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NON-STATIONARY GAS FLOW BETWEEN ROTATING CYLINDERS WHEN THE ACTIVE CYLINDER STOPS SUDDENLY *

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The viscous drag (wall shear stress) and heat flux on both cylinder walls, as well as macroscopic flow characteristics (velocity, density and temperature), are numerically studied in non-stationary Couette gas flow between rotating coaxial cylinders. The results are obtained using Direct Simulation Monte Carlo (DSMC) method and numerical solution of a continual model based on the Navier-Stockes-Fourier (NSF) method. The cases of suddenly stopped active outer or inner cylinder are considered. These studies have been accomplished for fixed Knudsen numbers. The NSF results have been obtained by setting a local value of a Knudsen number in the corresponding first order slip boundary conditions.

1. Introduction. In microflow device design, the viscous drag and heat flux "solid surface – gas" is often of critical importance. However, this depends on the characteristics of the flow – the region of local non-equilibrium existing up to one or two molecular mean free paths away from the wall in any gas flow near a surface.

The Couette cylindrical flow is a fundamental problem in rarefied gas dynamics [5, 8, 10]. Thus, its modeling and numerical solving is of great importance for microfluidics, which serves for theoretical background of the analysis of new emerging Micro Electro Mechanical Systems (MEMS) [3, 9].

We consider both molecular and continuum models treating the gaseous flow by using a different approach of mathematical description. Both models take into account the specific microfluidic effects of gas rarefaction and slip-velocity regime at the solid boundaries [7, 13].

In a previous paper we studied flow macro-characteristics in stationary cases [7, 12]. In this paper we study numerically the viscous drag (wall shear stress) and heat flux on both cylinder walls, as well as flow macroscopic characteristics (velocity, density and temperature), in non-stationary Couette gas flow between rotating cylinders. The results are obtained using Direct Simulation Monte Carlo (DSMC) method and numerical solution of a continual model based on the Navier-Stockes-Fourier (NSF) equations. The cases of suddenly stopped outer or inner active cylinder areconsidered: These studies have been accomplished for fixed Knudsen numbers. The NSF results have been obtained by setting a local value of a Knudsen number in the corresponding first order slip boundary

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conditions. A wave appears in the gas phase between cylinders, when the active cylinder suddenly stops. This causes changes in the macroscopic characteristics profiles. The macro characteristics variation in the inner rotating cylinder case is smaller than these in the outer rotating cylinder case.

2. Formulation of the problem and methods of solution. We study a rarefied gas between two coaxial unconfined cylinders (one-dimensional, axis-symmetrical problem).

2.1. Continuous model (NSF) and numerical simulation. The continuous model is based on the Navier-Stokes-Fourier equations for compressible fluid, completed with the equations of continuity and energy transport. For details see [7, 11, 12].

A rather standard notation is used. u and v are the velocity components in r and φ directions, ρ is density and T is the temperature, P is the pressure. ρ, P, T, u, $v = f(r,t)$. The stress tensor components are $\tau_{i,j}$ and Φ is the dissipation function [6]. For a perfect monatomic gas, the viscosity and the heat transfer coefficient read as [1, 4]:

$$\mu = \mu(T) = C_\mu \rho_0 l_0 V_0 \sqrt{T}, \quad C_\mu = \frac{5}{16}\sqrt{\pi} \tag{2.1}$$

$$\lambda = \lambda(T) = C_\lambda \rho_0 l_0 V_0 \sqrt{T}, \quad C_\lambda = \frac{15}{32}\sqrt{\pi} \tag{2.2}$$

The governing equations are normalized by using the following scales: for density, $\rho_0 = mn_0$ (m is the molecular mass, n_0 the average number density), for velocity $V_0 = \sqrt{2RT_0} - R$ is the gas constant, for length the distance between the cylinders $L = R_2 - R_1$, for time $t_0 = L/V_0$, for temperature T_0, the wall temperature of both cylinders. The Knudsen number is $\mathrm{Kn} = l_0/L$, where the mean free path is l_0 and $\gamma = c_P/c_V = 5/3$ (c_P and c_V are the heat capacities at constant pressure and constant volume respectively). In this way in the dimensionless model the characteristic number Kn and the constants C_μ and C_λ take part.

For the problem formulated first-order slip boundary conditions are imposed at both walls, which can be written directly in dimensionless form as follows [7]:

$$v \mp A_\sigma \mathrm{Kn}_{\mathrm{local}} \left(\frac{\partial v}{\partial r} - \frac{v}{r} \right) = V_i \tag{2.3}$$

$$u = 0 \tag{2.4}$$

$$T \pm \zeta_T \mathrm{Kn}_{\mathrm{local}} \frac{\partial T}{\partial r} = 1 \tag{2.5}$$

at $r = R_i$, $i = 1, 2$. In Eqs (2.3)–(2.5) $V_i = v_i/V_0$ is the dimensionless wall velocity (v_i, $i = 1, 2$ is the dimensional wall velocity). The dimensionless temperature for both cylinder walls is equal to 1. For diffuse scattering we have used the viscous slip and temperature jump coefficients $A_\sigma = 1.1466$ and $\zeta_T = 2.1904$ calculated, respectively in [2], from the kinetic BGK equation. The boundary conditions are modeled by using the local Knudsen number $\mathrm{Kn}_{\mathrm{local}}$.

$$\mathrm{Kn}_{\mathrm{local}} = \frac{l}{L} = \left(L\sqrt{2}\pi\sigma^2 \frac{\bar{\rho}}{\rho_0}.n_0 \right)^{-1} = \frac{\mathrm{Kn}}{\rho} \tag{2.6}$$

In (2.6) l denotes the local mean free path, σ the molecular diameter and $\bar{\rho}$ the dimensional density.

The dimensionless wall stress tensor component $\bar{\tau}_{r\varphi}$ can be expressed through the dimensionless quantities v, T [6, 11, 12]:

$$\bar{\tau}_{r\varphi} = -C_\mu \,\mathrm{Kn}_{\mathrm{local}} \sqrt{T}\left[r\frac{\partial}{\partial r}\left(\frac{v}{r}\right)\right]. \tag{2.7}$$

The dimensionless heat flux q is [6, 11]

$$q = -C_\lambda \,\mathrm{Kn}_{\mathrm{local}} \sqrt{T}\frac{\partial T}{\partial r}. \tag{2.8}$$

2.2. Direct simulation Monte Carlo (DSMC) method. The gas considered is simulated as a stochastic system of N particles [5]. All quantities used are non-dimensional, so that the mean free path at equilibrium is equal to 1. The basic steps of simulation are as follows:

A. The time interval $\left[0;\hat{t}\right]$ over which the solution is found is subdivided into subintervals with step Δt.

B. The space domain is subdivided into cells with sides Δz, Δr.

C. Gas molecules are simulated in gap G using a stochastic system of N points (particles) having positions $z_i(t), r_i(t)$ and velocities $\left(\xi_z^i(t), \xi_r^i(t), \xi_\varphi^i(t)\right)$,

D. N_m particles are located in the mth cell at any given time. This number varies during the computer simulation by the following two stages:

Stage 1. Binary collisions in each cell are calculated, whereas particles do not move. Collision modeling is implemented using Bird's scheme "no time counter".

Stage 2. Particles move with new initial velocities acquired after collisions, and no external forces act on particles. No collisions are accounted for at this stage.

E. Stage 1 and Stage 2 are repeated until $t = \hat{t}$.

F. Flow macro-characteristics (density, velocity, temperature) are calculated as time-averaged. The shear-stress calculated by $\tau_{r\varphi} = \rho\left(\int_S \xi_\varphi \xi_r ds - \overline{\xi_\varphi}.\overline{\xi_r}\right)$ and the heat flux calculated by

$$q_r = \frac{1}{2}\rho\left(\int_S \xi^2\xi_r ds - 2\left(\int_S \xi_x\xi_r ds - \overline{\xi_x}.\overline{\xi_r}\right)\overline{\xi_x} - 2\left(\int_S \xi_r^2 ds - \overline{\xi_r^2}\right)\overline{\xi_r} \right.$$
$$\left. -2\left(\int_S \xi_\varphi\xi_r ds - \overline{\xi_\varphi}.\overline{\xi_r}\right)\overline{\xi_\varphi} - \overline{\xi^2}.\overline{\xi_r}\right)$$

where S is the cylinder wall, $\overline{\xi_x}$, $\overline{\xi_r}, \overline{\xi_\varphi}$ are the average velocity components and $\xi^2 = \xi_x^2 + \xi_r^2 + \xi_\varphi^2$ [4].

G. Boundary conditions are diffusive at the cylinder walls and periodic along cylinder axes. The number of particles (simulators) used in DSMC calculations is 12800000.

3. Numerical results. The aim of the studies is to determine the influence of the Knudsen number and the cylindrical wall velocity on the drag and heat flux between gas and cylinder wall in the non-stationary case. In a previous research [6, 12] in the stationary case it has been found that only for Kn = 0.02 and lower both solutions were in an excellent agreement. The case of a suddenly stopped active outer or inner cylinder is considered:

case (A) $V_1 = 0$ and $V_2 = -0.5$

case (B) $V_1 = 0.5$ and $V_2 = 0$,

the wall temperatures are $T_1 = T_2 = 1$ at Kn = 0.02, 0.05, 0.08. The studied flow starts from a suddenly started active cylinder (outer – case (A) or inner – case (B)), the flow reached a steady state and then the active cylinders stopped suddenly.

The sudden start process: The drag variation on the stationary cylinder wall graph is visible section of the gas phase acceleration – see Figure 1, 2, 3. The sudden stop process: The active cylinder wall velocity becomes zero because it stops suddenly. The gas phase velocity on the wall greatly decreased so that the adjacent layers' velocity of the phase is higher in comparison with the layer to the wall. The wave appears from the active cylinder wall to the stationary cylinder wall and it reflects repeatedly from the two walls, most clearly visible for Kn = 0.02 – see Figure 4. This wave causes transient changes (disturbances) in the temperature profile and the relatively larger changes in the density profile in case (A) Figure 1 and this through the local Knudsen number affects the drag value on the inner cylinder wall. This effect is not observed in case (B) – suddenly stopped inner cylinder (Fig. 2). Let consider the behavior of the layer with thickness Δ, where $\Delta \ll (R_2 - R_1)$. The mass of the layer with thickness Δ adjacent to wall of the outer cylinder is $\dfrac{\rho_2.R_2\Delta}{\rho_1.R_1\Delta} = \dfrac{\rho_2.R_2}{\rho_1.R_1}$ times greater than the mass of the layer with the same thickness adjacent to inner cylinder. For this reason when the inner cylinder stops (case(B)) a smaller mass quantity is moving towards the stationary cylinder, then in the

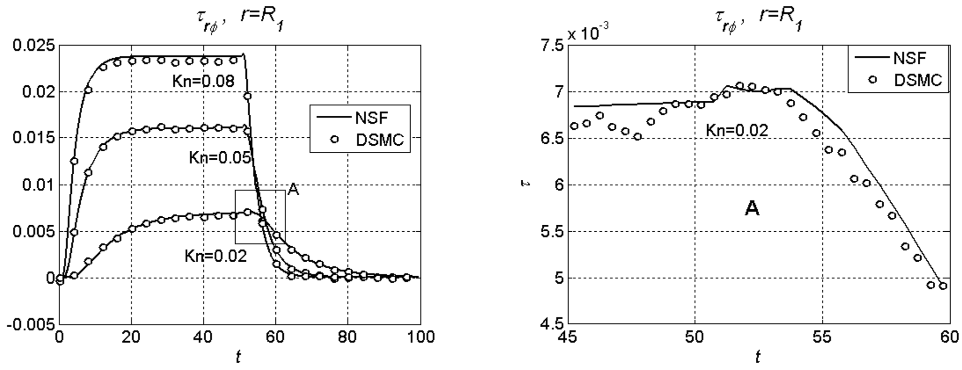

Fig. 1. The drag variation over the stationary cylinder in case (A)

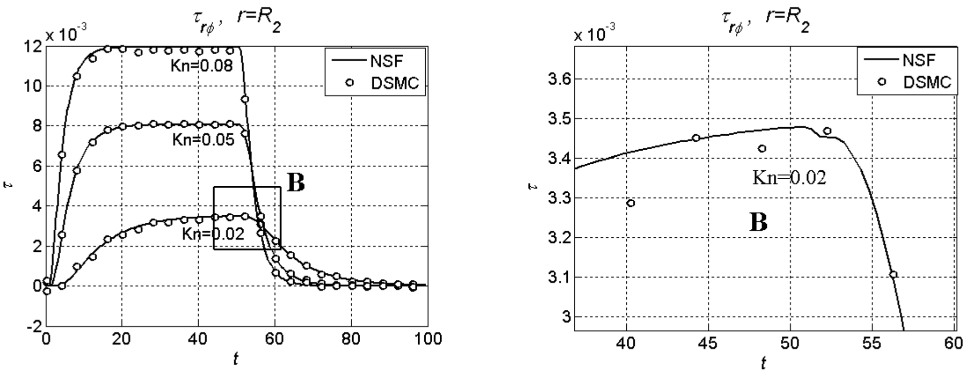

Fig. 2. The drag variation over the stationary cylinder case (B)

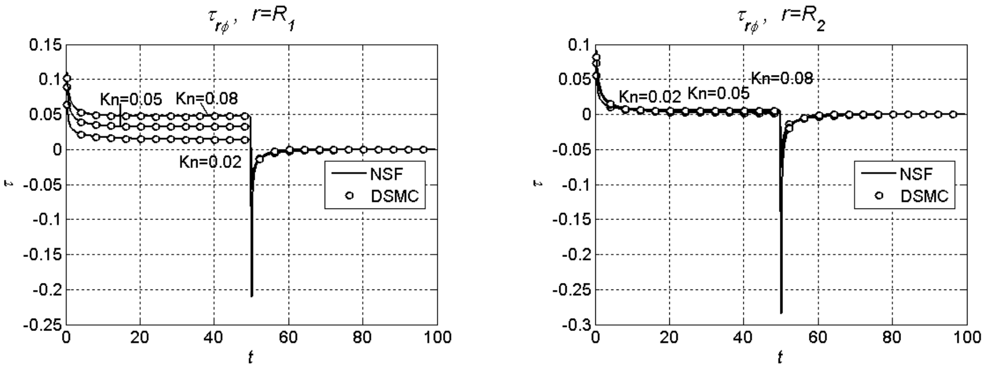

Fig. 3. The drag variation over the active cylinder, left-hand figure case (A), right-hand figure case (B)

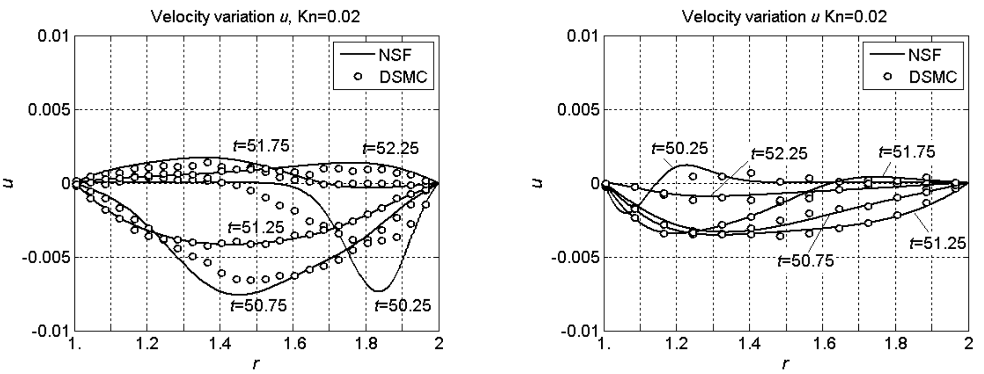

Fig. 4. The radial velocity profile, left-hand figure case (A), right-hand figure case (B)

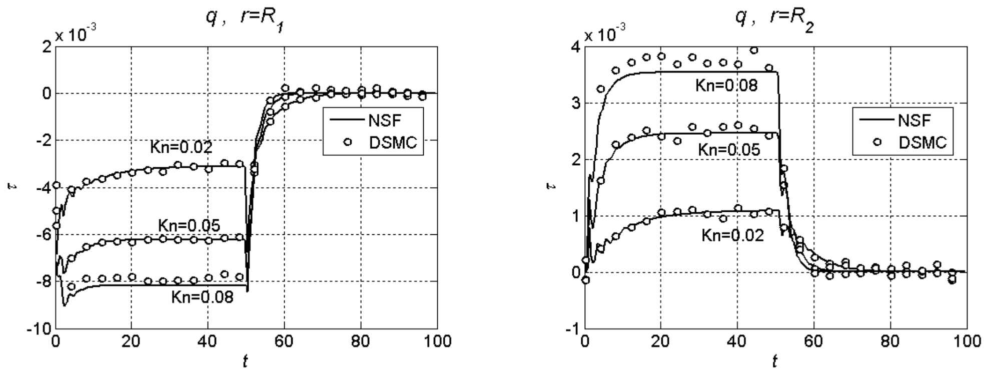

Fig. 5. The heat flux variation on the stationary cylinder, left-hand figure case (A), right-hand figure case (B)

opposite case (A). For this reason, the variation of the macro characteristics in case (B) is smaller than in case (A) and does not reflected visibly on the drag variation on the wall.

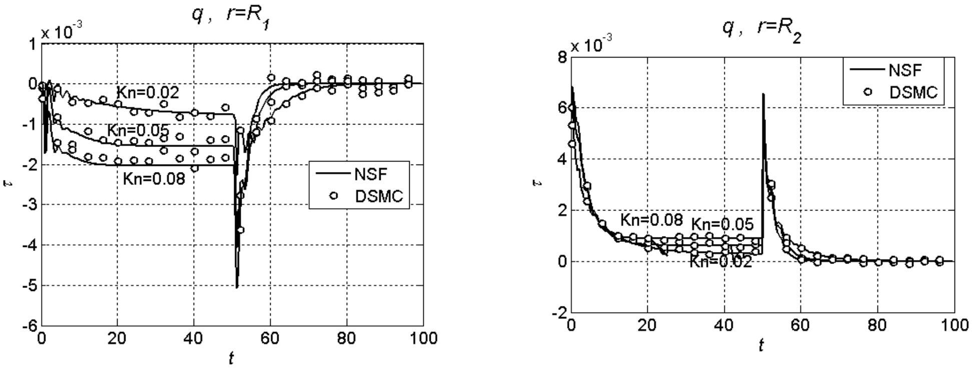

Fig. 6. The heat flux variation over the active cylinder, left-hand figure case (A), right-hand figure case (B)

The heat flux variation at both cylinder walls shows the suddenly stopped cylinder effect. The wave influence is not clearly visible in this case – Figure 5, 6.

4. Conclusion. The NSF model and the numerical solution found and the DSMC method enable one to investigate a cylindrical non-stationary Couette flow. After the sudden start of active cylinder, the flow reached a steady state and the active cylinder suddenly stops. The wave appears from the active cylinder wall to the stationary cylinder wall. It is reflected repeatedly by the two cylinder walls. The variation of the macro characteristics in the case of inner rotating cylinder is smaller than in case of outer rotating cylinder.

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НЕСТАЦИОНАРНО ГАЗОВО ТЕЧЕНИЕ МЕЖДУ ВЪРТЯЩИ СЕ ЦИЛИНДРИ ПРИ ВНЕЗАПНО СПИРАНЕ НА АКТИВНИЯ ЦИЛИНДЪР

Добри Данков, Петър Господинов, Владимир Русинов

Числено е изследвано вискозното триене и топлинните потоци върху стените, както и макроскопичните характеристики (скорост, плътност и температура) в нестационарно цилиндрично газово течение на Кует. Числените резултати са получени посредством решението на континуален модел, с използване на уравненията на Навие–Стокс–Фурие (NSF) и прякото статистическо моделиране (DSMC). Разгледани са случаите на внезапно спиране на вътрешния или на външния въртящ се цилиндър. Числените резултати са получени за няколко характерни стойности на числото на Кнудсен. В граничните условия на плъзгане, от първи ред в континуалния модел, е използвано локално число на Кнудсен.