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DISCRETE MONITORED BARRIER OPTIONS BY FINITE DIFFERENCE SCHEMES*

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The paper is devoted to pricing options characterized by discontinuities in the terminal condition. Finite difference schemes are examined to highlight how discontinuities can generate numerical drawbacks such as spurious oscillations. We propose a finite difference scheme that is free of spurious oscillations and satisfies both the positivity requirement and maximum principle, as it is demanded for the financial and diffusive solution of the original Black-Scholes partial differential equation. We explore examples of discrete double barrier knock-out call options and the results are in very good agreement with those in the literature.

1. Introduction. In the market of financial derivatives the most important problem is the so called *option valuation problem*, i.e. to compute a fair value for the option.

In this paper we investigate the use of various finite difference schemes to non-standard option pricing models, characterized by discontinuities in the terminal/boundary conditions. In order to make our analysis concrete we concentrate the attention on a *barrier option with a discrete monitoring clause*, but the presented analysis can easily be extended to many other exotic contracts (digital, supershare, binary and truncated payoff options, callable bonds and so on). For example for a double barrier knock-out call option, the payoff condition is continuous and equal to $(S-K)^+$ but the option expires worthless if before the maturity the asset price has fallen outside the corridor $[l,u]$ at the prefixed monitoring dates: at these dates the option price becomes zero if the asset falls out of corridor. If one of the barriers is touched by the asset price at the prefixed dates then the option is canceled, i.e. it becomes zero, but the holder may be compensated by a rebate payment. In the intermediate periods the Black-Scholes equation is applied over the real positive domain.

In recent years, numerical techniques for solving PDE's have found a large diffusion in Finance, and usually the choice goes towards methods with high order of accuracy (e.g. Crank-Nicolson method), and no attention is paid to how the financial provision of the contract can affect the reliability of the numerical solution. Indeed these schemes are adopted without considering the well-known problems that can arise in presence

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of discontinuities and deteriorate the numerical approximation. A discussion of these problems can be found in the classical books Smith (1985), [9], Tavella et al. (2000), [3], or in Zvan et al. (2000), [6]. The situation is even worse as we consider the Greeks Delta and Gamma.

In Section 2 is discussed the model for *discrete double barrier knock-out options*. In particular we discuss the main drawbacks like spurious oscillations, arising from centered difference discretization of the Black-Scholes PDE. Such numerical instabilities can arise because the finite difference equations do not satisfy the maximum principle that is of critical importance for the corresponding solutions of the differential equations. An important factor for numerical schemes is the condition of positivity of the solution that must be satisfied as a consequence of the financial meaning of the involved PDE. If the discrete approximation allows a negative solution, then numerical instabilities will occur.

In sections 3 we propose a variant of the well-known Crank-Nicolson finite difference schemes that enables to solve accurately the examined PDE. We show:

1. how an accurate choice of the numerical scheme depends strongly on the values of the parameters involved in the PDE;

2. how an accurate scheme is not necessarily the best, as it could require prohibitively small time steps;

In Section 4 we explore examples of discrete double barrier knock-out call options frequently used in literature [1], [5] and [6].

In the conclusion, we give some final remarks about spurious oscillations and convergence remedies in case of non-smooth payoffs in option pricing.

2. The Model. We consider as a model for the movement of the asset price a standard *geometric Brownian motion* diffusion process with constant coefficients r and σ

$$dS/S = rdt + \sigma dW_t. \tag{1}$$

The contract to be priced is a discretely monitored double barrier knock-out call option. If t is the time to expiry T of the contract, $0 \le t \le T$, the price $V(S,t)$ of the option satisfies the Black-Scholes PDE

$$-\frac{\partial V}{\partial t} + rS\,\frac{\partial V}{\partial S} + \frac{1}{2}\,\sigma^2 S^2\,\frac{\partial^2 V}{\partial S^2} - rV = 0 \tag{2}$$

endowed by its initial and boundary conditions

$$V(S,0) = (S-K)^+ 1_{[l,u]}(S) \tag{3}$$

$$V(S,t) \to 0 \ \text{ as } \ S \to 0 \ \text{ or } \ S \to \infty \tag{4}$$

with updating of the initial condition at the monitoring dates $t_i, i = 1, \ldots, F$:

$$V(S,t_i) = V(S,t_i^-) 1_{[l,u]}(S), \qquad 0 = t_0 < t_1 < \ldots < t_F = T \tag{5}$$

where $\mathbf{1}_{[l,u]}(x)$ is the indicator function, i.e. $\mathbf{1}_{[l,u]} = 1$ if $x \in [l,u]$, $\mathbf{1}_{[l,u]} = 0$ if $x \notin [l,u]$. It should be noted that away from the monitoring dates, the option price can move on the positive real axis interval $[0, +\infty)$.

Then the knock-out clause at the monitoring date introduces a discontinuity at the barriers (set at $l = 90$, $u = 110$). In order to give an idea of the numerical problems that can arise, in Figure 2 we present the solution of the above PDE when the Crank-Nicolson scheme is used. We remark the presence of undesired spurious oscillations near the barriers (set at $l = 90$ and $u = 110$ respectively) and near the strike ($K = 100$), where the Delta= $\dfrac{\partial V}{\partial S}$ is discontinuous.

These spikes which remain well localized, don't reflect instability but rather that the *discontinuities* that are periodically produced by the barriers at monitoring dates. The spikes cannot decay fast enough in the Crank-Nicolson solution. Mathematically, such spurious oscillations stem from the combined effect of several factors, such as

1. *positivity* of the solution $V(S,t)$ is not satisfied,
2. discrete version of the *maximum principle* is not guaranteed,
3. and a presence of *negative or complex eigenvalues* in the spectrum of the corresponding matrix, originating the finite difference equation.

Experimentally, the oscillations can be eliminated only by taking very small time steps. In Tavella et al. such a statement is quantified through the introduction of the so called characteristic diffusion time $\tau_d = \dfrac{\Delta S^2}{(\sigma S)^2}$, so that whenever $\Delta t \gg \tau_d$ is used, then an oscillating behavior close to barriers arises, [3].

Experimental evidence shows also that the spurious oscillations at time t, $t + \Delta t$, $t + 2\Delta t, \ldots$, are alternating in time, as a consequence of negative/complex eigenvalues in the corresponding iteration matrix. Indeed, under mild hypotheses the iteration matrix $A = P^{-1}N$ (see Tagliani, [2]) is similar to a diagonal matrix, so that $A = S\Lambda S^{-1}$, with S and Λ eigenvectors and eigenvalues matrices, respectively. The Crank-Nicolson scheme applied to equation (2) leads to the matrix equation $V^n = AV^{n-1} = A^n V^0 = S\Lambda^n S^{-1} V^0$. It is then the presence of negative/complex eigenvalues in Λ which leads to an alternating behaviour of the spurious oscillations. Thus spurious oscillations may disappear if the spectrum of the iteration matrix contains positive eigenvalues only, or negative eigenvalues far from -1. The former condition is feasible (see Theorem 3), whilst the latter remains a qualitative condition.

Then special finite difference schemes will be investigated where

1. the solution is positive;
2. the discrete version of the maximum principle is satisfied;
3. the spectrum contains uniquely positive eigenvalues.

3. Finite Difference Approach. As usual, in the finite difference approximation the S-domain is truncated at the cautelative value $S_{\max}$, sufficiently large such that computed values are not appreciably affected by the upper boundary. The computational domain $[0, S_{\max}] \times [0, T]$ is discretized by a uniform mesh with steps ΔS, Δt. Therefore we obtain the nodes S_j and t_n, where $(S_j = j\Delta S, t_n = n\Delta t)$, $j = 0, \ldots, M$, $n = 0, \ldots, N$ so that $S_{\max} = M\Delta S$ and $T = N\Delta t$.

The choice of a specific numerical scheme is based on its property of convergence. Such a requirement rests on the Lax's equivalence theorem.

The parabolic nature of the Black-Scholes equation ensures that:

1. Being the initial condition $V(S,0) = (S-K)^+ 1_{[l,u]}(S)$ square-integrable the solution is smooth in the sense that $V(S,t) \in C^\infty((t_{i-1} - t_i^-] \times I\!R^+)$, $i = 1, ..., F$. Thus rough initial data give rise to smooth solutions in infinitesimal time.
2. The solution obeys the maximum principle

$$\max_{S\in[0,S_{\max}]} | V(t_1, S) | \geq \max_{S\in[0,S_{\max}]} | V(t_2, S) | \qquad t_1 \leq t_2 \tag{6}$$

This inequality means that the maximum value of $V(S,t)$ should not increase as t

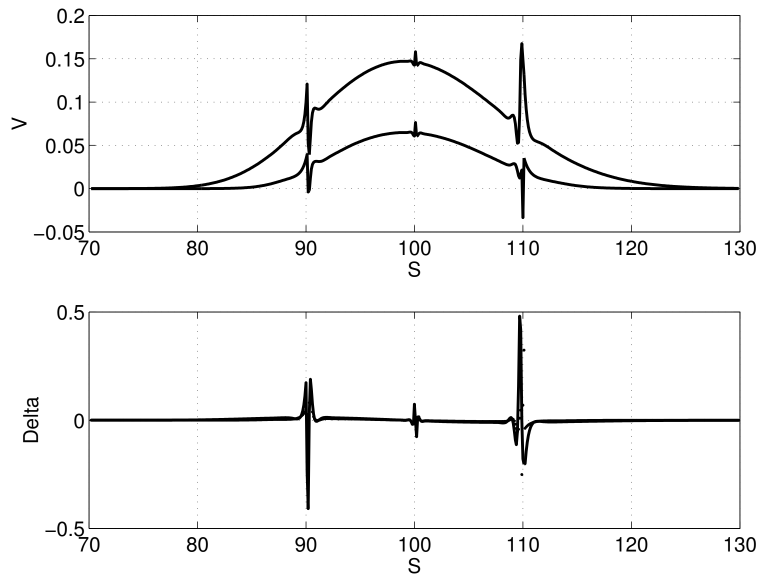

Fig. 1. Option pricing and Delta just before last monitoring date $t_F = T$. The solution is obtained using a Crank-Nicolson scheme with $\Delta S = 0.1$, $\Delta t = 0.01$. Upper and lower curves correspond to $F = 12$ and $F = 48$ monitoring dates respectively, equispaced in $[0, T]$, $T = 1$, $l = 90$, $u = 110$, $r = 0.05$, $\sigma = 0.2$.

increases. It is therefore reasonable to require that the adopted numerical scheme possesses a similar property. Unfortunately, the numerical solution does not always satisfy a corresponding discrete version of the maximum principle, especially in the presence of boundary layers. If that condition is violated then the numerical solution may exhibit *spurious wiggles near sharp gradients.* As a consequence, even though the numerical method converges, it often yields approximate solutions that differ qualitatively from corresponding exact solutions.

3.1. The Crank-Nicolson Scheme and Its Variant. It is known in literature that the stability of the Crank-Nicolson scheme could be explored only in the case $\sigma^2 > r$, but the case $\sigma^2 < r$ remains unsolved, [2].

To overcome this drawback we propose a variant of this scheme that differs from the usual Crank-Nicolson scheme in the discretization of the reaction term $-rV$ in the Black-Scholes equation (2) by six adjacent nodes (see fig. 3) through the following standard procedure

$$V(S,t) = a\left(V_{j-1}^{n} + V_{j+1}^{n}\right) + b\left(V_{j-1}^{n+1} + V_{j+1}^{n+1}\right) + \left(\frac{1}{2} - a - b\right)\left(V_j^{n+1} + V_j^{n}\right)$$

with a discretization error $o(\Delta S^2, \Delta t^2)$ if $a = b$ and $o(\Delta S^2, \Delta t)$ if $a \neq b$.

Here a and b are arbitrary constants to be determined below. In particular, setting $a = b = 0$ we obtain the standard Crank-Nicolson scheme.

The finite difference approximation provides the linear equation

$$PV_{n+1} = NV_n$$

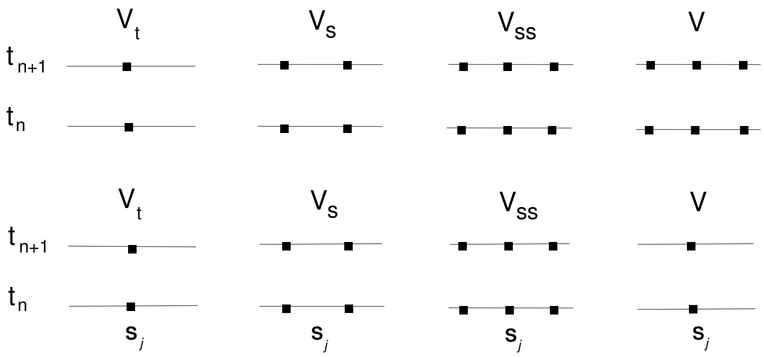

Fig. 2. The upper figure represents involved nodes in the *new variant of the Crank-Nicolson scheme*, the lower figure – the *standard Crank-Nicolson scheme*

with P and N the following tridiagonal matrices:

$$P = \left\{ rb + \frac{r}{4}\frac{S_j}{\Delta S} - \left(\frac{\sigma}{2}\frac{S_j}{\Delta S}\right)^2 ; \frac{1}{\Delta t} + \frac{1}{2}\left(\frac{\sigma S_j}{\Delta S}\right)^2 + r\left(\frac{1}{2} - a - b\right); rb - \frac{r}{4}\frac{S_j}{\Delta S} - \left(\frac{\sigma}{2}\frac{S_j}{\Delta S}\right)^2 \right\}$$

$$N = \left\{ \left(\frac{\sigma}{2}\frac{S_j}{\Delta S}\right)^2 - ra - \frac{r}{4}\frac{S_j}{\Delta S}; \frac{1}{\Delta t} - \frac{1}{2}\left(\frac{\sigma S_j}{\Delta S}\right)^2 - r\left(\frac{1}{2} - a - b\right); \left(\frac{\sigma}{2}\frac{S_j}{\Delta S}\right)^2 - ra + \frac{r}{4}\frac{S_j}{\Delta S} \right\}$$

The constants a and b are chosen according to the following criteria:

- P is an M-matrix (all off-diagonal entries should be negative, Windisch, [7]):

$$rb + \frac{r}{4}\frac{S_j}{\Delta S} - \left(\frac{\sigma}{2}\frac{S_j}{\Delta S}\right)^2 < 0, \ \ \forall S_j, \text{ from which } \quad b < -\frac{r}{16\sigma^2} \tag{7}$$

(as a consequence $rb - \frac{r}{4}\frac{S_j}{\Delta S} - \left(\frac{\sigma}{2}\frac{S_j}{\Delta S}\right)^2 < 0$ holds). Under the condition (7), P is irreducibly diagonally dominant and thus P is an M-matrix, so that $P^{-1} > 0$. In addition, we obtain that $||P^{-1}||_\infty \le \left(\frac{1}{\Delta t} + \frac{r}{2}\right)^{-1}$, (see Windisch, 1989, [7]).

- N has positive entries. Then:

$$\text{a)} \quad -ra - \frac{r}{4}\frac{S_j}{\Delta S} + \left(\frac{\sigma}{2}\frac{S_j}{\Delta S}\right)^2 > 0, \ \ \forall S_j, \text{ from which } a < -\frac{r}{16\sigma^2} \tag{8}$$

$$\text{b)} \quad \frac{1}{\Delta t} - \frac{1}{2}\left(\frac{\sigma S_j}{\Delta S}\right)^2 - r\left(\frac{1}{2} - a - b\right) > 0, \ \ \forall S_j,$$

from which

$$\Delta t < \frac{1}{r\left(\frac{1}{2} - a - b\right) + \frac{1}{2}(\sigma M)^2}. \tag{9}$$

Then $a = b = -\dfrac{r}{16\sigma^2}$ is chosen and such a value is adopted hereinafter. Thus, the scheme has the same accuracy as the standard Crank-Nicolson one.

By combining $N \ge 0$ and $P^{-1} > 0$ then $V^{n+1} = P^{-1}NV^n = (P^{-1}N)^n V^0$ is positive, since $V^0 \ge 0$. Then, under (9) the scheme is *positivity-preserving*.

Under the time step condition (9) we prove that the scheme satisfies the discrete maximum principle (6). Indeed by combining the norms $||N||_\infty = \dfrac{1}{\Delta t} - \dfrac{r}{2}$ and $||P^{-1}||_\infty \leq \left(\dfrac{1}{\Delta t} + \dfrac{r}{2}\right)^{-1}$ we get $||V_{n+1}||_\infty = ||(P^{-1}N)\, V_n||_\infty$ or

$$||V_{n+1}||_\infty \leq ||P^{-1}||_\infty ||N||_\infty ||V_n||_\infty \leq \left(\frac{1}{\Delta t} - \frac{r}{2}\right)\left(\frac{1}{\Delta t} + \frac{r}{2}\right)^{-1} ||V_n||_\infty \leq ||V_n||_\infty.$$

For the eigenvalues of the iteration matrix $P^{-1}N$ the following result holds:

Theorem 3.1. *Under the condition* $\Delta t < \dfrac{1}{r\left(\frac{1}{2} - 4b\right) + (\sigma M)^2}$*, then* $P^{-1}N$ *admits* M *real positive and distinct eigenvalues* $\lambda_i(P^{-1}N)$ *and* $\lambda_i(P^{-1}N) \in (0,1)$.

Proof. The matrices P and N may be written respectively as

$$P = \frac{1}{\Delta t} I + C \quad \text{and} \quad N = \frac{1}{\Delta t} I - C,$$

where C is the following tridiagonal matrix

$$C = \left\{ rb + \frac{r}{4}\frac{S_j}{\Delta S} - \left(\frac{\sigma}{2}\frac{S_j}{\Delta S}\right)^2 ; \frac{1}{2}\left(\frac{\sigma S_j}{\Delta S}\right)^2 + r\left(\frac{1}{2} - a - b\right) ; rb - \frac{r}{4}\frac{S_j}{\Delta S} - \left(\frac{\sigma}{2}\frac{S_j}{\Delta S}\right)^2 \right\}.$$

Then C is similar to a symmetric positive definite matrix C^{spd} (Jacobi matrix), (Ortega, [8]), with $C = D^{-1}C^{spd}D$ and D a diagonal matrix, whose entries are obtained by the off-diagonal entries of C. Thus, the matrix C^{spd}, and then also C, admits M distinct real and positive eigenvalues $\lambda_i(C)$. In addition

$$|\lambda_i(P^{-1}N)| = \left| \frac{1 - \Delta t\, \lambda_i(C)}{1 + \Delta t\, \lambda_i(C)} \right| < 1$$

so that the Crank-Nicolson variant scheme is *unconditionally stable* and then via the Lax-theorem *convergent* with a local truncation error $o(\Delta S^2, \Delta t^2)$.

The condition $\lambda_i(P^{-1}N) > 0$ requires $\Delta t < 1/\rho(C)$, where by the $\rho(\cdot)$ is denoted the spectral radius of the matrix. From the Gerschgorin theorem follows $\lambda_i(C) \in \left[\dfrac{r}{2}; (\sigma M)^2 + r\left(\dfrac{1}{2} - 4b\right)\right]$ and then we have the following condition

$$\Delta t < \frac{1}{r\left(\frac{1}{2} - 4b\right) + (\sigma M)^2} \tag{10}$$

that guarantees $0 < \lambda_i(P^{-1}N) < 1$, $\forall\, i$.

By summarizing, under condition (9) the scheme is positivity-preserving and satisfies a discrete maximum principle, under condition (10) the matrix $P^{-1}N$ has eigenvalues $\lambda_i(P^{-1}N) \in (0,1)$ and the proposed scheme is stable.

Then the absence of spurious oscillations requires restrictions on the time step Δt uniquely while restrictions on the financial parameters r and σ are not required. The method is not so fast as some algorithms in literature, [1, 11].

Remark 3.1. From (10) the absence of spurious oscillations is guaranteed if the following relationship holds $\Delta t < \Delta t_1 =: \dfrac{1}{r\left(\frac{1}{2} - 4b\right) + (\sigma M)^2}$. It is interesting to be observed that for large M values $\Delta t_1 \simeq \dfrac{1}{(\sigma M)^2} =: \tau_d$, i.e. the characteristic grid diffusion time, (see Tavella et al. 2000, p. 189, [3]).

Table 1

Prices of a discrete double knock-out call option monitored 5 times. The current price of the underlying asset is S_0. Parameters $K = 100$, $\sigma = 0.25$, $T = 0.5$, $r = 0.05$, $L = 95$, $U = 110$. The Crank-Nicolson scheme and its variant are both applied with $\Delta S = 0.5$, $\Delta t = 0.00001$, while the Duffy implicit scheme is applied for $\Delta S = 0.05$, $\Delta t = 0.001$, $S_{\max} = 200$

Underl. Asset S_0	Standard Implicit Scheme	Crank-Nicolson Scheme	Crank-Nicolson Variant	Duffy Implicit Scheme	Monte Carlo Method (st. err.) 10^7-asset paths
95	0.16564	0.16561	0.17398	0.17315	–
95.001	0.16904	0.16963	0.17412	0.17395	0.17486 (0.00064)
100	0.22123	0.22122	0.23171	0.23137	0.23263 (0.00036)
109.999	0.15982	0.15989	0.16719	0.16656	0.16732 (0.00062)
110	0.15906	0.15912	0.16703	0.16616	–

4. Numerical Results. We present numerical results for the most explored examples in literature for discrete barrier options that are discretely monitored. There are two examples, respectively when the two barriers are close each other, i.e. $L = 95$ and $U = 110$, as it is explored in [6] and the second case is when $L = 95$ and $U = 140$, [5].

Example 4.1. *Let price a discrete double barrier knock-out call option having a discontinuous payoff defined by conditions* (3)–(5) *and for which the strike price is* 100, *the volatility is* 25% *per annum, the option has six months remaining to maturity, the risk-free rate is* 5% *per annum (compounded continuously), the lower barrier is placed at* 95, *and the upper barrier is imposed at* 110.

We have applied the Crank-Nicolson variant scheme, see Table 4. The results are compared with those obtained by other standard numerical methods in Finance such as the Monte Carlo simulations, [4], the Crank-Nicolson method used in [2], the Duffy exponentially fitted finite difference scheme, [10].

It should be noted that the close distance of each of the barriers to the strike price is not an obstacle for the Crank-Nicolson variant scheme for obtaining a smooth numerical solution. This is one of the frequently met practical problems when finite difference schemes are applied in Finance because usually *oscillations* derive from an inaccurate approximation of the *very sharp gradient produced by the knock-out clause*, generating an error that is damped out very slowly, [2].

It should be noted that the advantage of the Crank-Nicolson variant scheme is that it works successfully both for the cases $\sigma^2 > r$ and $\sigma^2 < r$, i.e. the application of the variant scheme is *independent of the financial parameters* σ and r. Here, it should be remembered that the second case $\sigma^2 < r$ still remains unsolved when the Crank-Nicolson method is applied. In Finance this problem is known as a *low volatility problem*, i.e. when σ takes small values, and Duffy has proposed *exponentially fitted finite difference schemes* for such cases, [10].

Example 4.2. *Let price a discrete double barrier knock-out call option having a discontinuous payoff defined by conditions* (3)–(5) *and for which the strike price is* 100, *the volatility is* 20% *per annum, the option has six months remaining to maturity, the* *risk-free rate is* 10% *per annum (compounded continuously), the lower barrier is placed at* 95*, and the upper barrier is imposed at* 140*.*

In this example, i.e. when the barriers are set at $L = 95$ and $U = 140$, it should be noted that generally when the barriers are far away from the strike price the finite difference schemes give accurate results because the error produced by the knock-out clause is damped out quickly, [2].

The numerical solution of the proposed Crank-Nicolson variant scheme is *smooth*, *free of spurious oscillations*, and satisfies the *maximum principle*, i.e. the solution does not increase as t increases, as it can be seen on Fig. 4 that shows the Black-Scholes surface of a discrete double barrier knock-out call option monitored monthly, i.e. 6 times. It should be noted that $\sigma^2 < r$, i.e. the volatility takes a small value, but the proposed finite difference scheme is unaffected satisfying all the financial requirements of the option contract. In contrast, the standard Crank-Nicolson scheme is not stable and thus not efficient.

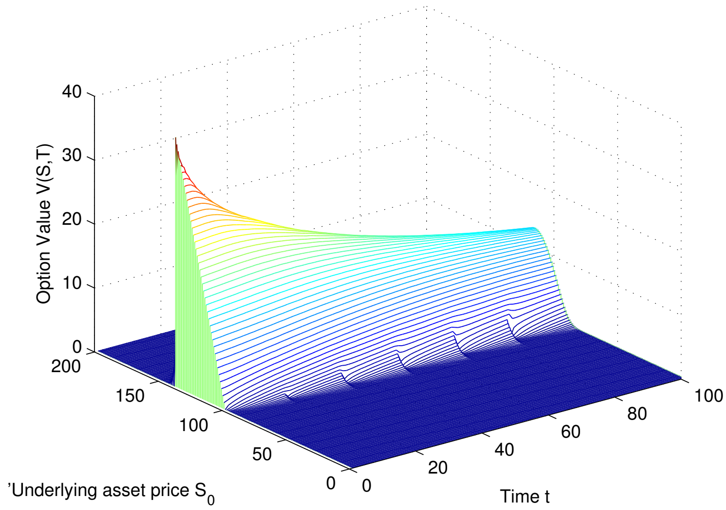

Fig. 3. The Black-Scholes surface of a discrete double barrier knock-out call option monitored monthly (6 times) with parameters $K = 100$, $\sigma = 0.2$, $T = 0.5$, $r = 0.1$, $L = 95$, $U = 140$. The solution is smooth and positive for every t

5. Conclusions. Options with discontinuous payoffs represent extreme cases. Then it is not surprising that more sophisticated discretization techniques are required than those commonly described in financial literature. The proposed scheme in the paper aims to damp fast oscillations more effectively by adjusting the spectrum of eigenvalues of finite difference matrix. The advantage of the new scheme is that it gives highly accurate results, guarantees smooth numerical solution free of undesired spurious oscillations and its application is unaffected of low volatility values. Thus, the proposed scheme guarantees convergence not only in the L^2-norm but also in the supremum norm that is most relevant in Finance.

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ДИСКРЕТНО НАБЛЮДАВАНИ БАРИЕРНИ ОПЦИИ СЪС СХЕМИ НА КРАЙНИ РАЗЛИКИ

Мариян Милев, Алдо Талиани

Статията е посветена на оценяване на опции, характеризиращи се с прекъсвания в крайните условия. Схеми на крайни разлики се изследват, за да се покаже как прекъсванията могат да предизвикат числени недостатъци като изкуствени осцилации. Ние предлагаме схема на крайни разлики без изкуствени осцилации, която удовлетворява условие за положителност и принцип на максимум, които се изискват поради финансовия и дифузионен характер на решението на уравнението на Black-Scholes. Ние изследваме примери за дискретни нокаут опции (discrete knock-out options) с две бариери и резултатите са в много добро съгласие с тези в литературата.