INTERACTIVE DYNAMICAL VISUALIZATION IN THE SUPPORT OF VISUAL THINKING

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The present article considers the role of interactive dynamic visualization in the solution of geometrical problems and in the study of geometry in general. Dynamic visualization enhances the visual thinking of students supporting its basic operations: ordering of the semantic meanings of visual images; variation of the structure of visual images; recoding of visual images; assessment and reassessment of the importance of visual image properties; reconstruction of visual images; selection of visual images. Examples of learning situations are presented, which demonstrate the advantages of interactive dynamical visualization compared to static one.

The term of "visual thinking" was introduced for scientific use by the American psychologist Rudolf Arnheim in 1969 [1]. He defined it as non-verbal thinking directed to processing of information that has been obtained through visual channels of perception. Essentially, Arnheim enlarged the scientific understanding of the role of images in knowledge. Among the most valuable visual thinking operations in learning process he included the following ones: ordering of the values of images; varying of the structures of images; recoding of images; assessing and reassessing of the importance of image properties; developing of the sensitively non-perceived parts of images under memorized information; selecting of visual images to help verbal thinking [2].

The research works of many scientists (V. Voytsehovich, V. Krutetsky, L. Fridman, etc.) proved that visual thinking is a significant component of the mathematical style of scientific thinking. According to S. Grozdev [3] it turns out to be of high importance for successful performance of students in mathematical competitions and contests improving essentially their skills in problem solving of geometric tasks mainly. The interactive dynamical visualization is an effective tool to support visual thinking. It could be created by means of dynamical geometry systems (DGS): Cabri Geometer, the Geometer's Sketchpad, GeoGebra, Crocodile, Cinderella, GeoNext, . . .). The main advantages of such a visualization over the static one are connected with the possibilities to integrate particular geometric images, which are separate representatives of the object of study enabling to allocate independent phases of the drawing state during its dynamic changes. Such features of the interactive dynamic images could be used at different stages of learning activities [4], [5]:
- at the stage of defining the object of geometric knowledge (as a search and optimization procedure in the selection of the property set in the algorithmic foundation for the construction of the concept dynamical model);

- at the stage of discovering the content and the volume of the new geometrical concept (as a defining procedure for invariant properties of changes allowed by the dynamic model and the determining one in connection with the boundaries and the conditions for the variation of changeable properties);
- at the stage of finding ideas for deductive grounding of the established properties (as a presentation procedure for hidden relations which are valuable for the explanation of reasons causing the existence of regularities);
- at the stage of generalizing or presenting particular cases of already solved problems or proved theorems (as a modification procedure for keeping or changing of the proved regularity).

We give examples of learning situations in which dynamic interactive visualizations are most important for visual thinking.
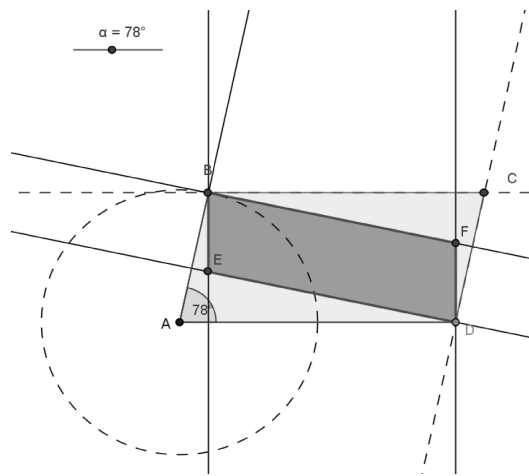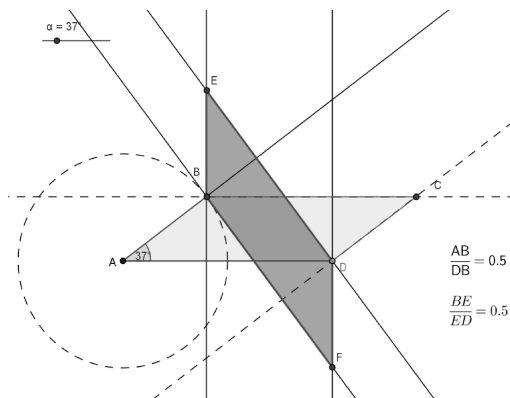
Situation 1. Overcoming stereotypes in the creation of images. Suppose that we have to solve the following problem: "Given is a parallelogram $ABCD$ with $AB = 1$, $BC = 2$ and obtuse angle ABC. Consider the lines through B and D, which are perpendicular to the side AB, and also the lines through B and D, which are perpendicular to the side BC. The intersection points of the four lines form a new parallelogram, which is homothetic to the initial one. Find the area of the new parallelogram."

If students will use a static schematic drawing, most likely they will propose the image in Figure 1. Solving the problem with the support of this image they will come to the equation: $\dfrac{1}{2} = \dfrac{1 - 2\cos\alpha}{2 - \cos\alpha}$, where $\alpha = \sphericalangle BAD$. The answer will be $\alpha = 90°$, which will contradict to the problem condition for the angle ABC to be obtuse. An interactive dynamic model allows overcoming the stereotype of visual thinking in this and similar situations. It gives possibilities for an accurate construction and variation of irrelevant data (Figure 2).
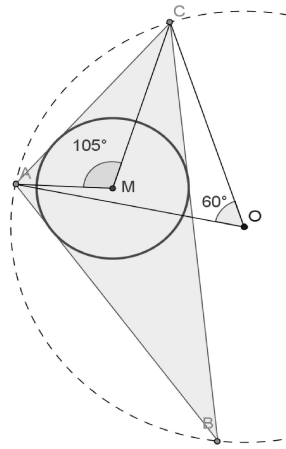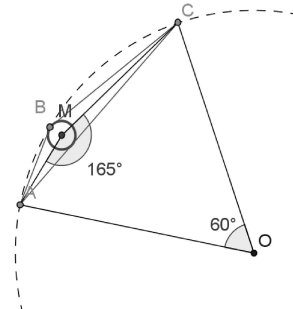

Fig. 1

Fig. 2

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Fig. 3

Fig. 4

Situation 2. Detecting ambiguity of images. Consider the following problem: "Given is a triangle ABC with circumcenter O and incenter M. Find $\sphericalangle AMC$ if $\sphericalangle AOC = 60°$."

The construction of a dynamic drawing starts with a circle with center O. Then, take points A and C on the circle such that $\sphericalangle AOC = 60°$. Let B be an arbitrary point on the circle different from A and C. Using ruler and compass rules construct the incircle of the triangle ABC and denote its center with M. The measurement of $\sphericalangle AMC$ is carried out on the screen (Figure 3). Experiments with the dynamic model show that $\sphericalangle AMC$ displays correctly the assertion of the problem if and only if M and O are located in different semi-planes with respect to the line AC. If this condition is not satisfied, then the answer is the angle, which is complimentary to the expected one (Figure 4).

Situation 3. Finding of hidden dependencies. Suppose that we want to find a way to construct a circle by ruler and compass through two given points A and B and tangent to a given circle (c). It is quite easy to solve the problem by computer experiments. It is enough to construct a dynamic model of a circle, determined by three points: A, B and an arbitrary point D on (c). Further, we look for a position of D, when the two circles have only one common point. However, this construction does not help much to find a conventional one, i.e. using the ruler

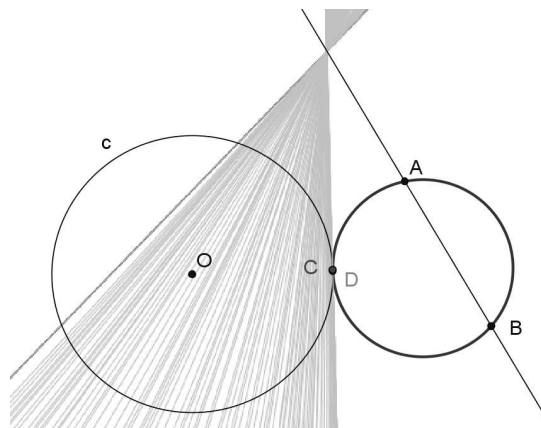
Fig. 5

and the compass. But still, take the lines AB and DC, where D and C are the intersection points of the two circles. Apply the function "leave a trail" (Figure 5) to visualize the line DC when displacing D on the circle moving it to C. The experimental result tells us that the intersection point of the lines AB and DC is invariant under these transformations and we use it as a base of the construction.
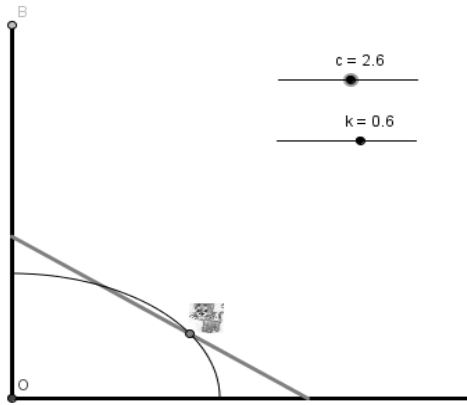
Situation 4. Help for imagination. Sometimes we face tasks for which it is difficult to construct an image of a geometric figure using the given properties. Usually, a large number of such problems concern locus problems. In such situations it is useful to apply the function "leave a trail" or the instrument "locus". Consider the following problem: "A ladder is standing against a wall and a kitten is sitting on one of its stairs. The ladder starts to slide down. What will be the trajectory of the kitten? How does it depend on the kitten's position on the ladder?"

Fig. 6

Let us construct a dynamic drawing of the problem. For the purpose we introduce two parameters: c – the height of the upper end of the ladder measured from the ground; k – the length of the kitten's trajectory. We apply the instrument "locus" (Figure 6), which visualizes the trajectory on the screen. Changing k, we conclude that the kitten is moving along an elliptical curve (a quarter of an ellipse). Note, that the value of the parameter k is determined by the compression ratio of the ellipse.

Situation 5. Integration and specialization of images. Suppose that we want to generalize or specialize the well-known Ceva theorem: "The segments, which connect the vertices of a triangle with the opposite sides (i.e. the cevians), have a common point if and only if the product of the quotients of the corresponding segments on the opposite sides is equal to 1." We construct a dynamic model illustrating the necessary condition for uniqueness of the intersection point.

A specification of Ceva theorem may be related to the gravity center of the triangle as a special case. Denote the midpoints of the sides of the triangle by A_2, B_2 and C_2 (Figure 7).

The experiment includes the case when the points B_1 and A_1 coincide with the points B_2 and A_2 respectively. The result is that the points C_1 and C_2 coincide too. For possible generalizations move the points A_1, B_1 and C_1 along the straight lines BC, AC and AB respectively (Figure 8).
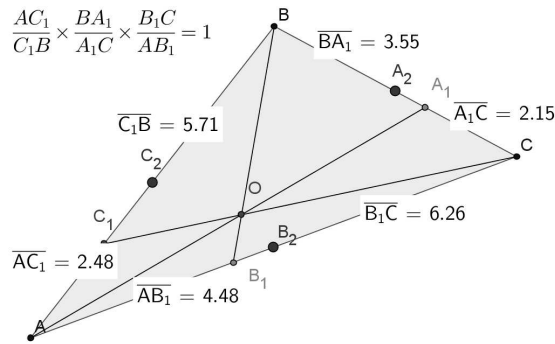
$$\frac{AC_1}{C_1B} \times \frac{BA_1}{A_1C} \times \frac{B_1C}{AB_1} = 1$$

$\overline{BA_1} = 3.55$

$\overline{A_1C} = 2.15$

$\overline{C_1B} = 5.71$

$\overline{B_1C} = 6.26$

$\overline{AC_1} = 2.48$

$\overline{AB_1} = 4.48$

Fig. 7

$$\frac{AC_1}{C_1B} \times \frac{BA_1}{A_1C} \times \frac{CB_1}{B_1A} = 1$$

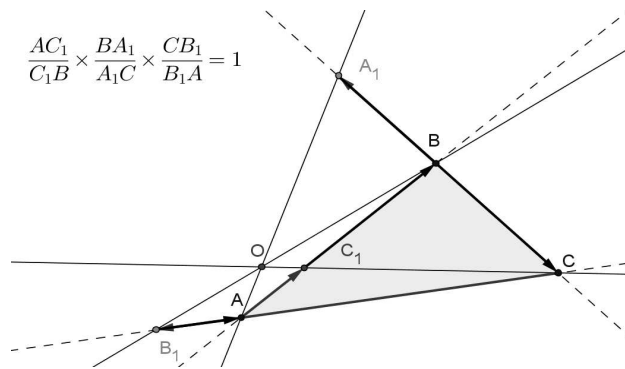Fig. 8

REFERENCES

[1] R. ARNHEIM. Visual thinking. Berkley, Univ. of California Press, 1969, 345 pp.

[2] R. ARNHEIM. New essays on the psychology of art. Moscow, Prometheus, 1998, 352 pp.

[3] S. GROSDEV. For High Achievements in Mathematics. The Bulgarian Experience (Theory and Practice). Sofia, ADE, 2007, 291 pp.

[4] I. S. KHRAPOVITSKY. Heuristic range for geometry. *Computer Tools in Education*, **1** (2003), 15–26.

[5] V. I. RYZHIK. Geometry and computer. *Computer Tools in Education*, **6** (2000), 7–11.

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ИНТЕРАКТИВНАТА ДИНАМИЧНА НАГЛЕДНОСТ В ПОДКРЕПА НА ВИЗУАЛНОТО МИСЛЕНЕ

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Настоящата статия е посветена на ролята на интерактивната динамична визуали-зация при решаването на геометрични задачи и при изучаването на геометрията изобщо. Динамичната нагледност повишава визуалното мислене на учениците, като подкрепя основните му операции: подреждане на семантичните значения на визуалните образи; вариране на структурата на визуалните образи; регистриране на визуални образи; оценяване и преоценяване на значимостта на свойствата на визуалните образи; реконструкция на визуални образи; подбор на визуални об-рази. Представени са примери на учебни ситуации, които показват предимствата на интерактивната динамична визуализация в сравнение със статичната.

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