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STOCHASTIC PROCESSES IN FINANCE AND INSURANCE*

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We consider an elementary definition of stochastic processes. The basic properties of random walks, Markov processes and martingales are given. As applications we consider the binomial model of financial markets and the basic risk model with an upper bound of ruin probability. The particular case of the classical risk model is given.

1. Introduction. The applications of stochastic processes and martingale methods have attracted much attention in recent years. In this paper we consider some elementary stochastic processes for modelling the basic properties in finance and insurance. The basic processes are given in Section 2. The applications of the martingale methods in finance and insurance risk model are discussed in Section 3 and Section 4.

2. Discrete time stochastic processes. In many cases we need a model for counting some events or to describe data collected from some process in fixed times. These are discrete time processes.

Definition 1. *The sequence of random variables* $X_1, X_2, \ldots$ *with well defined joint distribution, is called a stochastic process in discrete time.*

We write also $\{X_n\},\ n = 1, 2, \ldots$, or simply X_n, $n = 1, 2, \ldots$.

Example 1 (Pólya urn model)**.** *Consider an urn that contains* $b \geq 1$ *black balls and* $w \geq 1$ *white balls. After randomly drawing a ball from the urn it is put back into the urn together with an additional ball of the same color. The process of drawing continues to infinity. Let* $X_n = 1$ *if the* n*th ball is white and* $X_n = 0$ *if the ball is black. Then* $X_1, X_2, \ldots$ *is a stochastic process.*

It is of interest to consider also the partial sum $S_n = X_1 + X_2 + \cdots + X_n,\ \ n = 1, 2, \ldots$ *which is the number of white balls up to the* n*th drawing. The sequence* $S_1, S_2, \ldots$ *is also a stochastic process.*

2.1. Random Walks. Let $X_1, X_2, \ldots$ be a sequence of independent identically distributed random variables distributed as the random variable X with distribution function $F(x)$. If u is a real number we set

$$S_n = u + X_1 + \ldots + X_n, \quad S_0 = u.$$

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Definition 2. *The stochastic process* S_n, $n = 0, 1, 2, \ldots$, *is called a random walk starting at* u.

The random variables $X_1 = S_1 - u$, $X_2 = S_2 - S_1, \ldots$, $X_n = S_n - S_{n-1}, \ldots$, are called the increments of the process S_n.

According to Definition 2, the sequence $S_n = X_1 + X_2 + \cdots + X_n$, $n = 1, 2, \ldots$, is a random walk if and only if the increments are independent identically distributed random variables.

Example 2. *Let* X_n *be the average force of return of some stock between* $n-1$ *and* n, $n = 1, 2, \ldots$. *If* V_n *is the value of the stock at time* n, *then*

$$V_n = e^{X_n} V_{n-1} = e^{S_n} V_0.$$

The sum of returns S_n *is a random walk. If we suppose that the returns* X_n *are normally distributed, then* S_n *has also normal distribution. In this case the distribution of* e^{S_n} *is lognormal.*

2.2. Markov processes. Suppose that the process $S_0, S_1, \ldots$ has the following property. For any n, the joint conditional distribution of $S_n, S_{n+1}, \ldots$ given $S_0, \ldots, S_{n-1}$ is independent of $S_0, \ldots, S_{n-2}$. This property means that the future development of the process depends on the past only by the last value and is called a Markov property. The process S_n is a Markov process. The conditional distribution function is given by

$$F(x, y, n) = P(S_n \leq x | S_0, S_1, \ldots, S_{n-1} = y) = P(S_n \leq x | S_{n-1} = y) \tag{1}$$

and for all x, y, n is independent of $S_0, \ldots, S_{n-2}$.

If the distribution function (1) is independent of n, the Markov process is called *stationary.*

Example 3. *Let* S_n *be a random walk. Then the conditional distribution function is given by*

$$P(S_n \leq x | S_{n-1} = y) = P(S_n - S_{n-1} \leq x - y) = F_X(x - y),$$

where F_X *is the distribution function of the increments* X.

Definition 3. *The function*

$$F(x, y, n) = P(S_n \leq x | S_{n-1} = y)$$

is called the transition function of the Markov process. The functions

$$p(x, y, n) = P(S_n = x | S_{n-1} = y), \quad n = 1, 2, \ldots$$

are called transition probabilities.

2.3. Martingales. The martingales are stochastic processes, determined by the history of the process and suitable for modeling noises and sources of uncertainty if finance and insurance. In this note we use the terminology of [2]. Suppose that $H_1, H_2, \ldots$ are random vectors.

Definition 4. *The process* $S_n, n = 0, 1, \ldots$, *is called a martingale with respect to* $\{H_n\}$ *if for all* n

1) $E|S_n| < \infty$ **(integrability)**;
2) S_n *is a function of* $H_0, H_1, \ldots, H_n$ **(measurability)**;
3) $E(S_{n+1} | H_0, H_1, \ldots, H_n) = S_n$ **(martingale property)**.

The vectors H_n are interpreted as the state of the system at time n. Let denote

$$\mathcal{F}_n = (H_0, H_1, \dots, H_n), \quad n = 0, 1, \dots.$$

$\mathcal{F}_n$ represents the history of the system or the available information up to time n. The martingale property is equivalent to the following

$$E(S_{n+1}|\mathcal{F}_n) = S_n, \quad n = 0, 1, \dots. \tag{2}$$

Note that (2) implies

$$E(S_{n+k}|\mathcal{F}_n) = S_n, \quad n = 0, 1, \dots, \quad k = 1, 2, \dots. \tag{3}$$

Indeed, repeatedly using the basic properties of conditional expectation (2) we have

$$\begin{aligned} E(S_{n+k}|\mathcal{F}_n) &= E(E(S_{n+k}|\mathcal{F}_{n+k-1})|\mathcal{F}_n) = E(S_{n+k-1}|\mathcal{F}_n) \\ &= E(E(S_{n+k-1}|\mathcal{F}_{n+k-2})|\mathcal{F}_n) \\ &\dots \\ &= E(S_{n+1}|\mathcal{F}_n) = S_n. \end{aligned}$$

Taking expectations on both sides of (3) we get

$$ES_n = ES_0, \quad n = 1, 2, \dots. \tag{4}$$

When S_n is interpreted as the gain of the gambler at time n, the condition (2) means that the game is fair. If

$$E(S_{n+1}|\mathcal{F}_n) \geq S_n, \quad n = 0, 1, \dots$$

the game is favorable for the gambler and the process is called a **submartingale.**

Let $\{S_n\}$ be a martingale and let the increments $X_n = S_n - S_{n-1}$ have finite second moments $EX_n^2 < \infty$. Then

$$EX_n = 0, \quad Cov(X_n, X_{n+k}) = 0$$

and consequently

$$Var(X_n) = \sum_{i=0}^{n} Var(X_i).$$

Example 4. *Let* $\{S_n\}$, $S_n = \sum_{i=1}^{n} X_i$, *be a random walk with* $EX = 0$. $\{S_n\}$ *is a martingale with respect to* $\mathcal{F}_n = \mathcal{F}_n^X$, *the history generated by the increments up to time* n, *because*

$$E(S_{n+1}|\mathcal{F}_n) = E(S_n|\mathcal{F}_n) + E(X_{n+1}|\mathcal{F}_n) = S_n + EX_{n+1} = S_n.$$

Example 5. *Let* Y *be a random variable with finite expectation and* $H_1, H_2, \dots$ *random vectors with history* $\{\mathcal{F}_n\}$. *Denote*

$$S_n = E(Y|\mathcal{F}_n), \quad n = 1, 2, \dots, \ S_0 = EY.$$

The process $\{S_n\}$ *is a martingale with respect to* $\{\mathcal{F}_n\}$.

Example 6. *Suppose that* $Y_0, Y_1, \dots$ *is a sequence of random variables such that* Y_n *is a function of* H_n. *Let* r_n *be a function of* $\mathcal{F}_n$ *which is a solution of the equation*

$$E(Y_{n+1}|\mathcal{F}_n) = e^{r_n} Y_n. \tag{5}$$

Denote

$$S_n = \exp\left(-\sum_{i=0}^{n-1} r_i\right) Y_n, \quad n = 1, 2, \ldots, \quad S_0 = Y_0.$$

The process $\{S_n\}$ is a martingale with respect to $\{\mathcal{F}_n\}$. r_n can be interpreted as a force of interest between moments n and $n+1$ given the history $\{\mathcal{F}_n\}$. S_n is the present value of Y_n and is a martingale.

$$\begin{aligned} E(S_{n+1}|\mathcal{F}_n) &= E\left(\exp\left(-\sum_{i=0}^{n} r_i\right) Y_{n+1}|\mathcal{F}_n\right) = \exp\left(-\sum_{i=0}^{n} r_i\right) E\left(Y_{n+1}|\mathcal{F}_n\right) \\ &= \exp\left(-\sum_{i=0}^{n} r_i\right) \exp(r_n) Y_n = \exp\left(-\sum_{i=0}^{n-1} r_i\right) Y_n = S_n. \end{aligned}$$

Example 7. *Let $X_1, X_2, \ldots$ be a sequence of independent identically distributed random variables distributed as X and let $X_0 = 0$. Denote $M(t) = Ee^{tX}$ the moment generating function of X. For a fixed t and finite $M(t)$ we denote*

$$S_n = \frac{e^{tX_n}}{M(t)}, \quad n = 1, 2, \ldots. \tag{6}$$

The process $\{S_n\}$ is a martingale with respect to $\{X_n\}$. Indeed, the conditional expectation given the history $\mathcal{F}_n = (X_0, X_1, \ldots, X_n)$, is

$$\begin{aligned} E(S_n|\mathcal{F}_{n-1}) &= \frac{1}{M(t)} E(e^{tX_n}|\mathcal{F}_{n-1}) = \frac{1}{M(t)} E[e^{t(X_n - X_{n-1})} e^{tX_{n-1}}|\mathcal{F}_{n-1}] \\ &= \frac{e^{tX_{n-1}}}{M(t)} Ee^{t(X_n - X_{n-1})} = S_{n-1}. \end{aligned}$$

The martingale (6) is called an **exponential martingale**. The condition (4) in this case is

$$ES_n = 1, \quad n = 1, 2, \ldots. \tag{7}$$

Example 8. *Let $X_1, X_2, \ldots$ be the sequence defined in Example 7. The process*

$$S_n = \frac{e^{t\sum_{i=1}^{n} X_i}}{[M(t)]^n}, \quad n = 1, 2, \ldots, \quad S_0 = 1 \tag{8}$$

is a martingale with respect to $\{X_n\}$. Let denote

$$Y_n = e^{t\sum_{i=1}^{n} X_i}, \quad n = 0, 1, \ldots, \quad \sum_{1}^{0} = 0.$$

Then following relation

$$Y_{n+1} = e^{X_{n+1}} Y_n, \; n = 0, 1, \ldots$$

is true. For the martingale property we get

$$E(S_{n+1}|\mathcal{F}_n) = \frac{Y_n}{[M(t)]^{n+1}} E(e^{tX_{n+1}}|\mathcal{F}_n) = \frac{Y_n}{[M(t)]^{n+1}} M(t) = S_n.$$

2.4. Submartingales. Note that the process is called a submartingale, if it is integrable, measurable and

$$E(S_{n+1}|\mathcal{F}_n) \geq S_n, \quad n = 0, 1, \dots. \tag{9}$$

If the martingale condition is

$$E(S_{n+1}|\mathcal{F}_n) \leq S_n, \quad n = 0, 1, \dots,$$

the process is called **supermartingale**.

The condition (9) implies the more general condition

$$E(S_{n+k}|\mathcal{F}_n) \geq S_n, \quad n = 0, 1, \dots, \ k = 1, 2, \dots.$$

Taking expectation in (9) we get that $\{ES_n\}$ is a non-decreasing sequence.

Example 9. *Let $\{S_n\}$, $S_n = \sum_{i=1}^{n} X_i$, be a random walk with $EX \geq 0$. $\{S_n\}$ is a submartingale with respect to $\mathcal{F}_n = \mathcal{F}_n^X$, the history generated by the increments up to time n, because*

$$E(S_{n+1}|\mathcal{F}_n) = E(S_n|\mathcal{F}_n) + E(X_{n+1}|\mathcal{F}_n) = S_n + EX_{n+1} \geq S_n.$$

Example 10. *Let $\{S_n\}$ be a martingale with respect to $\{\mathcal{F}_n\}$. Then $\{S_n^2\}$ is a submartingale.*

$$E(S_{n+1}^2|\mathcal{F}_n) = S_n^2 + E(X_{n+1}^2|\mathcal{F}_n) \geq S_n^2,$$

where $X_{n+1} = S_{n+1} - S_n$.

Let $S_0, S_1, \dots, S_n$ be a finite sequence of random variables with submartingale property (9) and $\mathcal{F}_0, \mathcal{F}_1, \dots \mathcal{F}_n$ the corresponding histories.

Theorem 1 (Kolmogorov inequality for positive submartingales)**.** *If $\{S_k, \mathcal{F}_k\}$, $k = 0, 1, \dots, n$ is a submartingale, such that $S_k \geq 0$ for every k, then for $a > 0$*

$$P(\max(S_0, S_1, \dots, S_n) \geq a) \leq \frac{1}{a} ES_n. \tag{10}$$

Proof. Denote $\tau = k < n$ if $S_0 < a, S_1 < a, \dots, S_{k-1} < a, S_k \geq a$ and $\tau = n$ if $S_0 < a, S_1 < a, \dots, S_{n-1} < a$. The event $\{\tau = k\} \in \mathcal{F}_k$, consequently τ is a stopping time relative to $\{\mathcal{F}_k\}$.

Let A be the event $\{\max(S_0, S_1, \dots, S_n) \geq a\}$. Clearly $A \in \mathcal{F}_\tau$ since $A \cap \{\tau = k\} \in \mathcal{F}_k$. Hence

$$ES_n = E(S_n|A)P(A) + E(S_n|\overline{A})(1 - P(A)) \geq E(S_n|A)P(A) \geq aP(A),$$

which is equivalent to (10). □

In the case when $\{S_k\}$ is a martingale with $S_k \geq 0$ it follows that $ES_n = ES_0$. Then (10) implies that for any $a > 0$

$$P(\max(S_0, S_1, \dots, S_n) \geq a) \leq \frac{1}{a} ES_0.$$

3. Martingales in Finance. One of the most elementary market models is the single period binomial model. The reader can find more about this model in [4]. Suppose that the beginning of the period is at time $t = 0$ and the end of the period is at time $t = 1$. There are two securities: one risk-free bond B with interest rate r and a stock S. The market with two securities is called (B, S) market. At time zero the price of the stock is S_0. At time 1 the stock price will be one of the following positive values $(1+u)S_0$ or $(1+d)S_0$, where u denotes "up" and d denotes "down". Assume that the probability of "up" is $p > 0$ and the probability of "down" is $q = 1-p$. A trading strategy for a portfolio is a pair (B_0, γ_0), where B_0 is the money amounts in the bond and γ_0 is the number of shares of the stock at time zero. If we were to buy this portfolio at time zero, it would cost $V(0) = B_0 + \gamma_0 S_0$. At time 1 it would be worth one of the possible values

$$V_u(1) = B_0(1+r) + \gamma_0(1+u)S_0, \quad \text{and} \quad V_d(1) = B_0(1+r) + \gamma_0(1+d)S_0.$$

Definition 5. *The strategy (B_0, γ_0) admits arbitrage opportunity if*

$$V(0) = 0, \quad V(1) \geq 0 \quad \textit{and} \quad P(V(1) > 0) > 0.$$

The market is said to be arbitrage-free if there are not arbitrage opportunity.

Proposition 1. *The market is arbitrage-free if and only if $-1 < d < r < u$.*

Proof. Suppose that $r < d$. In this case $\gamma_0 = 1$ and $B_0 = S(0)$. This is an arbitrage strategy. The case $u \leq r$ is similar. Conversely, if $d < r < u$ and $V(0) = 0$, then

$$V_u(1) = \gamma_0 S_0(u-r) \quad \text{and} \quad V_d(1) = \gamma_0 S_0(d-r),$$

and $V(1)$ can not be non-negative, i.e. there is no arbitrage strategy. □

We assume that the market is arbitrage-free.

An European call (put) option, written on risky security gives its holder the right, but not obligation to buy (sell) a given number of shares of a stock for a fixed price at a future date T. The exercise date T is called maturity date and the price K – an exercise price. The problem of option pricing is to determine what value to assign to the option at a time zero. The writer of the option has to calculate the fair price as the smallest initial investment that would allow him to replicate the value of the option throughout time T. The replication portfolio can be used to hedge the risk inherent in writing the option.

Consider an option with payoff function f, which pays f_u at the upstate and f_d at the downstate at time 1. To determine the price of this option, we construct a portfolio, such that the expected payoff is the same as that of the option.

$$B_0(1+r) + \gamma_0 S_0(1+u) = f_u,$$

$$B_0(1+r) + \gamma_0 S_0(1+d) = f_d.$$

Solving these equations yields

$$B_0 = \frac{(1+u)f_d - (1+d)f_u}{(1+r)(u-d)}, \quad \gamma_0 S_0 = \frac{f_u - f_d}{u-d}. \tag{11}$$

The portfolio consists $\frac{(1+u)f_d - (1+d)f_u}{(1+r)(u-d)}$ units of bonds and $\frac{f_u - f_d}{u-d}$ units of stocks. The price of the option is given by

$$B_0 + \gamma_0 S_0 = \frac{1}{1+r}\left[\frac{r-d}{u-d}f_u + \frac{u-r}{u-d}f_d\right]. \tag{12}$$

Setting $q = \frac{r-d}{u-d} < 1$ and $1-q = \frac{u-r}{u-d}$ it follows that $V(0) = \frac{1}{1+r}[qf_u + (1-q)f_d]$.

It follows from (12) that the price of the option is the expected discounted payoff of the option under the probability measure Q, defined by $\{q, 1-q\}$. The new probability measure Q depends only on the returns of the stock and the bond.

It is easy to see that $q \in (0,1)$ if and only if $d < r < u$.

According the Proposition 1 the (B,S) market is arbitrage-free if and only if the defined probability measure Q exists.

The uncertainty of the (B,S) market is related to the risky asset S. The probability measure P can give some characteristics of S, that are incompatible with B. In order to compare the bond and the stock we need a new probability measure Q, such that the expected return of S relative to Q is equal to the risk-free return. In this reason we have

$$E_Q\left(\frac{S_1}{B_1}\right) = E_Q\left(\frac{S_1}{1+r}\right) = \frac{1}{1+r}E_Q S_1 = S_0, \tag{13}$$

where E_Q denotes the mathematical expectation with respect to Q and $B_1 = 1+r$ is the bond price at time 1. Let the measure Q be defined by the probabilities $\{q, 1-q\}$. According (13)

$$E_Q\left(\frac{S_1}{B_1}\right) = \frac{1}{1+r}[(1+u)q + (1+d)(1-q)]S_0 = S_0.$$

Consequently $(1+u)q + (1+d)(1-q) = 1+r$ and

$$q = \frac{u-r}{r-d}.$$

This measure coincides with the measure Q defined by the arbitrage-free price of the option (12). The new probability measure Q is called a **risk-neutral measure** or **martingale measure.**

Example 11 (European call option)**.** *Assume that the payoff function is* $f = (S_1 - K)^+$ *and*

$$(1+d)S_0 < K \le (1+u)S_0.$$

Then we have $f_u = (1+u)S_0 - K$ *and* $f_d = 0$, *so that* $\gamma_0 S_0 = \dfrac{(1+u)S_0 - K}{u-d}$. *The call option price is given by*

$$V(0) = \frac{1}{1+r}\left[\frac{r-d}{u-d}((1+u)S_0 - K)\right].$$

Differentiation relative to u and d shows that, under the above condition, the call option price is an increasing function of u and a decreasing function of d.

4. Insurance Risk Model. We consider the standard risk model, where the time until first claim and the times between claims $T_1, T_2, \dots$ are independent identically distributed random variables distributed as T. Let $Z_1, Z_2, \dots$ be a sequence of independent identically distributed random variables, distributed as Z, independent of T. Z_i denotes the ith claim amount with mean value $\mu = EZ_1 < \infty$. Let c be the constant insurer's premium income per unit time and $\sum_{i=1}^{n} Z_i$ the aggregate claim amount up to time n, called also loss process. We assume that for each i

$$cET_i > EZ_i$$

(see [1] and [5]).

The surplus of an insurance company at time of nth claim is given by

$$U_n = u + \sum_{i=1}^{n} cT_i - \sum_{i=1}^{n} Z_i, \quad n = 1, 2, \ldots U_0 = u,$$

where u is the initial surplus. The process $X_n = \sum_{i=1}^{n}(cT_i - Z_i)$ is called a risk process. The model could be applied to many non-insurance companies. The first sum in the model represents the incomes. The second sum is the loss process.

The probability of ruin in the infinite horizon case for this risk process is defined as

$$\Psi(u) = P(U_n < 0 \text{ for some } \ n, \quad n = 1, 2, \ldots) \tag{14}$$

We will show how to use martingale inequality to obtain some upper bounds for the ruin probability. Note that

$$S_n = \sum_{i=1}^{n}(Z_i - cT_i)$$

is a random walk. The ruin probability (14) can be written as

$$\Psi(u) = P\left(\bigcup_{n=1}^{\infty}\{S_n > u\}\right), \quad u \geq 0.$$

Theorem 2. *Assume that a constant $R > 0$ satisfies*

$$E\left(e^{RZ}\right) E\left(e^{-RcT}\right) = 1, \tag{15}$$

if the moment generating functions of Z and T exist. Then

$$\Psi(u) \leq e^{-Ru}. \tag{16}$$

Proof. Since $E\left(e^{RZ}\right) E\left(e^{-RcT}\right) = 1$ the process $e^{RS_n} = \prod_{i=1}^{n} e^{R(Z_i - cT_i)}$ is a martingale with $Ee^{RS_n} = 1$ (see Example 8 and [6]). According Kolmogorov inequality we get

$$\begin{aligned}
\Psi(u) &= P\left(\bigcup_{i=1}^{\infty}\{S_i > u\}\right) = P\left(\lim_{n\to\infty}\bigcup_{i=1}^{n}\{S_i > u\}\right) \\
&= \lim_{n\to\infty} P\left(\bigcup_{i=1}^{n}\{S_i > u\}\right) = \lim_{n\to\infty} P\left(\max(S_1, S_2, \ldots, S_n) > u\right) \\
&= \lim_{n\to\infty} P\left(\max(e^{RS_1}, e^{RS_2}, \ldots, e^{RS_n}) > e^{Ru}\right) \\
&\leq \lim_{n\to\infty} e^{-Ru} Ee^{RS_n} = e^{-Ru}.
\end{aligned}$$

□

The condition (15) is known as **Cramér condition.** Inequality (16) is called **Lundberg inequality** and the constant R is the adjustment coefficient or **Lundberg exponent** (see [3]).

Example 12 (Classical Risk Model)**.** *We consider the case of exponentially distributed inter-claim times* $F(t) = 1 - e^{-\lambda t},\ t \geq 0, \lambda > 0$. *This model is called also Cramér–Lundberg risk model. Suppose that the claim sizes are exponentially distributed with parameter* μ, *that is* $F(z) = 1 - e^{-\frac{z}{\mu}},\ z \geq 0, \mu > 0$. *In this case*

$$Ee^{RZ} = \frac{1}{1-\mu R} \quad \textit{and} \quad Ee^{-RcT} = \frac{\lambda}{\lambda + Rc}.$$

The solution of equation (12) *is*

$$R = \frac{c - \lambda\mu}{\mu c},$$

and consequently

$$\Psi(u) \leq e^{-\frac{c-\lambda\mu}{\mu c} u}.$$

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СЛУЧАЙНИ ПРОЦЕСИ ВЪВ ФИНАНСИ И ЗАСТРАХОВАНЕ

Леда Минкова

Разглежда се едно елементарно определение на случаен процес. Дадени са основните свойства на случайно блуждаене, Марковски процеси и мартингали. Като приложения се разглеждат биномен модел на финансов пазар и модел на риск с една горна граница на вероятността за фалит. Разгледан е частния случай на класически модел на риск.