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# A WORKSHOP AT THE SIGMA MATHEMATICAL CLUB AT HRISTO BOTEV SECONDARY SCHOOL

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This article examines the preparation and implementation of a lesson of the mathematical club Sigma at 21st Secondary School "Hr. Botev" – Sofia. The topic is "Progressions", with subtopics "Arithmetic Progression", "Geometric Progression", and "Application of Progressions in Other Sciences". The focus is on the selection of topics and materials, the team work, and the tasks that the individual members of the club must meet. Particular attention is paid to the clear and precise enunciation of the tasks, the criteria for their implementation and the evaluation of the final product. The article also shows the opinion for this extracurricular work of the participants, their parents, and some English teacher also present at the lesson.

Essential for the successful realization of the aims of mathematical education is that students work on their own with mathematical literature, understand it, and be able to prioritize and to extract the most important parts. Another goal is to teach students to present mathematical theories and conceptions in front of an audience. The members of the Sigma Club at 21st Secondary School "Hristo Botev" are high school students with exactly this aim and ambition. The members of this club work on their own over problems given by their teacher with great enthusiasm and accuracy; research mathematical literature; develop their own projects in different areas of mathematics and present them in front of their classmates, members of the club, and others. The best projects are presented at the national sessions of the Students Institute of Mathematics and Informatics.

The preparation of one lesson is a long, complicated, and heavy process. The first thing to be done is the choice of topic. Next comes the determination of the sub-topics and the students that are going to work over. The present lesson of the club takes place in 11th grade. That is why is chosen the topic of "Progressions" with sub-topics" "Arithmetic Progression", "Geometric Progression", and "Application of Progressions in Other Sciences". Each sub-topic is developed by a group of two students, as each group is given exact and clear mathematical problem. The first two groups have to investigate the chosen topics in their books, systematize and classify the types of problems, point out and solve some problems, and propose interesting problems for solution of each type. The task for the third group is to find and systematize the problems demonstrating the usage of progression in other sciences.

The six students who have been members of the club for four years and are very experienced now take the problem with enthusiasm and great passion for work. The first

and the second working groups make the research through their books, some sources of past entrance exams, the Internet, and other specialized literature. The third group discusses the problem with teachers in other sciences in order to find all the areas of application of Progressions. The students solve a lot of mathematical problems in order to classify the various types and to choose the most interesting for the coming presentation. The students also prepare theoretical basis and search for curious facts, and other materials to make the presentations more interesting and catch the attention of the audience.

Following the long-term work, together with the students we assigned and approved indices for evaluation of each project:

**1. Scientific**

- Deeply researched complicated extracurricular material; summary of the matter of instruction and the types of problems; classification; solved difficult problems – 5 points;
- Researched and presented non-sophisticated material at school level; solved easy mathematical problems – 3 points;
- Material on educative level; lack of solved problems, conclusion and summary – 1 point.

**2. Visualization**

- Small amount of text on the slides. The picture and the solution of one and the same mathematical problem are on one slide. The Presenter uses diagrams, pictures, etc. – 5 points;
- Fully used slides; lack of drawings and diagrams; no visualization. The picture and the solution of one and the same mathematical problem are on different slides – 3 points;
- No Power Point Presentation, boards, or other illustrative materials – 1 point.

**3. Presentation**

- The Presenter has good knowledge of the content and presents it in easy and understandable way; catches the audience – 5 points;
- The Presenter has good knowledge of content, but cannot explain some parts of it; the presentation is boring – 3 points;
- The Presenter does not know the content of the research and cannot explain on the presentation – 1 point.

**4. Level of attraction**

- Large quantity of interesting parts, curious facts, pieces of news, etc. – 5 points;
- Some interesting parts, curious facts, pieces of news, etc. – 3 points;
- No quantity of interesting parts, curious facts, or pieces of news – 1 point.

**5. Finalization**

- High quality of the presentation materials – folders, boards, Power Point Presentation – 5 points;
- Medium quality of the presentation materials – folders, boards, Power Point Presentation – 3 points;
- Poor quality of the presentation materials – folders, boards, Power Point Presentation – 1 point.

Very important for the excellent presentation of the students is that they receive the evaluation criteria in advance. Thus, they strive to reach the highest rate of every index.

However, the first drafts always need revision and precision, discussion and argue in order to reach the final successful product that the groups actually present.

**As a result of intensive and hard work, the first group offers the following classification of the types of problems of Arithmetic Progression.**

I. *Mathematical problems using the formula* $a_n = a_1(n-1)d$.
There are 4 parameters: $a_n$, $a_1$, $n$, $d$. If we know 3 of them we could find the 4th with a linear equation.

II. *Mathematical problems using the formula* $S_n = \frac{a_1 + a_n}{2}.n$.
There are 4 parameters $a_n$, $a_1$, $n$, $S_n$ again and we solve linear equation with 3 of them.

III. *Mathematical problems, using the formula* $S_n = \frac{2a_1 + (n-1)\,d}{2}.n$.
There are 4 parameters $S_n$, $a_1$, $d$, $n$ again. In the cases when we are looking for $n$, we solve quadratic equation. In the other cases is solved linear equation.

IV. *Solution of mathematical systems.*

V. *Usage of the properties of Arithmetic Progression.*

VI. *Text mathematical problems which can be solved with the help of Arithmetic Progression.*

VII. *Some exceptional mathematical problems (not necessarily difficult).*

**The second group offers classification of the types of problems of Geometric Progression.**

I. *Mathematical problems using the formula* $a_n = a_1.q^{n-1}$.
Four parameters are used: $a_n$, $a_1$, $q$, $n$. In the cases when we are looking for $q$ and $n$,we solve exponential equations.

II. *Problems using the formula* $S_n = a_1 d\frac{q^n - 1}{q - 1}$.
In the cases when we are looking for $q$ and $n$,we solve exponential equations again. The second group presents the formula's application with the following tale: As the legend says, the ruler of India charmed by the game of chess asked its inventor what award he wanted. The inventor asked to be paid with grains, as for the first square of the chess table to be given one grain, for the second square – two grains and so on, for each following square – two times more. The ruler answered that this request is very modest one and is ready to pay immediately. But did he manage to pay?
In order to find the quantity of grain, we have to calculate the sum of the first 64 terms of the Geometric Progression with a common ratio 2 and a scale factor 1. According to the formula for the sum of Geometric Progression

$$S_{64} = 1\frac{2^{64} - 1}{2 - 1} = 2^{64} - 1 = 18446744073709551615$$

Taking into consideration that 20 cereal grains weigh at average about 1 gram, the inventor of the chess asked approximately 900 billions tones of grain, which is 1500 times more than the quantity of wheat produced in the world now. Obviously the ruler who did not know mathematics strongly underestimated the request.

III. *Solution of mathematical systems.*

IV. *Text problems that are solved with the help of Geometric Progression.*

**The third group offers the following application of progressions in other sciences:**

I. *Application in physics*
   - Free fall
   - Problems containing radioactive decay as an element.

II. *Application in biology*

III. *Application in chemistry*
   For determination of the velocity of chemical reaction, depending on the change of temperature is used the following formula: $V_{t_2} = V_{t_1}.a^{\frac{t_2-t_1}{10}}$, where $2 \leq \ \leq 4$.
   The team proposes the following aspect of the formula: $V_{t_2} = V_{t_1}.(\sqrt[10]{a})^{t_2-t_1}$. If we use $a_1 = V_{t_1}$, $q = \sqrt[10]{a}$, $n = t_2 - t_1$ the result is Geometric Progression.

IV. *Application in geography*
   The group offers as an application the theory of Tomas Malthus, according to which the population of the Earth grows in geometric progression, as in the same time, the resources grow in arithmetical progression. This means that we need wars, epidemic, hunger, and natural disasters to regulate the number of people on the Earth.
   The members of the team point that they do not comment over the social effect of the theory, but rather show the application of progressions in the economic part of Geography.

For each type of the presented classifications, all three groups propose a lot of problems of past university exams, as well as of the school-leaving examination in 2008. A given part of them is solved, and the others are proposed for individual work. Along with the serious mathematical solutions the students also use a lot of examples for more attractive illustration.

The members of the club that do not make presentations are engaged in the evaluation of the projects, according to the above given criteria. Evaluation cards are also given to 10 British teachers in mathematics, who are guests of the meeting of the club. The two groups both ranked first the project of "Application of progressions in other sciences".

Along with the other worth of the project that won, the reason for this evaluation can be found in the following matter: the students need to see the practical application of mathematics. This motivates them to study more passionately and deeply, and to pay more attention on mathematics.

The work done on their own, with mathematical literature and its presentation in front of an audience is a way of work that gives opportunity for self expression to the students with special studying needs or other problems, as well. One of the members of the first group shutters, but he prepared in written wonderful presentation, which was then spoken by the second member of the group. One of the members in the third team has cerebral paralysis because of which he could not write well. However, he works excellently on the computer and took part in the preparation and presentation of the project.

The work that the students do during the development of the projects is hard, and intensive. Besides this, they do it with great pleasure and satisfaction of the results achieved. This way of work gives them also an opportunity for development of new research qualities. The students' parents also react positively, as they see strong interest

and will for development in their children.

The British teachers also applicate highly the work done:

**Anika Jones:** The class were very focussed and working at an advanced level in mathematics. The children spoke clearly and confidently. I will ask my pupils to use prezentations and evaluations.

**Deb Broun:** I was very impressed at the level the students were working at and the confident and interesting way they produced and presented their project. **Alison Day**: I was very impressed by the student's knowledge and confidence. You have inspired me to make more time to encourage my children in Math. I may also set up a Maths Club in my school.

**Maxine Leetham:** The level of knowledge shown by the students is very good.

**Joanne Ashton:** Your knowledge is very impressive. You are very commited and you will all do well in your careers. I like the idea of a Maths Club that has projects.

**Debbie Milligan:** You have a high knowledge and understanding of Maths and English.

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## ЕДНО ЗАНЯТИЕ НА МАТЕМАТИЧЕСКИ КЛУБ "СИГМА" ПРИ 21 СОУ "ХР. БОТЕВ"

**Иванка Марашева-Делинова**

В статията се разглежда подготовката и провеждането на едно занятие на клуба по математика "Сигма" при 21 СОУ "Хр. Ботев" – София. Занятието е на тема Прогресии с подтеми Аритметична прогресия, Геометрична прогресия и Приложение на прогресиите в другите науки. Акцентирано е на подбора на теми и материали; екипите за работа; задачите, които отделните членове на клуба трябва да изпълнят. Особено внимание се обръща на точната и ясна формулировка на поставените задачи; критериите за тяхното изпълнение и оценката на получения продукт. Отразено е мнението за клубната форма на работа на участниците, техните родители и присъствалите на занятието английски учители.