МАТЕМАТИКА И МАТЕМАТИЧЕСКО ОБРАЗОВАНИЕ, 2014
MATHEMATICS AND EDUCATION IN MATHEMATICS, 2014
Proceedings of the Forty Third Spring Conference
of the Union of Bulgarian Mathematicians
Borovetz, April 2–6, 2014

CNN MODELING OF TSUNAMI WAVES*

Angela Slavova

In this talk several models of tsunami waves are considered. Physical interpretation of this kind of waves is presented. Mathematical model of the long water waves with nonlinear vorticity is studied. For this model Cellular Neural Network (CNN) approach is applied. The dynamics of the CNN model is studied by means of describing function method. Computer simulations and discussions are provided.

1. Tsunami modeling. Physical regimes of long waves of small amplitudes. Tsunami are without a doubt among the most infamous and least understood natural disasters today. Often referred to in the popular literature by the misnomer “tidal wave”, tsunami are generated by large displacements in the sea level, often via seismic activity. Most tsunami – a term from the Japanese for “harbor wave” – are caused by vertical movement along a break in the earth’s crust. Other causes can include volcanic collapse, subsidence, as well as landslides. Contrary to popular imagination, a tsunami need be neither large nor destructive – classification is based on origin of the wave or wave period rather than on size. Though between 1861 and 1948 there were more than 15 000 earthquakes recorded, there were only 124 tsunami [5, 8]. Indeed, off the west coast of South America, 1 098 earthquakes have led to only 20 recorded tsunami [2]. As waves of such great scale, generated by complex movements of the earth, and with such devastating consequences for populations surrounding the world’s oceans, accurate modeling of tsunami is of utmost importance.

One question which has been raised repeatedly is whether the behavior of tsunami at sea can be described by the Korteweg-de Vries equation (see [13, 14]). We will pursue this question for one of the greatest tsunami of recorded history – generated by a series of earthquakes in southern Chile on May 22, 1960 – as it propagated from Chile to Hawaii. These earthquakes, among them the largest ever recorded, resulted from a rupture about 1000 km long and 150 km wide along the fault between the Nazca and South American plates, at a focal depth of 33 km. The principal shock occurring on May 22 at 19:11 GCT registered at 9.5 on the moment magnitude scale, and led to changes in land elevation ranging from 6 m of uplift to 2 m of subsidence – which has been modeled to correspond to an average dislocation of 20 m along the fault, with peaks of more than 30 m [4]. This subsidence extended as far as 29 km inland, resulting in some 10 km^2 of forest around the Rio Maullrin being submerged by the tides and consequently defoliated [10].

*2010 Mathematics Subject Classification:** 92B20, 35Q53, 35Q80.
Key words: waves, tsunami waves, KdV equation, Camassa-Hol; equation, BBM equation, Cellular Neural Network.

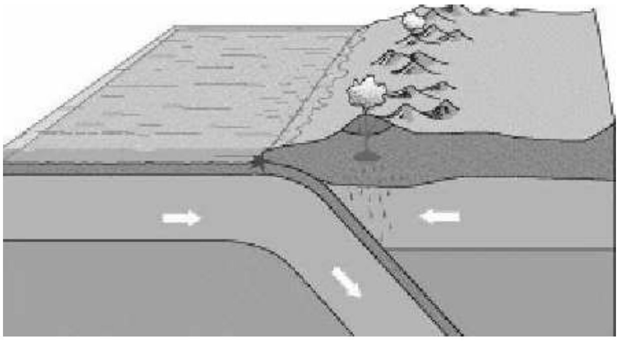

Fig. 1. Earthquake causing the tsunami

Not only was the principal earthquake at 39.5 S, 74.5 W especially powerful, it generated tsunami with an average run-up of 12.2 m and a maximal run-up on the adjacent Chilean coast of 25 m. Over the course of the next day, a number of tsunami wreaked havoc upon the Pacific, taking the lives of more than 2000 people and causing millions of dollars in damages. The initial wave traveled between 670 and 740 km/h, with a wavelength of between 500–800 km and a height in the open ocean of only 40 cm [4]. Borrowing an example from [10], sitting in a boat in the Pacific, the tsunami wave would take between 45 min to an hour to pass one by while raising the boat by less than one centimeter per minute – hardly noticeable on the open sea. Nevertheless, the tsunami reached amplitudes of 7 m in Kamchatka and 10.7 m in Hilo, Hawaii, where it caused widespread destruction after traveling 10 000 km in just under 15 hours. The Chilean tsunami of 1960 had wavelengths in excess of 500 km and amplitudes of less than one meter while propagating over the Pacific Ocean, which, though the deepest of the worlds' oceans, has an average depth of only 4.3 km. These scales lend themselves to modeling

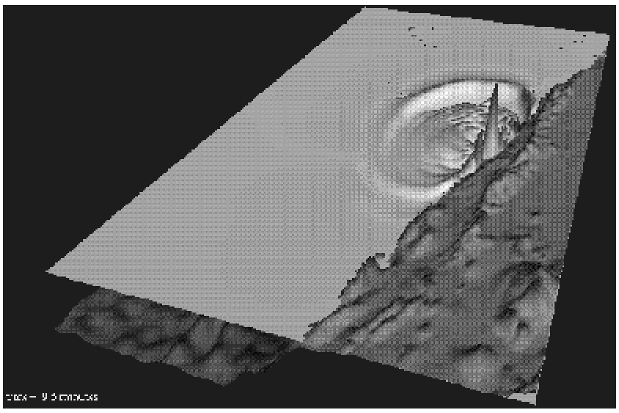

Fig. 2. Tsunami wave

with shallow-water long-wave theory, i.e. water depth is small compared to wavelength and depth is large compared to amplitude. We note also that the depth of open ocean across which the 1960 tsunami traveled is relatively uniform, and given that the rupture length exceeded the wavelength of the resulting tsunami, it is reasonable to assume the waves as two-dimensional; this is borne out (at least between Chile and Hawaii) by consulting travel time charts (see [7]).

The study of propagation of tsunami from their small disturbance at the sea level to the size they reach approaching the coast has involved the interest of several scientists. It is clear that in order to predict accurately the appearance of a tsunami it is fundamental to built up a good model. From this point of view the most important tool in the context of water waves is soliton theory [9]. Frequently in the literature it is stated that a tsunami is produced by a large enough soliton. Solitons arise as special solutions of a widespread class weakly nonlinear dispersive PDEs modeling water waves, such as the KdV or Camassa-Holm equation [6, 13], representing to various degrees of accuracy approximations to the governing equations for water waves in the shallow water regime. How the tsunami is initiated?

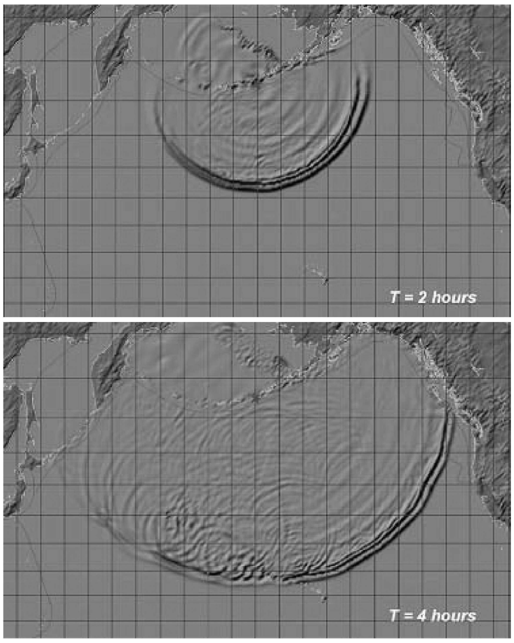

Fig. 3. Tsunami as large soliton

The thrust of a mathematical approach is to examine how a wave, once initiated, moves, evolves and eventually becomes such a destructive force of nature. We aim to describe how an initial disturbance gives rise to a tsunami wave.

Let h be the average depth of the water, λ be the typical wavelength of the wave and a be a typical amplitude. There are two important parameters: $\varepsilon = \dfrac{a}{h}$, called amplitude parameter, and the shallowness parameter $\delta = \dfrac{h}{\lambda}$. According to these parameters a rigorous validity ranges are obtained [1] for the main physical regimes encountered in modelling of two-dimensional water waves:

1. Shallow-water, large amplitude ($\delta \ll 1$, $\varepsilon \sim 1$), leading firstly to the shallow-water equations [14] and secondary to the Green-Naghdi model [11]. In this case, when the wavelength $\delta \to 0$ is increasing the stability properties of traveling water waves improve significantly. Moreover, one can prove orbital stability of these waves allowing their shape to be stable under small perturbations.

2. Shallow-water, medium amplitude regime ($\delta \ll 1$, $\varepsilon \sim \delta$) leading to the Serre equations [5] and to the Camassa-Holm equation (CH) [6]. Unlike KdV, which is derived by asymptotic expansions in the equation of motion, CH is obtained by using an asymptotic expansions directly in the Hamiltonian for Euler's equations in the shallow water regime. The novelty of the Camassa and Holm's work was the physical derivation of CH equation and the discovery that the equation has solitary waves (solitons) that retain their individuality under interaction and eventually emerge with their original shapes and speeds. For this reason CH is not appropriate for advancing insight into the propagation of tsunamis.

3. Shallow-water, small amplitude or long-wave regime ($\delta \ll 1$, $\varepsilon \sim \delta^2$) leading at first order to the linear wave equation

$$\varphi_{tt} - \varphi_{xx} = 0 \tag{1}$$

with general solution

$$\varphi(x,t) = \varphi_+(x-t) + \varphi_-(x+t), \tag{2}$$

where the sign $\pm$ refers to a wave profile $\varphi_\pm$ moving with unchanged shape to the right/left at constant unit speed. The small effects that were ignored at first order (small amplitude, long wave) build up on longer time/spatial scales to have a significant cumulative nonlinear effect so that on a longer time scale each of the waves that make up the solution (2) to (1) satisfies the KdV equation [13].

This regime leads to Boussinesq systems as well.

4. Deep-water, small steepness regime ($\delta \gg 1, \varepsilon\delta \ll 1$) leading to the full-dispersion Matsuno equations [16].

We are interested in small-amplitude long waves, in the limits $\varepsilon \to 0$ and $\delta \to 0$. The regime $\varepsilon = O(\delta^2)$ emerges naturally since one obtains a problem involving only small parameter, ε. It is in this regime that firstly the evolution of the waves is governed by the linear wave equation (1) with general solution (2). The corresponding dimensional speed is $\sqrt{gh_0}$. Let us choose the wave moving to right by means of the method of multiple scales [14]. It is possible to obtain in the region of (x,t)-space more precise information about the evolution of the water's free surface by taking into account weakly nonlinear interactions. This can be achieved by showing that to the next order of approximation, the evolution of the leading order of the free surface is described by the KdV equation instead of the linear equation (1). Allowing for waves traveling in both directions the Boussinesq system is obtained. For our purposes it is more important to specify how solutions of KdV or Boussinesq approximate the free surface. The sharpest rigorous result in this direction is given in [14] and ensures that, given $\varepsilon_0 > 0$, there exists $T_0 > 0$

such that if $\varepsilon = O(\delta^2)$, then if one defines

$$\varphi^\varepsilon(x,t) = \varphi^+(\tau, x-t),$$

where $\tau = t\varepsilon$ and $\varphi^+(\tau,\varphi)$ solves the KdV equation

$$\varphi^+_\tau + \frac{3}{2}\varphi^+_{\varphi\varphi\varphi} + \frac{1}{6}\varphi^+\varphi^+_\varphi = 0,$$

then for some $A > 0$ independent on $\varepsilon \in (0, \varepsilon_0)$ the following is satisfied:

$$|\varphi(x,t) - \varphi^\varepsilon(x,t)| \leq A\varepsilon^2 t, t \in [0, \frac{T_0}{\varepsilon}].$$

Similar approximation of order $O(\varepsilon^2 t)$ can be found for the solution of the Boussinesq system in the case when the wave propagation is not unidirectional [14]. Moreover, in the case of a non-flat bed with small variations of the order of the size of the surface waves, meaning that if b measures the amplitude of the variations of the bottom topography, then $\frac{b}{h_0} = O(\varepsilon)$, the constant-coefficient KdV equation may be replaced by a variable-coefficient KdV equation [13]; and similarly for the Boussinesq system with the same scaling and approximation properties.

2. Mathematical modeling of tsunami waves. In order to derive the model equation of tsunami wave we assume an initial disturbance of the form of a two-dimensional wave and we are interested in understanding the dynamics of the wave as it propagates across the ocean. Choose Cartesian coordinates (X,Y) with the Y-axis pointing vertically upwards, the X-axis being the direction of wave propagation, and with the origin located on the mean water level $Y = 0$. Let $(\Phi(X,Y,T), \Psi(X,Y,T))$ be the velocity field of the two-dimensional flow propagating in the X-direction over the flat bed $Y = -h$, and let $Y = H(X,T)$ be the water's free surface with mean water level $Y = 0$. The equation of mass conservation

$$\Phi_X + \Psi_Y = 0$$

is a consequence of assuming constant density, a physically reasonable assumption for gravity water waves.Under the assumption of inviscid flow (which is realistic since experimental evidence confirms that the length scales associated with an adjustment of the velocity distribution due to laminar viscosity or turbulent mixing are long compared to typical wave-lengths) the equation of motion is Euler's equation:

$$\begin{cases} \Phi_T + \Phi\Phi_X + \Psi\Phi_Y = -\frac{1}{\rho}P_X, \\ \Psi_T + \Phi\Psi_X + \Psi\Psi_Y = -\frac{1}{\rho}P_Y - g, \end{cases}$$

where P is the pressure, g is the constant acceleration of gravity and ρ is the constant density of water. We also have the boundary conditions $P = P_{atm}$ on $Y = H(X,T)$, where P_{atm} is the (constant) atmospheric pressure at the water's free surface, $\Psi = H_T + \Phi H_X$ on $Y = H(X,T)$, and $\Psi = 0$ on $Y = -h$. These conditions express the fact that the water particles cannot cross the free surface, respectively, the impermeable rigid bed, while $P = P_{atm}$ decouples the motion of the water from that of the air above it in the absence of surface tension; for wavelength larger than a few mm (and in our case we deal with hundreds of km) the effects of surface tension are known to be negligible. We will consider irrotational flows with zero vorticity

$$\Phi_Y - \Psi_X = 0,$$

a hypothesis that allows uniform currents but neglects the effects of non-uniform currents in the fluid.

Finding exact solutions to the nonlinear governing equations for water waves is not possible even with the aid of the most advanced computers. In order to derive approximations to the governing equations it is useful to write them in non-dimensional form. We assume that the two-dimensional waves under investigation have acquired a certain pattern. We assume that the wave pattern under investigation represents a weakly irregular perturbation of a wave train in the sense that averages over suitable times/distances resemble a wave train. Since h is the average depth of the water, the non-dimensionalisation Y_0 of Y should be $Y = hy$, which is to be understood as replacing the dimensional, physical variable Y by hy, where y is now a non-dimensional version of the original Y. The non-dimensionalisation of the horizontal spatial variable is also obvious; if λ is some average of typical wavelength of the wave, we set $X = \lambda x$. The corresponding non-dimensionalsation of time is $T = \dfrac{\lambda}{\sqrt{gh}}t$.

Then the governing equation for irrotational water waves equations in nondimensionalized form is:

$$\begin{cases} \delta^2 U_{xx} + U_{yy} = 0 & \text{in} \quad \Gamma(t), \\ U_y = 0, & \text{on} \quad y = -1, \\ \xi_t + \varepsilon \xi_x U_x + \dfrac{\varepsilon}{\delta^2} U_y = 0 & \text{on} \quad y = \varepsilon\xi, \\ U_t + \dfrac{\varepsilon}{2} U_x^2 + \dfrac{\varepsilon}{2\delta^2} U_y^2 + \xi = 0 & \text{on} \quad y = \varepsilon\xi, \end{cases}$$

where $x \mapsto \varepsilon\xi(x,t)$ is a parametrization on the free surface at time t, $\Gamma(t) = \{(x,y), -1 < y < \varepsilon\xi(x,t)\}$ is the fluid domain delimited above by the free surface and below by the flat bed $\{y = -1\}$, and where $U(\cdot,\cdot,t) : \Gamma(t) \to \mathbf{R}$ is the velocity potential associated to the flow, so that the two-dimensional velocity field is given by (U_x, U_y).

An interesting phenomena in water channels is the appearance of waves with length much greater than the depth of the water. Korteweg and de Vries started the mathematical theory of this phenomenon and derived a model describing unidirectional propagation of waves of the free surface of a shallow layer of water. This is the well known KdV equation:

$$\begin{cases} u_t - 6uu_x + u_{xxx} = 0, \quad t > 0, & x \in \mathbf{R}, \\ u(0,x) = u_0(x), & x \in \mathbf{R} \end{cases}$$

where u describes the free surface of the water; for a presentation of the physical derivation of the equation. The beautiful structure behind the KdV equation initiated a lot of mathematical investigations.

Recently, Camassa and Holm proposed a new model for the same phenomenon:

$$\begin{cases} u_t - u_{xxt} + 3uu_x = 2u_x u_{xx} + uu_{xxx}, \quad t > 0, & x \in \mathbf{R}, \\ u(0,x) = u_0(x), & x \in \mathbf{R}. \end{cases}$$

The variable $u(t,x)$ in the Camassa-Holm (CH) equation represents the fluid velocity at time t in the x direction in appropriate nondimensional units (or, equivalently, the height of the water's free surface above a flat bottom). Unlike KdV, which is derived by asymptotic expansions in the equation of motion, CH is obtained by using an asymptotic expansions directly in the Hamiltonian for Euler's equations in the shallow water regime. The novelty of the Camassa and Holm's work was the physical derivation of CH equation and the discovery that the equation has solitary waves (solitons) that retain their individuality under interaction and eventually emerge with their original shapes and speeds.

As an alternative model to KdV, Benjamin, Bona and Mahoney [3] proposed the so-called BBM-equation:

$$u_t + u_x + uu_x - u_{xxt} = 0, t > 0, x \in \mathbf{R}.$$

Numerical work of Bona, Pritchard and Scott shows that the solitary waves of the BBM-equation are not solitons.

As noted by Whitham [18], it is intriguing to find mathematical equations including the phenomena of breaking and peaking, as well as criteria for the occurrence of each. He observed that the solutions of the KdV-equation do not break as physical water waves do. Whitham suggested to replace the KdV-model by the nonlocal equation

$$u_t + uu_x + \mathbf{K} = 0, t > 0, x \in \mathbf{R},$$

for which he conjectured that breaking solutions exist. Here $\mathbf{K}$ is a Fourier operator with symbol $k(\xi) = \sqrt{(\tanh \xi)/\xi}$. Whitham's conjecture was proved in Naumkin, Shishmarev, Nonlinear Nonlocal Equations in the Theory of Waves, vol. 133, Transl. Math. Monographs, Providence, Rhode Island, 1994. The numerical calculations carried out for the Whitham equation do not support any strong claim that soliton interaction can be expected.

On the other hand, Camassa, Holm and Hyman [6] show that the solitary waves have a discontinuity in the first derivative at their peak and that soliton interactions occur in CH equation. The advantage of the new equation in comparison with the well-established models KdV, BBM and the Whitham equation is clear: The Camassa-Holm equation has peaked solitons, breaking waves, and permanent waves.

3. CNN modeling of tsunami waves. In this chapter we shall study a model of the motion of the water before arrival of a tsunami wave. We require that a flat free surface for the background state excludes linear vorticity functions, unless the flow is trivial. So, nonlinear vorticity distributions are introduced in order to admit nontrivial flows with a flat free surface. Consider the following dynamical system

$$\varphi_{tt} + \varphi_{xx} = -f(\varphi), \tag{3}$$

$$f(\varphi) = \begin{cases} \varphi - \varphi|\varphi|^{-1/2} & \text{if} \quad \varphi \neq 0 \\ 0 & \text{if} \quad \varphi = 0. \end{cases} \tag{4}$$

By applying Cellular Neural Networks (CNN) approach [17] we shall study the wave propagation of the model (3), (4).

CNN model of our system (3), (4) will be the following:

$$\begin{aligned} \frac{dv_j}{dt} &= A_1 * u_j + f(u_j), \\ \frac{du_j}{dt} &= v_j, \qquad\qquad 1 \leq j \leq N \end{aligned} \tag{5}$$

We shall study the dynamics of the above model (5) by applying the describing function method [17]. Applying the double Fourier transform:

$$F(s,z) = \sum_{k=-\infty}^{k=\infty} z^{-k} \int_{-\infty}^{\infty} f_k(t) exp(-st)dt,$$

to the CNN model (5) we obtain:

$$\begin{aligned} sV(s,z) &= (z^{-1}U(s,z) - 2U(s,z) + zU(s,z)) + F(U(s,z)) \\ sU(s,z) &= V(s,z). \end{aligned} \tag{6}$$

Without loss of generality we denote $F(U) = U(s,z) - U(s,z)|U(s,z)|^{-1/2}$. In the double Fourier transform we suppose that $s = i\omega_0$, and $z = \exp(i\Omega_0)$, where ω_0 is a temporal frequency, Ω_0 is a spatial frequency.

According to the describing function method [17], $H(s,z) = \dfrac{1}{s - (z^{-1} - 2z + z)}$ is the transform function, which can be presented in terms of ω_0 and Ω_0, i.e. $H(s,z) = H_{\Omega_0}(\omega_0)$.

We are looking for possible periodic state solutions of system (5) of the form:

$$X_{\Omega_0}(\omega_0) = X_{m_0} \sin(\omega_0 t + j\Omega_0), \tag{7}$$

where $X = (U, V)$. According to the describing function method we take the first harmonics, i.e. $j = 0 \Rightarrow$

$$X_{\Omega_0}(\omega_0) = X_{m_0} \sin \omega_0 t,$$

On the other side if we substitute $s = i\omega_0$ and $z = \exp(i\Omega_0)$ in the transfer function $H(s,z)$ we obtain:

$$H_{\Omega_0}(\omega_0) = \frac{1}{i\omega_0^2 - (2\cos\Omega_0 - 2)}. \tag{8}$$

According to (8) the following constraints hold:

$$\begin{aligned} \mathrm{Re}(H_{\Omega_0}(\omega_0)) &= \frac{U_{m_0}}{V_{m_0}} \\ \mathrm{Im}(H_{\Omega_0}(\omega_0)) &= 0. \end{aligned} \tag{9}$$

Since our CNN model (5) is a finite circular array of $L = N.N$ cells we have finite set of spatial frequencies:

$$\Omega_0 = \frac{2\pi k}{L}, 0 \le k \le L-1. \tag{10}$$

Thus, (8), (9) and (10) give us necessary set of equations for finding the unknowns U_{m_0}, V_{m_0}, ω_0, Ω_0. As we mentioned above, we are looking for a periodic wave solution of (5), therefore U_{m_0} and V_{m_0} will determine approximate amplitudes of the waves, and $T_0 = 2\pi/\omega_0$ will determine the wave speed.

Based on the above considerations the following proposition hold:

Proposition 1. *CNN model* (5) *of circular array of* L *identical cells has periodic state solution* $u_j(t)$, $v_j(t)$ *with a finite set of spatial frequencies* $\Omega_0 = 2\pi k/L$, $0 \le k \le L-1$.

After simulating our CNN model (5) we obtain the results on Figure 4.

4. Discussion on the wave dynamics. Let us conclude with a brief discussion of the wave dynamics as the tsunami propagates towards the coast. The previous considerations show that from initiation until reaching towards the costal region, a good approximation in non-dimensional variables of tsunami waves is provided by the solu-

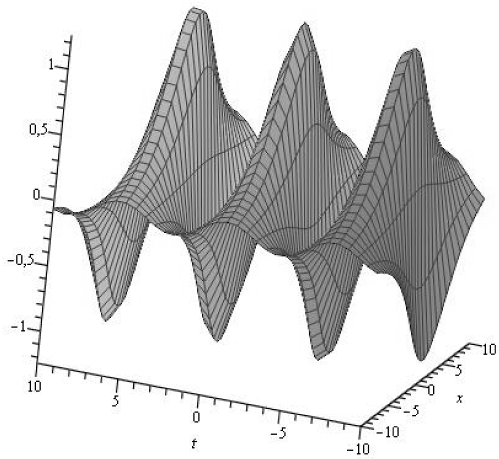

Fig. 4. Periodic wave solution of CNN model (5)

tions of the corresponding model equation. In the original physical variables this means that up until near-shore the wave profile remains unaltered propagating at constant speed $\sqrt{gh_0}$. The linear model breaks down when the tsunami waves enter the shallower water of the coastal regions and for understanding of the tsunamis close to the shore the appropriate equations are those modeling the propagation of long water waves over variable depth. Before the waves reach the breaking state, their front steepens and dispersion, no matter how weak, becomes relevant. In this region faster wave fronts can catch up slower ones(but they can never overtake them) as a manifestation of the "confluence of shocks" and can result in large amplitude wave fronts building up behind smaller ones.

Let us take for example the tsunami of 2004 in the Indian Ocean [12]. For modeling purposes, outside of the Bay of Bengal the two-dimensional character of the tsunami waves can not be taken any more for granted since diffraction around islands and reflection from steep shores alter this feature considerably (see Fig. 5). The earthquake that generated the tsunami changed the shape of the ocean floor by raising it by a few m to the west of the epicenter and lowering it to the east (over 100 km in the east-west direction and about 900 km in the north-south direction).

The initial shape of the wave pattern that developed into the tsunami wave featured therefore to the west of the epicenter a wave of elevation followed by a wave of depression (that is, with water levels higher, respectively lower than normal), while to the east of the epicenter the initial wave profile consisted of a depression followed by an elevation. The fact that as the tsunami waves reached the shore in either direction, the shape of the initial disturbance (first wave of elevation, then wave of depression, respectively vice-versa) was not altered is of utmost importance in validating a theory for the wave dynamics on this occasion. This observation suggests that perhaps the shape of the tsunami waves remained approximately constant as they propagated across the Bay of Bengal. These clearly show a leading wave of elevation, followed by a wave of depression, a feature common both to the initial wave profile west of the epicenter and to the tsunami as it

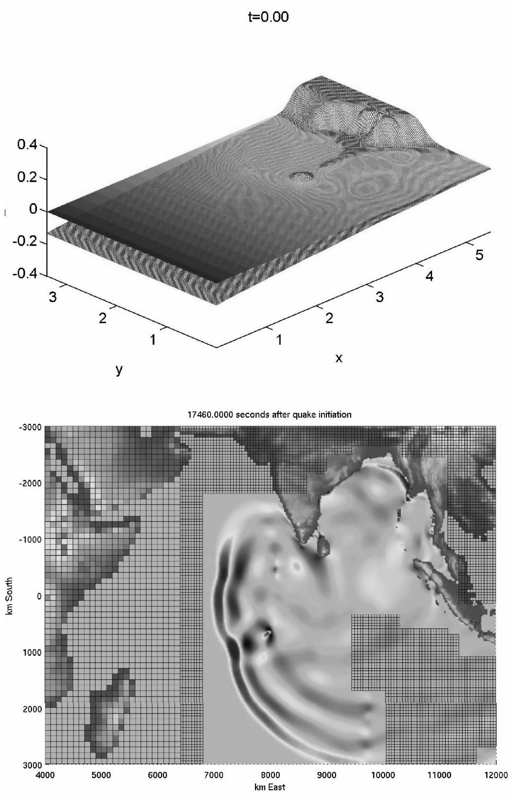

Fig. 5. 2004 tsunami in the Indian ocean

entered the coastal regions of India and Sri Lanka. These measurements also confirm another essential feature of tsunami waves: even though these waves reach large amplitudes due to the diminishing depth effect as they approach the shore (waves as high as 30m were observed near the city Banda Aceh on the west coast of the northern tip of Sumatra about 160 km away from the epicenter of the earthquake), tsunami waves are barely noticeable at sea due to their small amplitude. Indeed, the satellite data shows that the maximum amplitude of the waves, whether positive or negative with respect to the usual sea level, was less than 0.8 m over distances of more than 100 km.

REFERENCES

[1] B. Alvarez-Samaniego, D. Lannes. Large time exitence for 3D water-waves and asymptotics. *Invent.Math.*, **171** (2008), 485–541.

[2] S. Barrientos. Earthquakes – giant returns in time. *Nature*, **437** (2005), 329.

[3] T. B. Benjamin, J. I. Bona, J. J. Mahony. Model equations for long waves in nonlinear dispersive systems. *Philos. Trans. Roy. Soc. Lond. Ser. A*, **272** (1972), 47–78.

[4] S. C. Berkman, J. M. Symons. The tsunami of May 22, 1960, as recorded at tide stations. *US Coast and Geod. Survey*, 1960, 79.
[5] E. Bryant. Tsunami: The Underrated Hazard. Springer, Heidelberg, 2008.
[6] R. Camassa, D. Holm. An integrable shallow water equation with peaked solutions. *Phys. Rev. Letters*, **71** (1993), 1661–1664.
[7] M. Cisternas, B. F. Atwater et al. Predecessors of the giant 1960 Chile earthquake. *Nature*, **437** (2005), 404–407.
[8] W. Craig. Surface water waves and tsunamis. *J. Dynam. Diff. Eq.*, **18** (2006), 525-549.
[9] P. G. Drazin, R. S. Johnson. Solitons: An Introduction. Cambridge University Press, Cambridge, 1989.
[10] C. M. Duke. Chilean earthquakes of May 1960. *Science*, **132** (1960), 1797-1802.
[11] A. E. Green, P. M. Naghdi. A derivation of equations for wave propagation in water of variable deph. *J. Fluid Mech.*, **78** (1976), 237–246.
[12] J. Grue, E. N. Pelinovsky, D. Fructus, T. Talipova, C. Kharif. Formation of undular bores and solitary waves in the Strait of Malacca caused by the 26 December 2004 Indian Ocean tsunami. *J. Geophys. Res.*, doi:10.1029/2007JC004343.
[13] J. L. Hammack, H. Segur. The Korteweg-de Vries equation and water waves. *J. Fluid Mech.*, **84** (1978), 337–358.
[14] R. S. Johnson. A Modern Introduction to the Mathematical Theory of Water Waves. Cambridge University Press, Cambridge, 1997.
[15] C. Kharif, E. Pelinovsky. Asteroid impact tsunamis. *C. R. Phys.*, **6** (2005), 361–366.
[16] Y. Matsuno. Nonlinear evolutions of surface gravity waves on fluid of finite depth. *Phys. Rev. Lett.*, **69** (1992), 609–611.
[17] A. Slavova. Cellular Neural Networks: Dynamics and Modeling. Kluwer Academic Publishers, 2003.
[18] G. B. Whitham. Linear and Nonlinear Waves. J. Whiley and Sons, New York, 1980.

Angela Slavova
Institute of Mathematics and Informatics
Bulgarian Academy of Sciences
Acad. G. Bonchev Str., Bl. 8
1113 Sofia, Bulgaria
e-mail: slavova@math.bas.bg

МОДЕЛИРАНЕ НА ВЪЛНИ ЦУНАМИ С КЛЕТЪЧНО НЕВРОННИ МРЕЖИ

Анжела Славова

В този доклад се разглеждат няколко модела на вълни цунами. Представена е физическата интерпретация на такъв вид вълни. Изучен е математически модел на дълги вълни в нелинейност. За този модел се прилага подхода на Клетъчно Невронните Мрежи (КНМ). Изследва се динамиката на КНМ модел с метода на описващите функции. Представени са компютърни симулации и дискусия.