A'_1 and A'_2 are represented by the matrices $T'_1 = \begin{pmatrix} (a-b)k/\lambda & 0 \\ 0 & (a+b)k/\lambda \end{pmatrix}$ and $T'_2 = \begin{pmatrix} (1-a+b).l/\mu & 0 \\ 0 & (1-a-b).l/\mu \end{pmatrix}$, respectively. If F' is the attractor of the IFS $\{A'_1, A'_2\}$, then $\dim_{\mathcal{H}} F = \dim_{\mathcal{H}} F'$. Hence, the statement follows immediately from Corollary 3.2 in [5].

Example 1. Let us consider a concrete case of such attractor F. Suppose that $p = (0.45, 0.4)$, $k = 0.3$, $l = 0.2$. Solving the equations $0.0249136^{s_1} + 0.279382^{s_1} = 1$ and $0.0441129^{s_2} + 0.423532^{s_2} = 1$ we find $s_1 = 0.305999$ and $s_2 = 0.397442$. Hence, $\dim_{\mathcal{H}} F = 0.397442$.

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Radostina P. Encheva, Georgi H. Georgiev
Faculty of Mathematics and Informatics
Shumen University
Universitetska Str., No. 115
9712 Shumen, Bulgaria
e-mail: r.encheva@fmi.shu-bg.net
g.georgiev@fmi.shu-bg.net

НЯКОИ АВТО-АФИННИ МНОЖЕСТВА В ЕВКЛИДОВАТА РАВНИНА И ТЕХНИТЕ ФРАКТАЛНИ РАЗМЕРНОСТИ

Радостина Петрова Енчева, Георги Христов Георгиев

Разглеждаме клетъчната и Хаусдорфовата размерности на два вида авто-афинни множества. Първият вид е класът на авто-афинните криви, генерирани от афинни зипери. Вторият вид е тотално несвързан фрактал, получен чрез два свиващи афинитета. Тези афинитети са породени от произволен тъпоъгълен триъгълник.