МАТЕМАТИКА И МАТЕМАТИЧЕСКО ОБРАЗОВАНИЕ, 2007
MATHEMATICS AND EDUCATION IN MATHEMATICS, 2007
Proceedings of the Thirty Sixth Spring Conference of
the Union of Bulgarian Mathematicians
St. Konstantin & Elena resort, Varna, April 2–6, 2007

A NEW APPROACH TO THE PERTURBATION ANALYSIS FOR DISCRETE $\mathcal{H}_\infty$ SYNTHESIS PROBLEMS*

Andrey S. Yonchev, Mihail M. Konstantinov, Petko P. Petkov

The paper presents a new approach for obtaining local linear perturbation bounds for the discrete $\mathcal{H}_\infty$ synthesis problem based on linear matrix inequalities (LMI). The sensitivity analysis of the perturbed LMI is done by introducing a suitable slightly perturbed right-hand part. This approach leads to tight local linear perturbation bounds for the LMI solutions of the $\mathcal{H}_\infty$ synthesis problem. Numerical example illustrating the theoretical results is presented.

1. Introduction. In many control problems the design constraints have a simple reformulation in terms of linear matrix inequalities (LMI) [1, 6]. The $\mathcal{H}_\infty$ control problem is an example of this in this field. Indeed, the $\mathcal{H}_\infty$ constraints can be expressed as a single matrix inequality *via* the bounded real lemma [4]. It must be stressed that the $\mathcal{H}_\infty$ control problem has a solution in terms of Riccati equations [7], the LMI approach remains valuable.

In this paper we propose a new approach to the local sensitivity analysis of the LMI based $\mathcal{H}_\infty$ synthesis problem by introducing a suitable right-hand part in the corresponding matrix inequalities.

The paper is organized as follows. In Section 2 we shortly present the problem setup and objective. Section 3 describes the performed local linear sensitivity analysis of the LMI based $\mathcal{H}_\infty$ synthesis problem. Section 4 presents a numerical example. Section 5 contains some final remarks.

2. Problem Statement. Consider the discrete–time autonomous linear control system

$$\begin{aligned} x_{k+1} &= Ax_k + B_1 w_k + B_2 u_k, \\ z_k &= C_1 x_k + D_{11} w_k + D_{12} u_k, \\ y_k &= C_2 x_k + D_{21} w_k + D_{22} u_k, \end{aligned} \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the state vector, $w_k \in \mathbb{R}^l$ is the exogenous input vector, $u_k \in \mathbb{R}^m$ is the control input vector, $z_k \in \mathbb{R}^p$ is the error vector, $y_k \in \mathbb{R}^r$ is the measurement vector and A, B_1, B_2, C_1, C_2, D_{11}, D_{12}, D_{21}, D_{22} are constant matrices of compatible size.

*2000 Mathematics Subject Classification: 93B36

Key words: Perturbation analysis, $\mathcal{H}_\infty$ synthesis, LMI based synthesis, Linear systems

The formulation and solution of corresponding $\mathcal{H}_\infty$ control problems are well-known, see e.g. [7].

We consider an LMI approach to solve the $\mathcal{H}_\infty$ synthesis problem, as stated in [2]. More precisely, we are interested in the solution of the LMI [2, 6]

$$(2)\qquad \begin{bmatrix} \mathcal{N}_{12} & \vdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \vdots & I \end{bmatrix}^\top \begin{bmatrix} ARA^\top - R & ARC_1{}^\top & \vdots & B_1 \\ C_1RA^\top & -\gamma I + C_1RC_1{}^\top & \vdots & D_{11} \\ \cdots & \cdots & \cdots & \cdots \\ B_1{}^\top & D_{11}{}^\top & \vdots & -\gamma I \end{bmatrix} \begin{bmatrix} \mathcal{N}_{12} & \vdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \vdots & I \end{bmatrix} < 0,$$

$$(3)\qquad \begin{bmatrix} \mathcal{N}_{21} & \vdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \vdots & I \end{bmatrix}^\top \begin{bmatrix} A^\top SA - S & A^\top SB_1 & \vdots & C_1{}^\top \\ B_1{}^\top SA & -\gamma I + B_1{}^\top SB_1 & \vdots & D_{11}{}^\top \\ \cdots & \cdots & \cdots & \cdots \\ C_1 & D_{11} & \vdots & -\gamma I \end{bmatrix} \begin{bmatrix} \mathcal{N}_{21} & \vdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \vdots & I \end{bmatrix} < 0,$$

$$(4)\qquad \begin{bmatrix} R & I \\ I & S \end{bmatrix} > 0,$$

where $\mathcal{N}_{12}$ and $\mathcal{N}_{21}$ are the orthonormal bases of the null-spaces of $\begin{bmatrix} B_2{}^\top & D_{12}{}^\top \end{bmatrix}$ and $\begin{bmatrix} C_2 & D_{21} \end{bmatrix}$, respectively. Here we assume that the optimal closed–loop performance $\gamma_{\rm opt}$ of the system (1) is already obtained.

The main objective point of the paper is to perform a local linear sensitivity analysis of the LMI system (2)–(3) near the optimal value of γ.

Suppose that the matrices A, B_1, B_2, C_1, C_2, D_{11}, D_{12}, D_{21}, D_{22} are subject to perturbations ΔA, ΔB_1, ΔB_2, ΔC_1, ΔC_2, ΔD_{11}, ΔD_{12}, ΔD_{21}, ΔD_{22} and assume that these perturbations do not change the sign of the LMI (2)–(3).

3. Linear Sensitivity Analysis. First, we perform a sensitivity analysis of the LMI (3). The structure of this LMI allows to consider only the part

$$\begin{aligned} &(\mathcal{N}_{21} + \Delta\mathcal{N}_{21})^\top \\ &* \left\{ \begin{bmatrix} (A+\Delta A)^\top (S^* + \Delta S)(A + \Delta A) - (S^* + \Delta S) & 0 \\ (B_1 + \Delta B_1)^\top (S^* + \Delta S)(A + \Delta A) & 0 \end{bmatrix} \right. \\ &+ \left. \begin{bmatrix} 0 & (A+\Delta A)^\top (S^* + \Delta S)(B_1 + \Delta B_1) \\ 0 & -\gamma_{\rm opt} I - \Delta\gamma_{\rm opt} I + (B_1 + \Delta B_1)^\top (S + \Delta S)(B_1 + \Delta B_1) \end{bmatrix} \right\} \end{aligned}$$

$$(5)\qquad *(\mathcal{N}_{21} + \Delta\mathcal{N}_{21}) = \bar{\mathcal{H}}^* + \Delta\bar{\mathcal{H}}_1 < 0,$$

and to study the effect of the perturbations ΔA, ΔB_1, ΔB_2, ΔC_1, ΔC_2, ΔD_{11}, ΔD_{12}, ΔD_{21}, ΔD_{22} and $\Delta\gamma_{\rm opt}$ on the perturbed LMI solution $S^* + \Delta S$, where S^* and ΔS are the nominal solution of LMI (3) and the perturbation. The essence of our approach is to perform sensitivity analysis of the LMI (3) in a similar manner as for a proper matrix equation after introducing a suitable right–hand part, which is slightly perturbed. The matrix $\bar{\mathcal{H}}^*$ is obtained using the nominal LMI

$$(6)\qquad \mathcal{N}_{21}{}^\top \begin{bmatrix} A^\top S^* A - S^* & A^\top S^* B_1 \\ B_1{}^\top S^* A & -\gamma_{\rm opt} I + B_1{}^\top S^* B_1 \end{bmatrix} \mathcal{N}_{21} = \bar{\mathcal{H}}^* < 0.$$

The matrix $\bar{\mathcal{H}}_1$ is due to the data and closed–loop performance perturbations, the rounding errors during the numerical solution and the sensitivity of the interior–point method used to solve the LMI. It is important to mention that the $(1,2)$, $(2,1)$, $(2,2)$ blocks of the LMI (2) and (3) pose constraints on the size of the perturbations ΔB_1, ΔC_1, ΔD_{11} and $\Delta\gamma_{\mathrm{opt}}$, since the introduced right-hand part matrix must be negatively definite.

The perturbed relation (5) may be written as

$$\mathcal{N}_{21}{}^{\top}\overline{\mathcal{V}}\mathcal{N}_{21}+\mathcal{N}_{21}{}^{\top}\overline{\mathcal{V}}\Delta\mathcal{N}_{21}+\Delta\mathcal{N}_{21}{}^{\top}\overline{\mathcal{V}}\mathcal{N}_{21}+\Delta\mathcal{N}_{21}{}^{\top}\overline{\mathcal{V}}\Delta\mathcal{N}_{21}=\bar{\mathcal{H}}^{*}+\Delta\bar{\mathcal{H}}_1, \tag{7}$$

where

$$\overline{\mathcal{V}}=\begin{bmatrix} A^{\top}S^{*}A-S^{*}+A^{\top}\Delta SA-\Delta S+A^{\top}S^{*}\Delta A+\Delta A^{\top}S^{*}A & 0\\ B_1{}^{\top}S^{*}A+B_1\Delta SA+B_1{}^{\top}S^{*}\Delta A+\Delta B_1{}^{\top}S^{*}A & 0\end{bmatrix}$$
$$+\begin{bmatrix} 0 & A^{\top}S^{*}B_1+A^{\top}\Delta SB_1+A^{\top}S^{*}\Delta B_1+\Delta A^{\top}S^{*}B_1\\ 0 & -\gamma_{\mathrm{opt}}I-\Delta\gamma_{\mathrm{opt}}I+B_1{}^{\top}S^{*}B_1+B_1{}^{\top}\Delta S^{*}B_1+B_1{}^{\top}S^{*}\Delta B_1+\Delta B_1{}^{\top}S^{*}B_1\end{bmatrix}.$$

Here the terms of second and higher order are neglected. Next, we use relation (6) to obtain the expression

$$\begin{aligned}&\mathcal{N}_{21}{}^{\top}\Psi_S\mathcal{N}_{21}+\mathcal{N}_{21}{}^{\top}(\mathcal{H}^{*}+\Psi_S)\Delta\mathcal{N}_{21}+\\ &+\Delta\mathcal{N}_{21}{}^{\top}(\mathcal{H}^{*}+\Psi_S)\mathcal{N}_{21}+\Delta\mathcal{N}_{21}{}^{\top}(\mathcal{H}^{*}+\Psi_S)\Delta\mathcal{N}_{21}=\Delta\bar{\mathcal{H}}_1,\end{aligned}\tag{8}$$

where

$$\bar{\mathcal{H}}^{*}=\mathcal{N}_{21}{}^{\top}\mathcal{H}^{*}\mathcal{N}_{21},\ \Psi_S=\Theta_S+\Lambda_S,\ \Theta_S=\begin{bmatrix} A^{\top}\Delta SA-\Delta S & A^{\top}\Delta SB_1\\ B_1{}^{\top}\Delta SA & B_1{}^{\top}\Delta SB_1\end{bmatrix},$$

$$\Lambda_S=\begin{bmatrix} A^{\top}S^{*}\Delta A+\Delta A^{\top}S^{*}A & A^{\top}S^{*}\Delta B_1+\Delta A^{\top}S^{*}B_1\\ B_1{}^{\top}S^{*}\Delta A+\Delta B_1{}^{\top}S^{*}A & B_1{}^{\top}S^{*}\Delta B_1+\Delta B_1{}^{\top}S^{*}B_1-\Delta\gamma_{\mathrm{opt}}I\end{bmatrix}.$$

Since we are going to obtain linear perturbation bounds for the LMI based $\mathcal{H}_\infty$ synthesis problem, the terms of second and higher order in (8) will be neglected. Hence, (8) yields

$$\mathcal{N}_{21}{}^{\top}\Theta_S\mathcal{N}_{21}+\mathcal{N}_{21}{}^{\top}\Lambda_S\mathcal{N}_{21}+\Delta\mathcal{N}_{21}{}^{\top}\mathcal{H}^{*}\mathcal{N}_{21}+\mathcal{N}_{21}{}^{\top}\mathcal{H}^{*}\Delta\mathcal{N}_{21}=\Delta\bar{\mathcal{H}}_1. \tag{9}$$

We use the setting $\mathcal{H}^{*}\mathcal{N}_{21}=\tilde{\mathcal{N}}_{21}$, $\mathcal{N}_{21}{}^{\top}\mathcal{H}^{*}=\tilde{\mathcal{N}}_{21}^{*}$ $A^{\top}S^{*}=S^{*}_{A^{\top}}$, $S^{*}A=S^{*}_{A}$, $B_1{}^{\top}S^{*}=S^{*}_{B_1{}^{\top}}$, $S^{*}B_1=S^{*}_{B_1}$. In this way relation (9) may be written in a vector form as

$$(\mathcal{N}_{21}{}^{\top}\otimes\mathcal{N}_{21}{}^{\top})\mathrm{vec}(\Theta_S)+(\mathcal{N}_{21}{}^{\top}\otimes\mathcal{N}_{21}{}^{\top})\mathrm{vec}(\Lambda_S)+\mathcal{N}_{S\Theta}\mathrm{vec}(\Delta\mathcal{N}_{21})=\mathrm{vec}(\Delta\bar{\mathcal{H}}_1), \tag{10}$$

where

$$\mathrm{vec}(\Theta_S)=\begin{bmatrix} A^{\top}\otimes A^{\top}-I_{n^2}\\ B_1{}^{\top}\otimes A^{\top}\\ A^{\top}\otimes B_1{}^{\top}\\ B_1{}^{\top}\otimes B_1{}^{\top}\end{bmatrix}\mathrm{vec}(\Delta S)=N\Delta s,$$

$$\mathrm{vec}(\Lambda_S)=\begin{bmatrix}(I\otimes S^{*}_{A^{\top}})+(S^{*}_{A}\otimes I)\Pi_{n^2} & 0 & 0\\ (S^{*}_{B_1}\otimes I)\Pi_{n^2} & (I\otimes S^{*}_{A^{\top}}) & 0\\ (I\otimes S^{*}_{B_1{}^{\top}}) & (S^{*}_{A}\otimes I)\Pi_{l^2} & 0\\ 0 & (I\otimes S^{*}_{B_1{}^{\top}})+(S^{*}_{B_1}\otimes I)\Pi_{l^2} & -e_{l^3}\end{bmatrix}\begin{bmatrix}\mathrm{vec}(\Delta A)\\ \mathrm{vec}(\Delta B_1)\\ \Delta\gamma_{\mathrm{opt}}\end{bmatrix}$$

$$=\begin{bmatrix} N_{\mathrm{t}1} & N_{\mathrm{t}2} & N_{\mathrm{t}3}\end{bmatrix}\Delta_{\gamma ab_1}=N_{\mathrm{t}}\Delta_{\gamma ab_1},\ \mathcal{N}_{S\Theta}=(\tilde{\mathcal{N}}_{21}{}^{\top}\otimes I)\Pi_{(n+l),n^2}+(I\otimes\tilde{\mathcal{N}}_{21}^{*}{}^{\top}).$$

Further, we obtain the expression

$$N_{\rm s}\Delta s + N_{\rm ts1}\mathrm{vec}(\Delta A) + N_{\rm ts2}\mathrm{vec}(\Delta B_1) + N_{\rm ts3}\Delta\gamma + \mathcal{N}_{S\Theta}\mathrm{vec}(\Delta\mathcal{N}_{21}) = \mathrm{vec}(\Delta\bar{\mathcal{H}}_1), \tag{11}$$

where

$$N_{\rm s} = (\mathcal{N}_{21}{}^{\top}\otimes\mathcal{N}_{21}{}^{\top})N,\ N_{\rm ts1} = (\mathcal{N}_{21}{}^{\top}\otimes\mathcal{N}_{21}{}^{\top})N_{\rm t1},$$
$$N_{\rm ts2} = (\mathcal{N}_{21}{}^{\top}\otimes\mathcal{N}_{21}{}^{\top})N_{\rm t2}\ N_{\rm ts3} = (\mathcal{N}_{21}{}^{\top}\otimes\mathcal{N}_{21}{}^{\top})N_{\rm t3}.$$

It is well-known [5] that the perturbation bound for the projector $\mathcal{N}_{21}$ may be written as

$$\|\Delta\mathcal{N}_{21}\|_2 \le \|[C_2,\ D_{21}]^{\dagger}\|_2\|[\Delta C_2,\ \Delta D_{21}]\|_2. \tag{12}$$

Using also the fact that $\|\mathrm{vec}(M)\|_2 = \|M\|_{\mathcal{F}}$, we can finally obtain that the relative perturbation bound for the solution S^* of the LMI (3) has the form

$$\begin{aligned}\frac{\|\Delta s\|_2}{\|S^*\|_{\mathcal{F}}} &\le \frac{1}{\|S^*\|_{\mathcal{F}}}\left(N_{\rm ab1}\frac{\|\Delta A\|_{\mathcal{F}}}{\|A\|_{\mathcal{F}}} + N_{\rm ab2}\frac{\|\Delta B_1\|_{\mathcal{F}}}{\|B_1\|_{\mathcal{F}}} + N_{\rm ab3}\frac{|\Delta\gamma_{\rm opt}|}{|\gamma_{\rm opt}|}\right)\\ &+ \frac{1}{\|S^*\|_{\mathcal{F}}}\left(N_{\rm cd}\frac{\|[\Delta C_2,\ \Delta D_{21}]\|_{\mathcal{F}}}{\|[C_2,\ D_{21}]\|_{\mathcal{F}}} + N_1\frac{\|\Delta\bar{\mathcal{H}}_1\|_{\mathcal{F}}}{\|\bar{\mathcal{H}}^*)|_{\mathcal{F}}}\right),\end{aligned} \tag{13}$$

where

$$\frac{N_{\rm ab1}}{\|S^*\|_{\mathcal{F}}} = \frac{\|N_{\rm s}^{\dagger}\|_2\|N_{\rm ts1}\|_2\|A\|_{\mathcal{F}}}{\|S^*\|_{\mathcal{F}}},\ \frac{N_{\rm ab2}}{\|S^*\|_{\mathcal{F}}} = \frac{\|N_{\rm s}^{\dagger}\|_2\|N_{\rm ts2}\|_2\|\mathrm{vec}(B_1)\|_2}{\|S^*\|_{\mathcal{F}}},$$

$$\frac{N_{\rm ab3}}{\|S^*\|_{\mathcal{F}}} = \frac{\|N_{\rm s}^{\dagger}\|_2\|N_{\rm ts3}\|_2|\gamma|}{\|S^*\|_{\mathcal{F}}},\ \frac{N_1}{\|S^*\|_{\mathcal{F}}} = \frac{\|N_{\rm s}^{\dagger}\|_2\|\bar{\mathcal{H}}^*\|_{\mathcal{F}}}{\|S^*\|_{\mathcal{F}}}$$

$$\frac{N_{\rm cd}}{\|S^*\|_{\mathcal{F}}} = \frac{\|N_{\rm s}^{\dagger}\|_2\|\mathcal{N}_{S\Theta}\|_2\|[C_2,\ D_{21}]^{\dagger}\|_{\mathcal{F}}\|[C_2,\ D_{21}]\|_{\mathcal{F}}}{\|S^*\|_{\mathcal{F}}},$$

may be considered as individual relative condition numbers of the LMI (3) with respect to the perturbations ΔA, ΔB_1, ΔB_2, ΔC_1, ΔC_2, ΔD_{11}, ΔD_{12}, ΔD_{21} and $\Delta\gamma_{\rm opt}$.

In a similar way the relative perturbation bounds for the solution R^* of the LMI (2) may be obtained using the expression

$$\begin{aligned}\frac{\|\Delta r\|_2}{\|R^*\|_{\mathcal{F}}} &\le \frac{1}{\|R^*\|_{\mathcal{F}}}\left(M_{\rm ac1}\frac{\|\Delta A\|_{\mathcal{F}}}{\|A\|_{\mathcal{F}}} + M_{\rm ac2}\frac{\|\Delta C_1\|_{\mathcal{F}}}{\|C_1\|_{\mathcal{F}}} + \|M_{\rm ac3}\|_2\frac{|\Delta\gamma_{\rm opt}|}{|\gamma_{\rm opt}|}\right)\\ &+ \frac{1}{\|R^*\|_{\mathcal{F}}}\left(M_{\rm bd}\frac{\|[\Delta B_2{}^{\top},\ \Delta D_{12}{}^{\top}]\|_{\mathcal{F}}}{\|[B_2{}^{\top},\ D_{12}{}^{\top}]\|_{\mathcal{F}}} + M_1\frac{\|\Delta\bar{\mathcal{E}}_1\|_{\mathcal{F}}}{\|\bar{\mathcal{E}}^*\|_{\mathcal{F}}}\right),\end{aligned} \tag{14}$$

where $\frac{M_{\rm ac1}}{\|R^*\|_{\mathcal{F}}}$, $\frac{M_{\rm ac2}}{\|R^*\|_{\mathcal{F}}}$, $\frac{M_{\rm ac3}}{\|R^*\|_{\mathcal{F}}}$, $\frac{M_1}{\|R^*\|_{\mathcal{F}}}$ and $\frac{M_{\rm bd}}{\|R^*\|_{\mathcal{F}}}$ may be considered as individual relative condition numbers of the LMI (2) with respect to the perturbations ΔA, ΔB_1, ΔB_2, ΔC_1, ΔC_2, ΔD_{11}, ΔD_{12}, ΔD_{21} and $\Delta\gamma_{\rm opt}$. These condition numbers can be obtained in a similar way to $\frac{N_{\rm ab1}}{\|S^*\|_{\mathcal{F}}}$, $\frac{N_{\rm ab2}}{\|S^*\|_{\mathcal{F}}}$, $\frac{N_{\rm ab3}}{\|S^*\|_{\mathcal{F}}}$, $\frac{N_1}{\|S^*\|_{\mathcal{F}}}$ and $\frac{N_{\rm cd}}{\|S^*\|_{\mathcal{F}}}$.

4. Numerical Example. Consider the following system description

$$A_c = \begin{bmatrix} 0 & 1\\ -k/m & -c/m\end{bmatrix},\ B_{1c} = \begin{bmatrix} 0 & 0 & 0\\ -pm & -pc/m & -pk/m\end{bmatrix},\ B_{2c} = \begin{bmatrix} 0\\ 1/m\end{bmatrix},$$

$$C_{1c} = \begin{bmatrix} -k/m & -c/m \\ 0 & c \\ k & 0 \end{bmatrix},\ C_{2c} = \begin{bmatrix} 1 & 0 \end{bmatrix},\ D_{11c} = \begin{bmatrix} -pm & -pc/m & -pk/m \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$D_{12c} = \begin{bmatrix} 1/m \\ 0 \\ 0 \end{bmatrix},\ D_{21c} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

and $m = 3$, $c = 1$, $k = 2$, $pm = 0.4$, $pc = 0.2$, $pk = 0.3$. Here A_c, B_{1c}, B_{2c}, C_{1c}, C_{2c}, D_{11c}, D_{12c}, D_{21c} are the system matrices of a continuous-time system, which for the aim of the analysis is turned into a discrete one using sampling time of 0.01s.

The perturbations in the system matrices of the system are chosen as

$$\Delta A = A \times 10^{-i},\ \Delta B_1 = B_1 \times 10^{-i},\ \Delta B_2 = B_2 \times 10^{-i},$$

$$\Delta C_1 = C_1 \times 10^{-i},\ \Delta C_2 = C_2 \times 10^{-i}, \Delta D_{11} = D_{11} \times 10^{-i},\ \Delta D_{12} = D_{12} \times 10^{-i},$$

$$\Delta\gamma_{\rm opt} = 10^{-i} * \gamma_{\rm opt},\ \Delta\bar{\mathcal{H}}_1 = 10^{-i} * \bar{\mathcal{H}}^*,\ \Delta\bar{\mathcal{E}}_1 = 10^{-i} * \bar{\mathcal{E}}^*.$$

The perturbed solutions $R^* + \Delta R$ and $S^* + \Delta S$ are computed based on the method derived in [2] and using the software [3]. In our experiments we obtain $\gamma_{\rm opt} = 0.4191$. The relative perturbation bounds for the solutions R^* and S^* of the LMIs (2)–(3) are obtained by the linear bounds (14) and (13), respectively.

The results obtained for different values of i are shown in the following table:

i	$\frac{\lVert\Delta s\rVert_2}{\lVert\mathrm{vec}(S^*)\rVert_2}$	Граница (13)	$\frac{\lVert\Delta r\rVert_2}{\lVert\mathrm{vec}(R^*)\rVert_2}$	Граница (14)
8	0.1 10^{-6}	0.5 10^{-5}	0.7 10^{-6}	6.3 10^{-5}
7	0.1 10^{-5}	0.5 10^{-4}	0.7 10^{-5}	6.3 10^{-4}
6	0.1 10^{-4}	0.5 10^{-3}	0.7 10^{-4}	6.3 10^{-3}
5	0.1 10^{-3}	0.5 10^{-2}	0.7 10^{-3}	6.3 10^{-2}
4	0.1 10^{-2}	0.5 10^{-1}	0.7 10^{-2}	6.3 10^{-1}

5. Conclusions. A complete local linear sensitivity analysis of the discrete LMI based on $\mathcal{H}_\infty$ synthesis problem is done. Tight perturbation bounds, which are linear functions of the data perturbations, are obtained for the matrix inequalities determining the problem solution. A numerical example is presented which explicitly reveals the performance and applicability of the proposed approach to analyze the sensitivity of discrete LMI based $\mathcal{H}_\infty$ synthesis problems.

REFERENCES

[1] S. Boyd, L. El GHaoui, F. Feron, V. Balakrishnan. Linear Matrix Inequalities in Systems and Control Theory. *SIAM Philladelphia*, **41**, No 3 (1996), 358–367.

[2] P. Gahinet, P. Apkarian. A Linear Matrix Inequality Approach to H_∞ Control. *Int. J. Robust Non. Contr.*, **4** (1994), 421–448.

[3] P. Gahinet, A. Nemirovski, A. Laub, M. Chilali. LMI Control Toolbox for Use with MATLAB. The MathWorks, Inc., 2000.

[4] I. R. Peterson, B. D. O. Anderson, E. A. Jonkheere. A first principles solution to the non-singular H_∞ control problem. *Int. J. Robust Non. Contr.*, **1** (1991), 171–185.

[5] G. Steward and J. G. Sun, Matrix Perturbation Theory. Academic Press, N.Y., 1990.

[6] A. Yonchev, M. Konstantinov, P. Petkov. Linear Matrix Inequalities in Control Theory. Demetra – Sofia, 2005, ISBN 954-9526-32-1 (in Bulgarian).
[7] K. Zhou, J. C. Doyle, K. Glover. Robust and Optimal Control. PrenticeHall, Upper Saddle River, New Jersey 07458, 1995.

A. S. Yonchev
P. H. Petkov
Department of Systems and Control
Technical University of Sofia
1756 Sofia, Bulgaria
e-mail: ajonchev@mail.bg
php@tu-sofia.bg

M. M. Konstantinov
Department of Mathematics
University of Architecture
Civil Engineering and Geodesy
e-mail: mmk_fte@uacg.bg

ПЕРТУРБАЦИОННИ ГРАНИЦИ ЗА ДИСКРЕТНАТА $\mathcal{H}_\infty$ ЗАДАЧА ЗА СИНТЕЗ

Андрей С. Йончев, Михаил М. Константинов, Петко Х. Петков

В работата са намерени линейни пертурбационни граници за дискретната задача за синтеза, основана на линейни матрични неравенства (ЛМН). Направен е анализ на чувствителността на смутените матрични уравнения след въвеждане на подходящо избрани смутена дясна страна.