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EXPLICIT SOLUTION OF BITSADZE-SAMARSKII PROBLEM*

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In this paper we find an explicit solution of Bitsadze-Samarskii problem for Laplace equation using operational calculus approach, based on two non-classical one-dimensional convolutions and a two-dimensional convolution. In fact, the explicit solution obtained is a way for effective summation of a solution obtained in the form of non-harmonic Fourier sine-expansion. This explicit solution is suitable for numerical calculation too.

In [1] it is posed the following nonlocal boundary value problem:

$$\begin{aligned} &\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0, \quad -l<x<l, 0<y<1 \\ &u(x,0)=0, \quad u(x,1)=f(x) \\ &u(-l,y)=g(y), u(l,y)=u(0,y). \end{aligned}$$

More elaborately, this problem is studied in A. Bitsadze's book [2], p. 214–219. Some generalisations are proposed by A. Skubachevskii in [3].

In [4], p. 175–176 one of the authors proposed an explicit solution of the problem

$$\begin{aligned} &\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0, \quad 0<x<1, \quad 0<y<1 \\ &u(x,0)=u(0,y)=0, u(x,1)=f(x) \\ &u(1,y)-u(\tfrac{1}{2},y)=0 \end{aligned} \tag{1}$$

which is only a slight modification of Bitsadze-Samarskii's problem.

This solution has the form

$$u(x,y)=-\int_{\frac{1}{2}}^{1} d\xi \left\{ \int_x^{\xi} U(x+\xi-\eta,y)f^{(4)}(\eta)d\eta - \int_{-x}^{\xi} U(\xi-x-\eta,y)f^{(4)}(|\eta|)\operatorname{sgn}\eta\, d\eta \right\} \tag{2}$$

where

$$U(x,y)=\sum_{n=1}^{\infty}\frac{\mathrm{sh}4n\pi y\sin 4n\pi x}{32\pi^3 n^3 \mathrm{sh}4n\pi}+\sum_{n=1}^{\infty}\frac{9\,\mathrm{sh}\frac{2}{3}(2n-1)\pi y\sin\frac{2}{3}(2n-1)\pi x}{4\,\pi^3(2n-1)^3\cos\frac{2}{3}(1+n)\pi\,\mathrm{sh}\frac{2}{3}(2n-1)\pi} \tag{3}$$

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is the solution of the same problem, but for the special choice of $f(x) = \frac{x^3}{6} - \frac{7x}{24}$. It is a classical solution of (1) under the assumptions $f(0) = f''(0) = 0$, $f(1) - f\left(\frac{1}{2}\right) = f''(1) - f''\left(\frac{1}{2}\right) = 0$.

Our aim here is to simplify (2) to the form

(4)

$$u(x,y) = \int_x^{\frac{1}{2}} U_x\left(\frac{1}{2} + x - \xi, y\right) f''(\xi)\, d\xi - \int_{-x}^{\frac{1}{2}} U_x\left(\frac{1}{2} - x - \xi, y\right) f''(|\xi|) \operatorname{sgn}\xi\, d\xi -$$
$$- \int_x^1 U_x(1 + x - \xi, y) f''(\xi)\, d\xi + \int_{-x}^1 U_x(1 - x - \xi, y) f''(|\xi|) \operatorname{sgn}\xi\, d\xi$$

where

$$U_x(x,y) = \frac{\partial U(x,y)}{\partial x} =$$

(5)
$$= \sum_{n=1}^{\infty} \frac{\operatorname{sh} 4n\pi y \cos 4n\pi x}{8\pi^2 n^2 \operatorname{sh} 4n\pi} + \sum_{n=1}^{\infty} \frac{3 \operatorname{sh} \frac{2}{3}(2n-1)\pi y \cos \frac{2}{3}(2n-1)\pi x}{2\pi^2 (2n-1)^2 \cos \frac{2}{3}(1+n)\pi \operatorname{sh} \frac{2}{3}(2n-1)\pi}$$

In a sense (4) is simpler than (2) since it uses only second derivatives of f instead of fourth ones and only simple integrals instead of repeated. The boundary value restrictions on f are also relaxed to $f(0) = f(1) - f\left(\frac{1}{2}\right) = 0$. Then, (4) is a generalised solution of (1) in the following sense:

Definition 1. *A function $u(x,y) \in C([0,1]\times[0,1])$ is said to be a generalised solution of Bitsadze-Samarskii problem (1), iff $u(x,y)$ satisfies the integral equation*

(6)
$$L_x u + L_y u = L_x f(x).y$$

where

(7)

$$L_x\{u(x,y)\} = \int_0^x (x-\xi)u(\xi,y)d\xi - 2x\left(\int_0^1 (1-\xi)u(\xi,y)d\xi - \int_0^{\frac{1}{2}} \left(\frac{1}{2} - \xi\right) u(\xi,y)d\xi\right)$$
$$L_y\{u(x,y)\} = \int_0^y (y-\eta)u(x,\eta)d\eta - y\left(\int_0^1 (1-\eta)u(x,\eta)d\eta\right).$$

The right inverse operators L_x and L_y of $\frac{\partial^2}{\partial x^2}$ and $\frac{\partial^2}{\partial y^2}$ are defined in $C([0,1] \times [0,1])$ by

$$v = L_x u : \frac{\partial^2}{\partial x^2} v = u, \quad v(0,y) = v(1,y) - v\left(\frac{1}{2}, y\right) = 0$$

and

$$w = L_y u \; : \quad \frac{\partial^2}{\partial y^2} w = u \qquad w(x,0) = w(x,\,1) = 0,$$

correspondingly.

Formally, (6) could be obtained from the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ by applying to it the operator $L_x L_y$ and using the boundary value conditions.

Lemma 1. *If $u(x,y) \in C([0,1] \times [0,1])$ satisfies (6), then $u(x,y)$ satisfies the boundary value conditions:*

$$u(x,0) = u(0,y) = 0, \quad u(x,1) = f(x), \quad u(1,y) - u\left(\frac{1}{2}, y\right) = 0$$

Proof. For $y = 0$ from (6) we obtain $L_x u(x,0) = 0$. Applying the operator $\frac{\partial^2}{\partial x^2}$ to this equation we find $u(x,0) = 0$. In a similar way for $y = 1$ we find $u(x,1) = f(x)$.

Next, for $x = 0$ from (6) we obtain $L_y u(0,y) = 0$. Applying the operator $\frac{\partial^2}{\partial y^2}$ to this equation we find $u(0,y) = 0$. Analogically, we find $u(1,y) - u\left(\frac{1}{2}, y\right) = 0$. □

Example. If $f(x) = \frac{x^3}{6} - \frac{7x}{24}$, then (3) is a generalized solution of boundary value problem (1) (see [4], p. 175).

Lemma 2. *If a function $u(x,y) \in C^2([0,1] \times [0,1])$ satisfy (6), then it is a classical solution of (1).*

Proof. We apply the operator $\frac{\partial^4}{\partial x^2 \partial y^2}$ to (6) and obtain $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$. As for the boundary value conditions, they are satisfied by Lemma 1. □

In order to elucidate our approach for obtaining of an explicit solution, we consider the following extension of Bitsadze-Samarskii problem (1):

$$\begin{aligned} &\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = F(x,y), \quad 0 < x < 1, \quad 0 < y < 1 \\ &u(x,0) = u(0,y) = 0 \\ &u(x,1) = f(x), u(1,y) - u\left(\frac{1}{2}, y\right) = g(y), \end{aligned} \tag{8}$$

where $f(x)$, $g(y) \in C([0,1])$, $F(x,y) \in C([0,1] \times [0,1])$.

Definition 2. *A function $u(x,y) \in C([0,1] \times [0,1])$ is said to be a generalised solution of problem (8), iff $u(x,y)$ satisfies the integral equation*

$$L_x u + L_y u = L_x f(x).y + L_y g(y).x + L_x L_y F(x,y) \tag{9}$$

Formally, (9) could be obtained easily from the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = F(x,y)$ applying the operator $L_x L_y$ to it and using the boundary value conditions.

Lemma 3. *If a function* $u(x,y) \in C([0,1]\times[0,1])$ *satisfy* (9)*, then* $u(x,y)$ *fulfils the boundary value conditions:*

$$u(x,0) = u(0,y) = 0, \quad u(x,1)\} = f(x), \quad u(1,y) - u\left(\frac{1}{2}, y\right) = g(y).$$

Proof. Analogically to the proof of Lemma 1. □

Lemma 4. *If a function* $u(x,y) \in C^2([0,1]\times[0,1])$ *satisfies* (9)*, then it is a classical solution of* (1)*.*

Proof. Applying the operator $\dfrac{\partial^4}{\partial x^2 \partial y^2}$ to (9), we obtain $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = F(x,y)$. The boundary value conditions are satisfied by Lemma 3. □

In order to obtain an explicit solution of (1) or (9) we outline an operational calculus approach to Bitsadze-Samarskii problem. To this end, we introduce three convolution algebras: $(C[0,1] \overset{x}{*})$, $(C[0,1], \overset{y}{*})$ and $(C([0,1]\times[0,1])*)$.

Theorem 1. *The operation*

$$(f \overset{x}{*} g)(x) = \left(\int_0^{\frac{1}{2}} h(x,\eta)d\eta - \int_0^1 h(x,\eta)d\eta \right), \tag{10}$$

where

$$h(x,\eta) = \int_x^\eta f(x+\eta-\xi)g(\xi)d\xi - \int_{-x}^{\eta} f(|\eta - x - \xi|)g(|\xi|)\, sgn\,(\xi(\eta - x - \xi))d\xi,$$

is a bilinear, commutative and associative operation on $C[0,1]$*, such that* $L_x f = \{x\} \overset{x}{*} f$.

This a special case of a more general operation $(f \overset{x}{*} g)(x) = -\dfrac{1}{2}\Phi_\xi \left\{ \int_0^\xi h(x,\eta)d\eta \right\}$ in $C[0,a]$ where Φ is a linear functional in $C^1[0,a]$ for the special choice $\Phi\{f\} = 2\left(f(1) - f\left(\dfrac{1}{2}\right)\right)$ and $a = 1$ (see [4], p. 119).

Theorem 2. *The operation*

$$(f \overset{y}{*} g)(y) = -\frac{1}{2}\int_0^1 h(y,\eta)d\eta, \tag{11}$$

where

$$h(y,\eta) = \int_y^\eta f(y+\eta-\xi)g(\xi)d\xi - \int_{-y}^{\eta} f(|\eta - y - \xi|)g(|\xi|)\, sgn\,(\xi(\eta - y - \xi))d\xi,$$

is a bilinear, commutative and associative operation on $C[0,1]$*, such that* $L_y f = \{y\} \overset{y}{*} f$.

This again is a special case of the above mentioned general operation for the special choice $a = 1$ and $\Phi\{f\} = f(1)$.

We may combine both one-dimensional convolutions into one two-dimensional convolution.

Theorem 3 [5]. *The operation*

$$(12)\qquad (f \overset{g}{*})(x,y) = \frac{1}{2}\int_0^1 \left(\int_0^1 h(x,y,\xi,\eta)d\xi - \int_0^{\frac{1}{2}} h(x,y,\xi,\eta)d\xi \right) d\eta,$$

where

$$h(x,y,\xi,\eta) = \int_x^\xi \int_y^\eta f(\xi+x-\sigma, \eta+y-\tau)g(\sigma,\tau)d\sigma d\tau -$$

$$-\int_{-x}^\xi \int_y^\eta f(|\xi-x-\sigma|, \eta+y-\tau)g(|\sigma|,\tau)\,\mathrm{sgn}\,(\xi-x-\sigma)\sigma d\sigma d\tau -$$

$$-\int_x^\xi \int_{-y}^\eta f(\xi+x-\sigma, |\eta-y-\tau|)g(\sigma,|\tau|)\,\mathrm{sgn}\,(\eta-y-\tau)\tau d\sigma d\tau +$$

$$+\int_{-x}^\xi \int_{-y}^\eta f(|\xi-x-\sigma|, |\eta-y-\tau|)g(|\sigma|,|\tau|)\,\mathrm{sgn}\,(\xi-x-\sigma)(\eta-y-\tau)\sigma\tau d\sigma d\tau,$$

is a bilinear, commutative and associative operation, in $C = C([0,1]\times[0,1])$ *such that the product* $L_x L_y$ *has the representation*

$$(13)\qquad L_x L_y u = \{xy\} * u.$$

Lemma 5.

$$(14)\qquad L_x \left\{ \frac{\partial^2 u}{\partial x^2} \right\} = u(x,y) - u(0,y) - 2x[u(1,y) - u(\frac{1}{2},y)]$$

and

$$(15)\qquad L_y \left\{ \frac{\partial^2 u}{\partial y^2} \right\} = u(x,y) + (y-1)u(x,0) - y\,u(x,1).$$

The proof is immediate.

In order to outline our operational calculus approach to the extended Bitsadze-Samarskii problem, we start with the general definition of a multiplier of convolutional algebra.

Definition 3 [7]. *A linear operator* $M : C \to C$ *is said to by a multiplier of the convolutional algebra* $(C,*)$ *if* $M(u*v) = (Mu)*v$ *for all* $u,\ v \in C$.

We introduce some notations. The multipliers of the form $\{u(x,y)\}*$ are denoted as $\{u\}$. Let $f = \{f(x)\}$ be a function of the variable x only and $g = \{g(y)\}$ be a function of the variable y only, but both considered as elements of C. The operators $[f]_y$ and $[g]_x$ defined by $[f]_y u = f \overset{x}{*} u$ and $[g]_x u = g \overset{y}{*} u$ are said to be partial numerical operators with respect to y and x correspondingly. In this notations we have $L_x = [x]_y$ and $L_y = [y]_x$.

The set of all the multipliers of the convolutional algebra $(C,*)$ is a commutative ring $\mathfrak{M}$. The multiplicative set $\mathfrak{N}$ of the non-zero non-divisors of 0 in $\mathfrak{M}$ is non-empty, since at least the operators $\{x\} \overset{x}{*} = [x]_y$ and $\{y\} \overset{y}{*} = [y]_x$ are non-divisors of 0.

Next we introduce the ring $\mathcal{M} = \mathcal{N}^{-1}\mathcal{M}$ of the multiplier fractions of the form $\dfrac{A}{B}$ where $A \in \mathfrak{M}$ and $B \in \mathfrak{N}$. The standard algebraic procedure named "localization" of constructing of this ring, is described, e.g. in Lang [8]. Most important for our considerations are the algebraic inverses $S_x = \dfrac{1}{L_x}$ and $S_y = \dfrac{1}{L_y}$ of the multipliers L_x and L_y correspondingly.

Lemma 6. *If $u \in C^2([0,a]\times[0,b])$, then*

$$u_{xx} = S_x u + S_x\{u(0,y)\} - 2\left[\left(u(1,y) - u\left(\frac{1}{2}, y\right)\right)\right]_x,$$

$$u_{yy} = S_y u + S_y\{(y-1)u(x,\,0)\} - [u(x,\,1)]_y.$$

Proof. By multiplication of (14) and (15) by S_x and S_y, correspondingly. □

Let us consider problem (1). Using boundary value conditions, the equation $u_{xx} + u_{yy} = 0$ together with the boundary conditions can be reduced to a single algebraic equation in $\mathcal{M}$. Indeed, then $u_{xx} = S_x u - [g(y)]_x$, $u_{yy} = S_y u - [f(x)]_y$ and the BVP (8) takes the algebraic form:

$$(S_x + S_y)u = [f(x)]_y + [g(y)]_x + \{F(x,y)\}.$$

If $S_x + S_y$ is a non-divisor of zero, then the last equation has a solution in $\mathcal{M}$:

$$u = \frac{1}{(S_x+S_y)}[f(x)]_y + \frac{1}{(S_x+S_y)}[g(y)]_x + \frac{1}{(S_x+S_y)}\{F(x,y)\}.$$

In order to show that the element $S_x + S_y$ is a non-divisor of zero in $\mathcal{M}$, we consider the following eigenvalue problem:

$$v''(y) + \mu^2 v(y) = 0, \quad y \in (0,1), \quad v(0) = 0, \quad v(1) = 0. \tag{16}$$

The eigenvalues of (16) are $\mu_m = m\pi$, $m \in \mathbb{N}$, with corresponding eigenfunctions $\sin m\pi x$.

Lemma 7. *The element $S_x + S_y$ is a non-divisor of zero in $\mathcal{M}$.*

Proof. Assume the contrary, i.e. that there exists a non-zero multipliers fraction $\dfrac{A}{B} \neq 0$ with $(S_x + S_y)\dfrac{A}{B} = 0$. The last relation is equivalent to $(S_x + S_y)A = 0$. Since $A \neq 0$, then there exist a function $v \in C$ such that $Av = u \neq 0$. Then, $(S_x + S_y)A = 0$ implies $(S_x + S_y)\ u = 0$ which is equivalent to

$$(L_x + L_y)u = 0. \tag{17}$$

We show that the only solution of this equation is the trivial one, i.e. $u \equiv 0$, which would be a contradiction. To this end we multiply (17) by the eigenfunction $\varphi_n(y) = \sin m\pi y$ of the eigenvalue problem (16) using the convolution product $f \overset{y}{*} g$, defined by

(11). It easy to see that

$$\{u(x,y) \overset{y}{*} \{\sin m\pi\, y\} = \left\{ \gamma_m \int_0^1 u(x,\eta) \sin m\pi\, \eta\, d\eta \right\} \sin m\pi\, y$$

with a constant $\gamma_m \neq 0$, the exact value of which is unessential for us. The function

$$A_m(x) = \gamma_m \int_0^1 u(x,\eta) \sin m\pi\, \eta\, d\eta$$

up to a non-zero constant is the m-th finite Fourier sine-transform of the function $u(x,y)$ with respect to y. From $(L_x + L_y)[u \overset{y}{*} \varphi_m(y)] = 0$ we obtain

$$[L_x A_m(x)] \sin m\pi y + A_m(x) L_y \sin m\pi y = 0$$

But $L_y \sin m\pi y = -\dfrac{1}{(m\pi)^2} \sin m\pi y$ and thus we obtain the following simple integral equation for $A_m(x)$:

$$L_x A_m(x) = \frac{1}{(m\pi)^2} A_m(x).$$

It is equivalent to the BVP

$$A''_m(x) = (m\pi)^2 A_m(x), \quad A_m(0) = 0, \quad A_m(1) = 0. \tag{18}$$

The only solution of (18) is the trivial one: $A_m(x) \equiv 0$.

Thus we proved that $\displaystyle\int_0^1 u(x,\eta) \sin m\pi\eta d\eta = 0$ for arbitrary $x \in [0,1]$ and $\forall n \in \mathbb{N}$.

From a basic property of the Fourier sine-transform it follow $u(x,y) \equiv 0$ for arbitrary $x \in [0,1]$ and $y \in [0,1]$.

This is a contradiction with the assumption $u(x,y) \neq 0$ and it proves the Lemma. Along with this, it is proven the uniqueness of the extended Bitsadze-Samarskii problem. □

Let us consider Bitsadze-Samarskii problem (1) for $f(x) = \dfrac{x^3}{6} - \dfrac{7x}{24} = L_x\{x\} = \dfrac{1}{S_x^2}$. In [4] a representation of the solution $U(x,\, y)$ of this problem by the series (3) is found. The same solution has the algebraic representation

$$U = \frac{1}{(S_x + S_y)} \left[\frac{x^3}{6} - \frac{7x}{24} \right]_y = \frac{1}{(S_x + S_y)} L_x\{x\} = \frac{1}{(S_x + S_y)} L_x^2 = \frac{1}{(S_x + S_y) S_x^2}.$$

Then, the solution of Bitsadze-Samarskii problem (1) for arbitrary f can be represented in the form:

$$u = \frac{1}{(S_x + S_y)} [f(x)]_y = S_x^2 \frac{1}{(S_x + S_y) S_x^2} [f(x)]_y = \frac{\partial^4}{\partial x^4} (U \overset{x}{*} f(x)). \tag{19}$$

In [4] one of the authors had shown that for $f(x) \in C^4[0,1]$ which satisfies the conditions $f(0) = f(1) - f\left(\dfrac{1}{2}\right) = f''(0) = f''(1) - f''\left(\dfrac{1}{2}\right) = 0$, (19) is a representation of the classical solution of (1). Indeed, since $U(x,\, y)$ is a (generalised) solution of problem

(1), we have $U(1,y) - U\left(\frac{1}{2}, y\right) = 0$.

Assuming that $f(x) \in C^2[0,1]$ with $f(0) = f(1) - f\left(\frac{1}{2}\right) = 0$ and using $U(1,y) - U\left(\frac{1}{2}, y\right) = 0$, we obtain

$$u(x,y) = \frac{\partial^4}{\partial x^4}(U(x,y) \overset{x}{*} f(x))$$

$$= -\left(\int_0^x (U_x(\xi+1-x,y) - U_x(x+1-\xi,y) - U_x(\xi+\frac{1}{2}-x,y) + U_x(x+\frac{1}{2}-\xi,y)\right) f''(\xi)d\xi + \tag{20}$$

$$+ \int_0^1 (U_x(x+1-\xi,y) - U_x(1-x-\xi,y))f''(\xi)d\xi -$$

$$- \int_0^{\frac{1}{2}} (U_x(x+\frac{1}{2}-\xi,y) - U_x(\frac{1}{2}-x-\xi,y))f''(\xi)d\xi)$$

with U_x (x, y) given by (5).

It is easy to see that this representation of the solution of (1) is equivalent to (4).

Theorem 4. *If* $f(x) \in C^2[0,1]$*,* $f(0) = 0$*, and* $f(1) - f\left(\frac{1}{2}\right) = 0$*, then* (19) *is a generalised solution of the boundary value problem* (1)*. If* $f(x) \in C^4[0,1]$ *and* $f(0) = f''(0) = 0$*,* $f(1) - f\left(\frac{1}{2}\right) = f''(1) - f''\left(\frac{1}{2}\right) = 0$*, then* $u(x,y) = \frac{\partial^4}{\partial x^4}(U(x,y) \overset{x}{*} f(x))$ *is a classical solution of* (1)*.*

The proof of the first part is a matter of a direct check. The second is proved in [4]. □

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ТОЧНО РЕШЕНИЕ НА ЗАДАЧАТА НА БИЦАДЗЕ-САМАРСКИ

Иван Димовски, Юлиан Цанков

В статията е намерено точно решение на задачата на Бицадзе-Самрски (1) за уравнението на Лаплас, като е използвано операционно смятане основано на некласическа двумерна конволюция. На това точно решение може да се гледа като начин за сумиране на нехармоничния ред по синуси на решението, получен по метода на Фурие.