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MATHEMATICAL MODELING OF THE WEAVING STRUCTURE DESIGN*

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An equivalence relation in the set of all square binary matrices is described in this paper. It is discussed a combinatoric problem about finding the cardinal number and the elements of the factor set according to this relation. We examine the possibility to get some special elements of this factor set. We propose an algorithm, which solves these problems. The results we have received are used to describe the topology of the different weaving structures.

1. Introduction. The present paper develops the ideas in the papers [8] and [3] and in that sense it is an extention of them.

As we know [9, 10], the interweaving of the fibres in certain weaving structure can be coded using square *binary* (or (0,1), or *boolean*) matrix, i.e. all elements of this matrix are 0 or 1. The fabric represented by this matrix exists if and only if in each row and in each column of the matrix there is at least one zero and at least one one. Two different matrices correspond to the same weaving structure if and only if one matrix is obtained from the other one by several consecutive cycle moves of the first row or column to the last place.

Let n be a positive integer. Let us denote by $\mathcal{B}_n$ the set of all $n \times n$ binary matrices, and by $\mathcal{Q}_n$ the set of all $n \times n$ binary matrices which have at least one 1 and one 0 in every row and every column. It is obvious, that $\mathcal{Q}_n \subset \mathcal{B}_n$. About the necessary definitions from the theory of matrices we refer [11] and [13]. It is not difficult to see, that

$$|\mathcal{B}_n| = 2^{n^2} \tag{1}$$

If $A = (a_{i\,j}) \in \mathcal{B}_n$, then $A^T = (a_{j\,i})$, $1 \le i, j \le n$ denotes the transposed matrix A.

We are interested in the subset $\mathcal{P}_n \subset \mathcal{Q}_n$ of all *permutating* matrices, i.e. binary matrices which have exactly one 1 in every row and column. As it is well known [13], the set $\mathcal{P}_n$ together with the operation multiplication of matrices is a group, isomorphic to the symmetric group $\mathcal{S}_n$, i.e. the set

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$$(2)\qquad \mathcal{S}_n = \left\{ \left(\begin{array}{cccc} 1 & 2 & \cdots & n \\ i_1 & i_2 & \cdots & i_n \end{array} \right) \;\middle|\; 1 \le i_k \le n,\ k = 1,2,\dots,n,\ i_k \ne i_l \text{ for } k \ne l \right\}$$

of all one to one maps of the set $\{1, 2, \dots, n\}$ to itself. If $M \in \mathcal{P}_n$ and the corresponding element in this isomorphism is $\left(\begin{array}{cccc} 1 & 2 & \cdots & n \\ i_1 & i_2 & \cdots & i_n \end{array} \right) \in \mathcal{S}_n$, then this means that the only one 1 in the first row of M to be on the i_1-st place, the 1 in the second row of M to be on the i_2-nd place, and so on, the 1 in the n-th row of M to be on the i_n-th place.

Let $t \in \{1, 2, \dots, n\}$ and let $\rho = \left(\begin{array}{cccccc} 1 & 2 & \cdots & t & \cdots & n \\ i_1 & i_2 & \cdots & i_t & \cdots & i_n \end{array} \right) \in \mathcal{S}_n$. We denote $(t)\rho = i_t$ the image i_t of the number t by the map ρ. For arbitrary $\rho_1, \rho_2 \in S_n$ by definition $(t)\rho_1\rho_2 = ((t)\rho_1)\rho_2$ (see [11]).

As it is well-known [13], if we multiply $n \times n$ matrix A from the right with permutational matrix $M \in \mathcal{P}_n$, then this is the same as changing the columns of A. And if the corresponding element of $M \in \mathcal{P}_n$ in the above-described isomorphism is $\left(\begin{array}{cccc} 1 & 2 & \cdots & n \\ i_1 & i_2 & \cdots & i_n \end{array} \right) \in \mathcal{S}_n$, then after the multiplication we get a matrix with k column equal to i_k column of A, $k = 1, 2, \dots, n$. Analogously, when we want to exchange the rows we multiply A from the left with M^T.

Identity element of the group $\mathcal{P}_n$ is the identity matrix E_n, consisting of 1's in the leading diagonal and zeros everywhere else. The identity element of the group $\mathcal{S}_n$ is the element $\left(\begin{array}{cccc} 1 & 2 & \cdots & n \\ 1 & 2 & \cdots & n \end{array} \right)$,

We say that the binary $n \times n$ matrices A and B are equivalent and we write $A \sim B$, if one of the matrix can be transformed by the other after several consecutive cycle moves of the first row or column to the last place. In other words, if $A, B \in \mathcal{Q}_n$ and $A \sim B$, then with the help of these matrices we code the same weaving structure (fabric). It is obvious that this relation in the set $\mathcal{B}_n$ is an equivalence relation. The equivalence class according to the relation $\sim$ with the matrix A we denote by $\overline{A}$, and the sets of equivalence classes in $\mathcal{B}_n$ and $\mathcal{Q}_n$ (factor set) according to $\sim$, by $\overline{\mathcal{B}_n}$ and $\overline{\mathcal{Q}_n}$. We consider that $\overline{\mathcal{B}_n}$ and $\overline{\mathcal{Q}_n}$ are described if there is a representative of each equivalence class. The equivalence classes of $\mathcal{B}_n$ by the equivalence relation $\sim$ are particular kind of *double coset* (see [5] §1.7, or [12] v. 1, ch. 2, §1.1). They make use of substitution groups theory ([6] §1.12, §2.6) and linear representation of finite groups ([4] §§44–45).

We call the elements of $\overline{\mathcal{Q}_n}$ *interweavings*. In that case the number n is called *repeating* of the interweavings of $\overline{\mathcal{Q}_n}$. These notions are taken from the Interweaving-knowing - the science which examines the design, physical and mechanical properties of the different interweavings of the fibres after the given textile structure is weaved.

It is naturally to arise a lot of combinatoric problems which take place in practice in the weaving industry, connected with the different subsets of $\overline{\mathcal{Q}_n}$, i.e. with the different classes of interweavings. We examine some of these classes in the present paper.

2. Some classes of interweavings. It is easy to see, that if $A \in \mathcal{P}_n$ and $B \sim A$, then $B \in \mathcal{P}_n$. Interweavings whoose representatives are elements of the set $\mathcal{P}_n$ of all permutational matrices, are called *primary* interweavings. A formula and an algorithm to calculate the number of all primary interweavings with a random repetition n are given in [7, 10].

We examine the matrix

$$P = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 1 & 0 & 0 & \cdots & 0 \end{pmatrix} = (p_{i\,j}) \in \mathcal{P}_n, \tag{3}$$

where $p_{1\,2} = p_{2\,3} = \cdots = p_{i\,i+1} = \cdots = p_{n-1\,n} = p_{n\,1} = 1$ and these are the only 1's in P, and all the other elements are zeros. In the above described isomorphism of the group of permutational matrices with the symmetric group, the matrix P corresponds to the element

$$\pi = \begin{pmatrix} 1 & 2 & 3 & \cdots & n-1 & n \\ 2 & 3 & 4 & \cdots & n & 1 \end{pmatrix} \in \mathcal{S}_n. \tag{4}$$

It is not difficult to calculate that $P^t \neq E_n$ when $1 \le t < n$, $P^n = E_n$, where E_n is the identity matrix of order n and $P^{k+n} = P^k$ for every natural number k.

Let $A \in \mathcal{B}_n$ and let

$$B = PA$$

and

$$C = AP.$$

It is easy to prove [13] that the first row of B is equal to the second row of A, the second row of B is equal to the third row of A and so on, the last row of B is equal to the first row of A, i.e. the matrix B is obtained from the matrix A by moving the first row to the last place, and the other rows are moved one level upper.

Analogously, we convince that C is obtained from from A by moving the last column to the first place, and the other column move one position to the right.

Having in mind what is described above, it is easy to prove the following:

Lemma 1. *Let $A, B \in \mathcal{B}_n$, then $A \sim B$ if and only if there exist natural numbers k, l, so that*

$$A = P^k B P^l,$$

where P is the matrix given by the formula (3). From the equality $P^{n+t} = P^t$ it follows that for each natural number t, is enough to find the numbers k and l in the interval $[0, n-1]$.

Corollary 1. *Each equivalence class of $\mathcal{B}_n$ according to the relation $\sim$ contains no more than n^2 elements.*

Corollary 2. *All elements of the given equivalence class of $\mathcal{B}_n$ according to the relation $\sim$ can be placed in a $s \times t$ rectangular table, where s and t are divisors of n.*

We use some matrix operations similar to transposition of matrix.

Let

$$A = \begin{pmatrix} a_{1\,1} & a_{1\,2} & \cdots & a_{1\,n} \\ a_{2\,1} & a_{2\,2} & \cdots & a_{2\,n} \\ \vdots & \vdots & & \vdots \\ a_{n\,1} & a_{n\,2} & \cdots & a_{n\,n} \end{pmatrix} \in \mathcal{B}_n \tag{5}$$

If A is a square binary matrix, represented by the formula (5), then by definition

$$A^S = \begin{pmatrix} a_{1\,n} & a_{1\,n-1} & \cdots & a_{1\,1} \\ a_{2\,n} & a_{2\,n-1} & \cdots & a_{2\,1} \\ \vdots & \vdots & & \vdots \\ a_{n\,n} & a_{n\,n-1} & \cdots & a_{n\,1} \end{pmatrix}, \tag{6}$$

i.e. A^S is obtained from A as the last column of A becomes the first, the column before last - second and so on, the first column becomes last. In other words, if $A = (a_{i\,j})$, then $A^S = (a_{i\,n-j+1})$, $1 \le i, j \le n$.

It is obvious that

$$\left(A^S\right)^S = A.$$

We say, that the matrix $A \in \mathcal{B}_n$ is *a mirror image* of the matrix $B \in \mathcal{B}_n$, if $A^S = B$.

It is easy to see, that if the matrix A is a mirror image of the matrix B, then B is a mirror image of A, i.e. the relation "mirror image" is symmetric.

In the general case $A \neq A^S$. If $A = B^S$ and $B = C^S$, then in the general case we have $A = B^S = \left(C^S\right)^S = C \neq C^S$. Therefore, the relation "mirror image" is not reflexive and is not transitive.

We consider the matrix

$$S = \begin{pmatrix} 0 & 0 & \cdots & 0 & 0 & 1 \\ 0 & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & \cdots & 1 & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots \\ 1 & 0 & \cdots & 0 & 0 & 0 \end{pmatrix} = (s_{i\,j}) \in \mathcal{P}_n, \tag{7}$$

where for each $i = 1, 2, \ldots, n$ $s_{i\,n-i+1} = 1$ and $s_{i\,j} = 0$ as $j \neq n - i + 1$. According to the above described isomorphism of the group of permutational matrices with symmetric group, the matrix S corresponds to the element

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & \cdots & n \\ n & n-1 & n-2 & \cdots & 1 \end{pmatrix} \in \mathcal{S}_n. \tag{8}$$

Obviously, S is a symmetric matrix, i.e. $S^T = S$. We check directly that $S^2 = E_n$.

It is not difficult to see [13], that for each $A \in \mathcal{B}_n$ e.g.

$$A^S = AS. \tag{9}$$

Lemma 2. *If P and S are the matrices given by the formulas (3) and (7), then for each $l = 0, 1, 2, \ldots, n-1$ the following is true:*

$$P^l S = SP^{n-l} \tag{10}$$

Proof. Let us denote by $\oplus$ and $\ominus$ the operations corresponding to addition and subtraction in the ring $\mathcal{Z}_n = \{1, 2, \ldots, n \equiv 0\}$ of the remainders modulus n. If $\pi \in \mathcal{S}_n$ and $\sigma \in \mathcal{S}_n$ are elements corresponding to the matrices $P \in \mathcal{P}_n$ and $S \in \mathcal{P}_n$ by the isomorphism of the groups $\mathcal{P}_n$ and $\mathcal{S}_n$ described by the formulas (4), (8), (3) and (7), then for each $t = 1, 2, \ldots, n$:

$$(t)\pi = t \oplus 1 \quad \text{(by definition)} \tag{11}$$

(12) $$(t)\sigma = n \oplus 1 \ominus t \quad \text{(by definition)}$$

We prove by induction, that for each positive integer l

(13) $$(t)\pi^l = t \oplus l$$

If $l = 1$, then the proposition follows from (11). Let the equation $(t)\pi^l = t \oplus l$ be true. Then, we get $(t)\pi^{l+1} = ((t)\pi^l)\pi = (t \oplus l)\pi = t \oplus l \oplus 1$, and it follows that the equation (13) is true for every positive integer l.

Using the equations (11), (12) and (13) we get:

$$(t)\pi^l\sigma = ((t)\pi^l)\sigma = (t \oplus l)\sigma = n \oplus 1 \ominus (t \oplus l) = n \oplus 1 \ominus t \ominus l$$

$$(t)\sigma\pi^{n-l} = ((t)\sigma)\pi^{n-l} = (n \oplus 1 \ominus t)\pi^{n-l} = (n \oplus 1 \ominus t) \oplus (n-l) = 2n \oplus 1 \ominus t \ominus l = n \oplus 1 \ominus t \ominus l.$$

The last equation is true, because $2n \equiv n \equiv 0 \pmod{n}$. We see, that $(t)\pi^l\sigma = (t)\sigma\pi^{n-l}$ for every $t = 1, 2, \ldots, n$ and therefore, $\pi^l\sigma = \sigma\pi^{n-l}$. Having in mind the isomorphism of the groups $\mathcal{S}_n$ and $\mathcal{P}_n$, we get that the proposition in the lemma is true. $\square$

Theorem 1. *If $A \sim A^S$ and $B \sim A$, then $B \sim B^S$.*

Proof. Since $B \sim A$, according to Lemma 1 there exist $k, l \in \{1, 2, \ldots, n\}$, such that $B = P^k A P^l$. Applying Lemma 2, we get $BS = P^k A P^l S = P^k A S P^{n-l}$, and then it follows that $BS \sim AS$, i.e. according to (9), $B^S \sim A^S$. But $A^S \sim A \sim B$ and because of the transitiveness of the relation $\sim$, we get $B^S \sim B \Rightarrow B \sim B^S$. $\square$

Theorem 1 gives us a rise to give the following definition:

Definition 1. *Let $A \in \mathcal{Q}_n$. We say that A is a representative of* ***self-mirrored image*** *(or* ***mirror image to itself****) interweaving, if $A \sim A^S$.*

Let us denote the set $\overline{\mathcal{M}_n} \subset \overline{\mathcal{Q}_n}$ including all self-mirrored interweavings with repetition equal to n.

For arbitrary $A \in \mathcal{B}_n$ we define the operation

(14) $$A^R = \begin{pmatrix} a_{1\,n} & a_{2\,n} & \cdots & a_{n\,n} \\ a_{1\,n-1} & a_{2\,n-1} & \cdots & a_{n\,n-1} \\ \vdots & \vdots & & \vdots \\ a_{1\,1} & a_{2\,1} & \cdots & a_{n\,1} \end{pmatrix} = \left(A^S\right)^T = (AS)^T = S^T A^T = SA^T$$

In other words, the matrix A^R is received from matrix A by 90° clockwise rotating. Obviously,

$$\left(\left(\left(A^R\right)^R\right)^R\right)^R = A.$$

In the general case $A^R \neq A$.

Lemma 3. *If P is a binary matrix, defined by the formula* (3)*, then*

$$P^T = P^{n-1}$$

Proof. If $P = (p_{ij})$ and $P^T = (p'_{ij})$, then by definition $p'_{ij} = p_{ji}$ for each $i, j \in \{1, 2, \ldots n\}$. Let $PP^T = Q = (q_{i,j}) \in \mathcal{P}_n$. Then, for each $i = 1, 2, \ldots, n$ there is

$q_{ii} = \sum_{k=1}^{n} p_{ik}p'_{ki} = \sum_{k=1}^{n} p_{ik}^2 = (n-1)0 + 1 = 1$ and it is the unique one in ith row of the matrix $Q = PP^T$. Therefore, $PP^T = E_n$, where E_n is the identity matrix. We multiply from the left the two sides of the last equation with P^{n-1} and having in mind, that $P^n = E_n$, we get $P^{n-1}PP^T = P^{n-1}E_n$, and then finally we get, that $P^T = P^{n-1}$. $\square$

Theorem 2. *If $A \sim A^R$ and $B \sim A$ then $B \sim B^R$.*

Proof. $B \sim A$, hence according to the Lemma 1 there exist natural numbers k and l, such that $B = P^kAP^l$. If we apply Lemma 2 and Lemma 3, we get $B^R = SB^T = S(P^kAP^l)^T = S(P^T)^lA^T(P^T)^k = S(P^{n-1})^lA^T(P^{n-1})^k = SP^{nl-l}A^TP^{kn-k} = SP^{n-l}A^TP^{n-k} = P^lSA^TP^{n-k}$. Therefore, $B^R \sim A^R \sim A \sim B$. $\square$

Theorem 2 allows us to give the following definition:

Definition 2. *Let $A \in \mathcal{Q}_n$. If $A \sim A^R$, then we say that A is a representative of* ***rotation stable*** *interweaving.*

Let us denote the set $\overline{\mathcal{R}_n} \subset \overline{\mathcal{Q}_n}$ of all the rotation stable interweavings with repetition equal to n.

The rotation stable interweavings play important role in practice. That means, if a fabric is weaved which weaving structure is coded with a matrix, representative of rotation stable interweaving, then this fabric have the same operating characteristics (except of course the color) after a rotation by 90°.

3. Quantity evaluation of the sets of all self-mirrored and all rotation stable interweavings with given repetition n. In [3] it is described a representation of the elements of $\mathcal{B}_n$ using ordered n-tuples of natural numbers $< k_1, k_2, \ldots, k_n >$, where $0 \le k_i \le 2^n - 1$, $i = 1, 2, \ldots, n$. The one to one corresponding is based on the definite representation of the natural numbers in binary number system, i.e. the number k_i in binary number system (having eventually some 0's at the beginning) is the ith row of the corresponding binary matrix. In [3] is proved that using this representation there are faster and saving memory algorithms. Having this in mind, we create an algorithm, which finds just one representative of each equivalence class to the factor sets $\overline{\mathcal{Q}_n}$, $\overline{\mathcal{M}_n}$ and $\overline{\mathcal{R}_n}$. And the representative we receive is the minimal of the equivalence class with regard to the lexicographic order, this order is naturally brought in the set $\mathbb{N}^n$ of all ordered n-tuples of whole nonnegative numbers. Therefore, we get an algorithm to solve the combinatorial problem to find the number of the equivalence classes in the sets $\mathcal{Q}_n$, $\mathcal{M}_n$ and $\mathcal{R}_n$ relevant to the relation $\sim$ with given natural number n.

The matrices P and S given by the formulas (3) and (7) are coded using ordered n-tuples as follows:

$$P \ : \ \langle 2^{n-2}, 2^{n-3}, \ldots, 2^1, 2^0, 2^{n-1} \rangle \tag{15}$$

$$S \ : \ \langle 2^0, 2^1, 2^2, \ldots, 2^{n-2}, 2^{n-1} \rangle \tag{16}$$

In some programming languages (for example C, C++, Java [3, 8]) the number $x = 2^k$ is calculated using the operation bitwise shift left "<<" and the operator (statement) $x = 1 << k$.

In [3] it is defined the operation *logical multiplication* of two binary matrices, which we denote by "$*$". Let $A = (a_{ij})$ and $B = (b_{ij})$ be matrices of $\mathcal{B}_n$. Then,

$$A * B = C = (c_{ij}) \in \mathcal{B}_n \tag{17}$$

and by definition for each $i, j \in \{1, 2 \ldots, n\}$,

$$c_{ij} = \bigvee_{k=1}^{n} (a_{ik} \;\&\; b_{kj}), \tag{18}$$

where we denote by & and $\vee$ the operations conjunction and disjunction in the boolean algebra $\mathcal{B}_n(\&, \vee)$.

Analogously to the classical proof that the operation multiplication of matrices is associative (see for example. [11]) it is proved that the operation logical multiplication of binary matrices is associative. Therefore, $\mathcal{B}_n$ with the entered operation logical multiplication is monoid with identity – the identity matrix E_n. However, $\mathcal{P}_n$ is not a trivial subgroup of this monoid.

In [3, 8] it is described an algorithm, which need $O(n^2)$ operations to perform the operation logical multiplication of two binary matrices, which are represented by using ordered n-tuple. But to get the product of two matrices according to the classical definition we need $O(n^3)$ operations.

If the binary matrix A is represented using the ordered n-tuple of numbers, then to check whether A belongs to the set $\mathcal{Q}_n \subset \mathcal{B}_n$ we can use the following obvious proposition:

Lemma 4. *Let $A \in \mathcal{B}_n$ and let A be represented by the ordered n-tuple $\langle k_1, k_2, \ldots, k_n \rangle$, where $0 \leq k_i \leq 2^n - 1$, $i = 1, 2, \ldots, n$ and let us denote by $|$ and & the operations bitwise "or" and bitwise "and" (to get detailed definitions see for example* [1], [2] *or* [8]). *Then:*

(*i*) *The number k_i, $i = 1, 2, \ldots, n$ represents row of 0's if and only if $k_i = 0$;*

(*ii*) *The number k_i, $i = 1, 2, \ldots, n$ represents row of 1's if and only if $k_i = 2^n - 1$;*

(*iii*) *j-th column of A contains only 0's if and only if*

$$(k_1 \,|\, k_2 \,|\, \cdots \,|\, k_n) \,\&\, 2^j = 0;$$

(*iv*) *j-th column of A contains only 1's if and only if*

$$(k_1 \,\&\, k_2 \,\&\, \cdots \,\&\, k_n) \,\&\, 2^j \neq 0.$$

The algorithm, just described is based on the following propositions:

Lemma 5. *If $A \in \mathcal{B}_n$, $B \in \mathcal{P}_n$, then*

$$A * B = AB$$

and

$$B * A = BA.$$

Proof. Let $A = (a_{ij})$, $B = (b_{ij})$, $U = A * B = (u_{ij})$ and $V = AB = (v_{ij})$, $i, j = 1, 2, \ldots, n$. Let the unique 1 in the j-th column of $B \in \mathcal{P}_n$ is on the s-th place, i.e. $b_{sj} = 1$ and $b_{kj} = 0$ when $k \neq s$. Then, by definition

$$u_{ij} = \bigvee_{k=1}^{n} (a_{ik} \;\&\; b_{kj}) = \begin{cases} 1 & \text{for} \quad a_{is} = 1 \\ 0 & \text{for} \quad a_{is} = 0 \end{cases}$$

and

$$v_{ij} = \sum_{k=1}^{n} (a_{ik} b_{kj}) = \begin{cases} 1 & \text{for} \quad a_{is} = 1 \\ 0 & \text{for} \quad a_{is} = 0 \end{cases}$$

Therefore, $u_{ij} = v_{ij}$ for each $i, j \in \{1, 2, \ldots, n\}$.
Analogously it can be proved, that $B * A = BA$. $\square$

Lemma 6. *Let $A \in \mathcal{B}_n$ be represented by using ordered n-tuple $< k_1, k_2, \ldots, k_n >$ and let A be a minimal element of the equivalence class corresponding to the lexicographic order in $\mathbb{N}^n$. Then, $k_1 \leq k_t$ for each $t = 2, 3, \ldots, n$.*

Proof. We assume that there exists $t \in \{2, 3, \ldots, n\}$ such that $k_t < k_1$. Then, if we move the first row on the last place $t-1$ times, we get a matrix $A' \in \mathcal{B}_n$, such as $A' \sim A$ and A' is represented using the n-tuple $< k_t, k_{t+1}, \ldots, k_n, k_1, \ldots, k_{t-1} >$. Then, obviously $A' < A$ according to the lexicographic order in $\mathbb{N}^n$, which runs counter to the minimum of A in the equivalence class $\overline{A}$. $\square$

We can propose the following generalized algorithm to obtain just one representative of each equivalence class in the factor sets $\overline{\mathcal{Q}_n}$, $\overline{\mathcal{M}_n}$ and $\overline{\mathcal{R}_n}$

Algorithm 1.

1. Generating all ordered n-tuples of natural numbers $< k_1, k_2, \ldots, k_n >$ such that $1 \leq k_i \leq 2^n - 2$ and $k_1 \leq k_i$ as $i = 2, 3, \ldots, n$;
2. Check whether the elements obtained in 1. belong to the set $\mathcal{Q}_n$ according to the Lemma 4 (iii) (iv) (cases (i) and (ii) we reject when we generate the elements in point 1 according to the Lemma 6);
3. Check whether the element, obtained in point 2. is minimal in the equivalence class. According to Lemmas 1 and 5 A is minimal in $\overline{A}$ if and only if $A \leq P^k * A * P^l$ for each $k, l \in \{0, 1, \ldots, n-1\}$, where P is the matrix represented by n-tuple (15);
4. Check whether the elements obtained in point 3. belong to the set $\mathcal{M}_n$ according to Definition 1 and apply Lemmas 1 and 5;
5. Check whether the elements obtained in point 3. belong to the set $\mathcal{R}_n$ according to Definition 2 and apply Lemmas 1 and 5.

The results of the Algorithm 1 for are presented in the next table:

n	2	3	4	5
$\lvert\overline{\mathcal{Q}_n}\rvert$	1	14	1 446	705 366
$\lvert\overline{\mathcal{M}_n}\rvert$	1	2	142	1 302
$\lvert\overline{\mathcal{R}_n}\rvert$	1	2	18	74

When $n \geq 6$, then too large values are got (see (1)) and to avoid "overloading" it is necessary to be used some special programming techniques which is not the task in this report.

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МАТЕМАТИЧЕСКО МОДЕЛИРАНЕ ПРИ ДИЗАЙНА НА ТЪКАЧНИ СТРУКТУРИ

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В работата се разглежда една релация на еквивалентност в множеството от всички квадратни бинарни матрици. Обсъдена е комбинаторната задача за намиране мощността и елементите на фактормножеството относно тази релация. Разгледана е и възможността за получаване на някои специални елементи на това фактормножество. Предложен е алгоритъм за решаване на поставените задачи. Получените в статията резултати намират приложение при описанието топологията на различните тъкачни структури.