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THE CURVATURE EFFECT ON THERMOACOUSTIC WAVES IN A COUETTE RAREFIED GAS FLOW*

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The thermoacoustic waves arising in cylindrical or planar Couette rarefied gas flow between rotating cylinders or moving plates is studied in the cases of suddenly wall velocity (cylinder or plate) direction turn on the active walls. Based on the developed in previous publications Navier-Stockes-Fourier (NSF) model and Direct Simulation Monte Carlo (DSMC) method and their numerical solutions, are considered non-stationary transients process in the gas phase. Macroscopic flow characteristics (velocity, density, temperature) are received. The cylindrical and planar flow cases for fixed velocity and temperature of the both walls are considered. The curvature effects over the wave's distribution and attenuation are studied numerically.

1. Introduction. In microflow device design, the thermoacoustic waves are often of critical importance. However, this depends on the characteristics of the flow – the region of local non-equilibrium existing up to one or two molecular mean free paths from the wall in any gas flow and the boundary conditions changes – wall velocity and temperature.

The Couette flow is a fundamental problem in the rarefied gas dynamics [3, 5, 6, 9, 10, 11, 12]. Thus, its modeling and numerical solving is of a great importance for microfluidics, which serves for theoretical background of the analysis of new emerging Micro Electro Mechanical Systems MEMS.

Heat flux through a rarefied gas confined between two coaxial cylinders is calculated on the basis of the kinetic equation [16]. Acoustic waves propagating through a rarefied gas between two plates induced by both oscillation and unsteady heating of one of them are considered [17] on the basis of a model of the linearized Boltzmann equation. The gas flow is considered as fully established so that the dependence of all quantities on time is harmonic.

The design of adequate mathematical models of gaseous flows in micro devices is one of the most important tasks of the studies. We consider both molecular and continuum models treating the gaseous flow by using different approach of mathematical description. Both models take into account the specific microfluidic effects of gas rarefaction and slip-velocity regime at the solid boundaries [13, 14, 15].

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In present paper we consider the thermoacoustic waves arising on the cases of suddenly walls velocity direction (cylinder or plate) turn on the active walls and the curvature influence on these waves. The arising wave starts from the active wall it reaches the other wall and it subsequently repeatedly reflected by the two walls.

The results can be used to resolve the non-stationary wave processes in MEMS.

2. Formulation of the problem and methods of solution. We study a rarefied gas between two coaxial unconfined cylinders or plates (one dimensional problem).

Continuous model (NSF) and numerical simulation. The continuous model is based on the Navier-Stokes-Fourier equations for compressible fluid, completed with the equations of continuity and energy transport. For details see [8, 12].

A rather standard notation is used. u and v are the velocity components in r and φ directions, ρ is density and T is the temperature, P is the pressure. $\rho,\ P,\ T,\ u, v = f(r,t)$. The stress tensor components are $\tau_{i,j}$ and Φ is the dissipation function [4]. For a perfect monatomic gas, the viscosity and the heat transfer coefficient read as [1, 3]:

$$\mu = \mu(T) = C_\mu \rho_0 l_0 V_0 \sqrt{T}, \quad C_\mu = \frac{5}{16}\sqrt{\pi} \tag{2.1}$$

$$\lambda = \lambda(T) = C_\lambda \rho_0 l_0 V_0 \sqrt{T}, \quad C_\lambda = \frac{15}{32}\sqrt{\pi} \tag{2.2}$$

The governing equations are normalized by using the following scales: for density, $\rho_0 = mn_0$ (m is the molecular mass, n_0 – the average number density), for velocity $V_0 = \sqrt{2RT_0}$ – R is the gas constant, for length – the distance between the cylinders $L = R_2 - R_1$, for time $t_0 = L/V_0$, for temperature T_0 – the wall temperature of both cylinders. The Knudsen number is $\mathrm{Kn} = l_0/L$, where the mean free path is l_o and $\gamma = c_P/c_V = 5/3$ (c_P and c_V. are the heat capacities at constant pressure and constant volume respectively). In this way in the dimensionless model the characteristic number Kn and the constants C_μ and C_λ take part.

For the problem formulated first-order slip boundary conditions are imposed at both walls, which can be written directly in dimensionless form as follows [7]:

$$v \mp A_\sigma \mathrm{Kn}_{local}\left(\frac{\partial v}{\partial r} - \frac{v}{r}\right) = V_i \tag{2.3}$$

$$u = 0 \tag{2.4}$$

$$T \pm \zeta_{\mathrm{T}} \mathrm{Kn}_{local}\frac{\partial T}{\partial r} = 1 \tag{2.5}$$

at $r = R_i$, $i = 1, 2$. In Eqs. (2.3)–(2.5) $V_i = v_i/V_0$ is the dimensionless wall velocity, (v_i, $i = 1, 2$ is the dimensional wall velocity). The dimensionless temperature T_i, $i = 1, 2$ for both cylinder walls is equal to 1. For diffuse scattering we have used the viscous slip and temperature jump coefficients $A_\sigma = 1.1466$ and $\zeta_T = 2.1904$ calculated, respectively in [2], from the kinetic BGK equation. The boundary conditions are modeled by using the local Knudsen number Kn_{local}.

$$\mathrm{Kn}_{local} = \frac{l}{L} = \left(L\sqrt{2}\pi\sigma^2 \frac{\bar{\rho}}{\rho_0}.n_0\right)^{-1} = \frac{\mathrm{Kn}}{\rho}. \tag{2.6}$$

In (2.6) with l is denoted the local mean free path, σ – molecular diameter and $\bar{\rho}$ is the dimensional density.

Direct simulation Monte Carlo (DSMC) method. The gas considered is simulated as a stochastic system of N particles [6, 7]. All quantities used are non-dimensional, so that the mean free path at equilibrium is equal to 1. The basic steps of simulation are as follows:

A. The time interval $\left[0;\hat{t}\right]$ over which the solution is found, is subdivided into subintervals with step Δt.

B. The space domain is subdivided into cells with sides $\Delta z, \Delta r$.

C. Gas molecules are simulated in gap G using a stochastic system of N points (particles) having position $z_i\left(t\right), r_i\left(t\right)$ and velocities $\left(\xi_z^i\left(t\right), \xi_r^i\left(t\right), \xi_\varphi^i\left(t\right)\right)$.

D. N_i particles are located in the i-th cell at any given time. This number varies during the computer simulation by the following two stages:

Stage 1. Binary collisions in each cell are calculated, whereas particles do not move. Collision modeling is realized using Bird's scheme "no time counter".

Stage 2. Particles move with new initial velocities acquired after collisions, and no external forces act on particles. No collisions are accounted for at this stage.

E. Stage 1 and Stage 2 are repeated until $t = \hat{t}$.

F. Flow macro-characteristics (density, velocity, temperature) are calculated as time-averaged.

G. Boundary conditions are diffusive at the cylinder walls and periodic along axis Oz. The number of particles (simulators) used in DSMC calculations is 12800000.

3. Numerical results. In present paper we consider the thermoacoustic waves arising on the cases of suddenly wall velocity direction (cylinder or plate) turn on the both active walls. The arising wave starts from the active cylinder wall it reaches the other cylinder wall and subsequently repeatedly reflected by the two walls. The value of $\mathrm{Kn} = 0.02$ is used in the numerical calculations. And here, our studies are the cases at: $R_1 = 1$, 5, 9, 9999 respectively $R_2 = 2$, 6, 10, 10000. The case $R_1 = 9999$, $R_2 = 10000$ is interpreted as a planar case. The cylinder velocities are $V_1 = 0.5$, $V_2 = -0.5$ and after impact for t=50, the velocities suddenly change to $V_1 = -0.5$, $V_2 = 0.5$. The wall temperatures are $T_1 = T_2 = 1$. The density (ρ), temperature (T) and tangential velocity (v) profiles at time $t = 47.75$, 50.25, 55.25 and 62.75 for all studied cases are shown on the Figs 1–3. The difference between DSMC and NSF results for $t = 51.25$ is due to the different hypothesis on which are based two methods. No such difference was observed for $t > 52$. We have studied in our previous paper the cases of different velocities of rotation of the cylinder [12, 13, 14]. It was found in these studies that in the case $V_1 = -0.5$, $V_2 = 0.5$ the wave has a pronounced character. The numerical results obtained by NSF are presented with a solid line and the corresponding results of these DSMC are representing with symbols ($\circ$, $\times$, $\Box$, ∇) in all figures. The effect of curvature on the density, temperature and tangential velocity distribution at the steady-state regime is more noticeable near to the outer cylinder wall Figs 1–3 ($t = 47.75$). The macro-characteristics has similar character at the suddenly cylinders velocity direction turn on, such as the quantitative differences can be explained by the varying curvature of the cylinders. There is a tendency for the curves representing the ρ, T and v to coincide at the certain time value Fig. 2 ($t = 55.25$). The process for tangential velocity develops late in time Fig. 3 ($t = 62.75$). With increasing the time after the wall velocity direction turn on macro characteristics go back to those in the steady-state case for $t < 50$. The proposed models and numerical

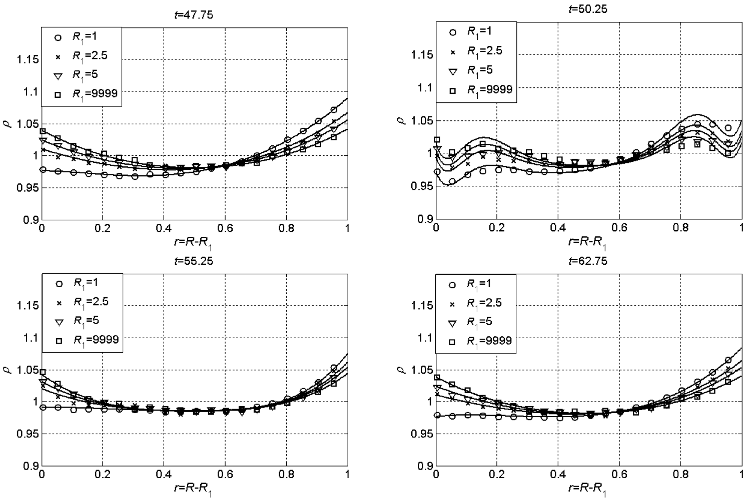

Fig. 1. The density profile

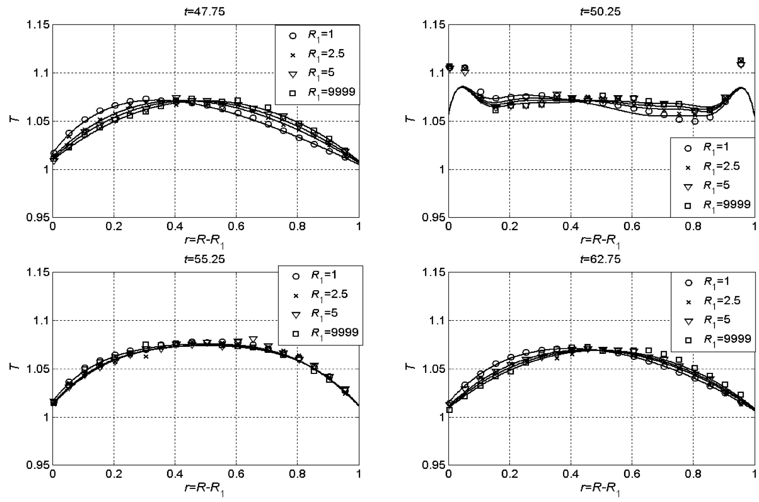

Fig. 2. The temperature profile

solutions allow studying transition processes for rarefied gas between cylinders when the kinematic boundary conditions are varied.

4. Conclusions. In this paper are studied the macro-characteristics varying at different values of the walls curvature at the steady-state regime and the wall velocity direction suddenly turn on. The thermoacoustic waves propagation in a cylindrical or a planar rarefied gas Couette flow is investigated on the basis of NSF and DSMC models. The proposed solutions allow to estimate the application limits of the various models NSF, DSMC, and to study the transition processes in some MEMS taking into account the curvature effect of the two walls restricting the gas flow.

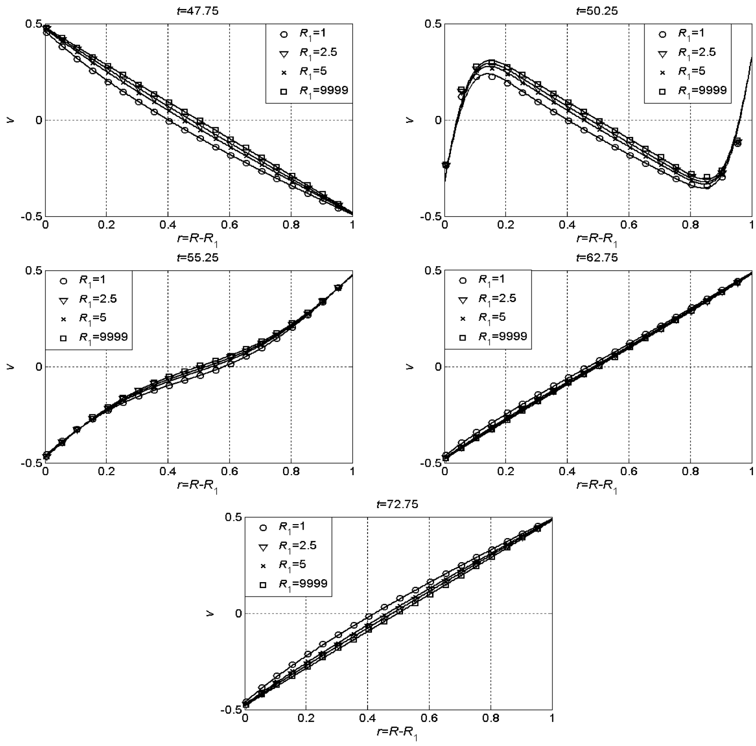

Fig. 3.The tangential v-velocity profile

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ИЗСЛЕДВАНЕ ЕФЕКТА НА КРИВИНАТА ВЪРХУ ТЕРМОАКУСТИЧНИ ВЪЛНИ В ТЕЧЕНИЕ НА РАЗРЕДЕН ГАЗ НА КУЕТ

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Изследвано е възникването на термоакустични вълни в цилиндрично или равнинно течение на Кует на разреден газ в случая на внезапна промяна на посоката на скоростта на движение на стената. На базата на разработените в предишни публикации модели на Навие-Стокс-Фурие и Direct Simulation Monte Carlo (DSMC) метод и техните числени решения е изследван нестационарен преходен процес в газовата среда. Получени са макрохарактеристиките на течението (скорост, плътност, температура). Разгледани са случаите на цилиндрично и равнинно течение за фиксирани скорости и температури на стените. Числено е изследвано влиянието на кривината върху възникването и затихването на вълната.